$K_L^0 \to \pi^+\pi^-e^+e^-$ in χPT

J. Elwood, J. Walden, M. Wise and MJS

1. Leading order in χPT
2. Sub-leading order in χPT

(i) Diagrams

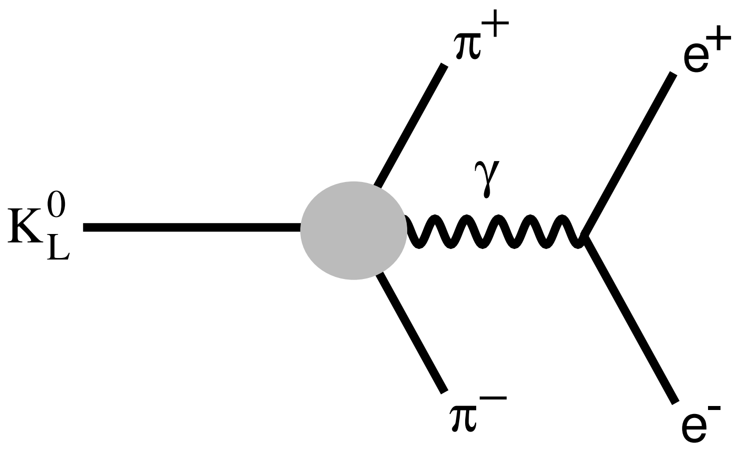
(ii) Counterterms

(iii) Final State Interactions

3. Differential decay rate $\frac{d\Gamma}{dq^2}$
4. CP-violating asymmetry, $B_{\rm CP}$

KAON 1999, U. of. Chicago. June 1999.

$$K_L^0 \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$$



$$\begin{aligned} p_{+,-} &= \pi^{+,-} \text{ four momentum} \\ q &= e^+e^- \text{ four momentum} \end{aligned}$$

$$\begin{aligned} \mathcal{M} &= H^\mu L_\mu \\ H^\mu &= \frac{s_1 G_F \alpha}{4\pi f_\pi q^2}\left[iG\varepsilon^{\mu\lambda\rho\sigma}p_{+\lambda}p_{-\rho}q_\sigma + F_+p_+^\mu + F_-p_-^\mu\right] \\ L_\mu &= \overline{u}(k_-)\gamma_\mu v(k_+) \end{aligned}$$

CP even $\quad G \to +G\ ,\ F_{+,-} \to +F_{-+}$

CP odd $\quad F_{+,-} \to -F_{-+}$

$$\begin{aligned} F_{+,-} &= F_{+,-}^{(1)} + F_{+,-}^{(2)} + F_{+,-}^{(3)} + \dots \\ G &= G^{(2)} + G^{(3)} + \dots \end{aligned}$$

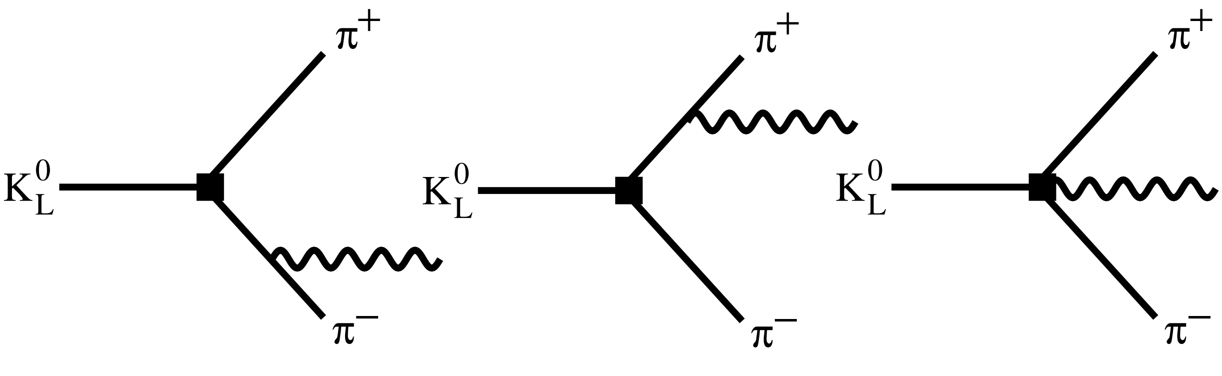
Leading Order in χPT

$$\mathcal{L} = g_8 \frac{G_F s_1 f_\pi^4}{4\sqrt{2}} \left[Tr \left[D_\mu \Sigma D^\mu \Sigma^\dagger H_w \right] + \text{h.c.} \right]$$

$$\Sigma = Exp \left[-\frac{2i}{f_\pi} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix} \right]$$

$$K_L = K_2 + \epsilon K_1$$

$$K_1 = \frac{1}{\sqrt{2}} \left[K^0 - \overline{K}^0 \right] \quad , \quad K_2 = \frac{1}{\sqrt{2}} \left[K^0 + \overline{K}^0 \right]$$



Leading Order in χPT

$$
\begin{aligned}
F_+^{(1)} &= -\epsilon\ g_8 \frac{32 f_\pi^2 (m_K^2 - m_\pi^2)\pi^2}{q^2 + 2q\cdot p_+} \\
F_-^{(1)} &= +\epsilon\ g_8 \frac{32 f_\pi^2 (m_K^2 - m_\pi^2)\pi^2}{q^2 + 2q\cdot p_-} \\
G^{(1)} &= 0
\end{aligned}
$$

g_8 , ϵ , f_π known $\quad\rightarrow\quad$ Prediction at LO

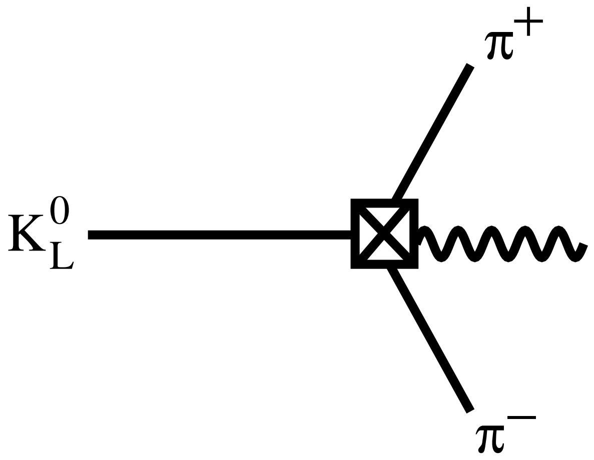
Next-to-Leading Order in χPT: $G^{(2)}$

(i) $G^{(2)} \rightarrow$ Counterterm Only

(ii) $\Gamma_{M1}(K_L^0 \rightarrow \pi^+\pi^-\gamma) \propto |G|^2$

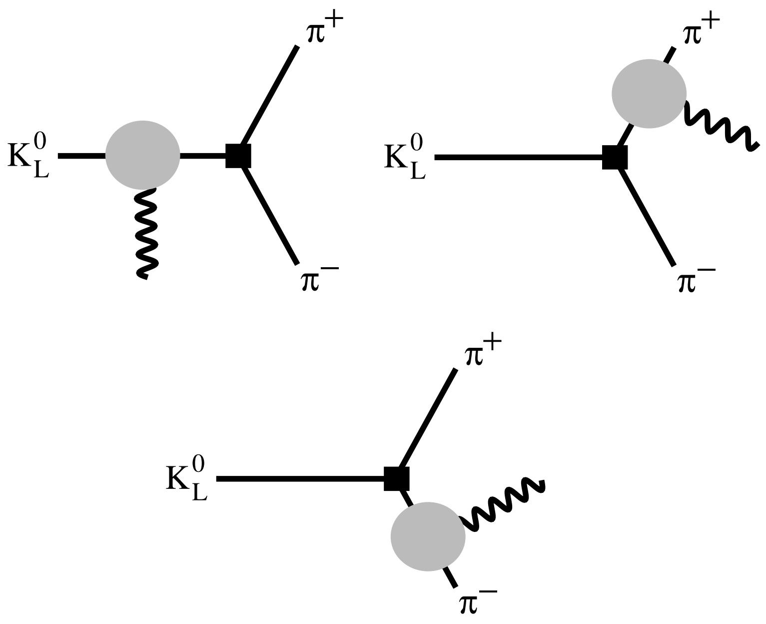
(iii) $G^{(2)} = 39.3$

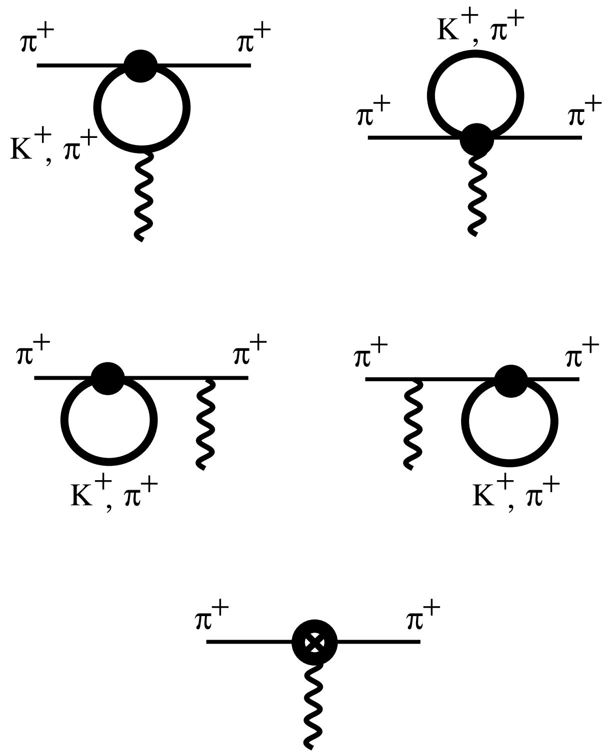
(iv) $p_{+,-}$ dependence in $K_L^0 \rightarrow \pi^+\pi^-\gamma \rightarrow G^{(3)}$.



Next-to-Leading Order in χPT: $F^{(2)}_{+,-}$

Charge Radii

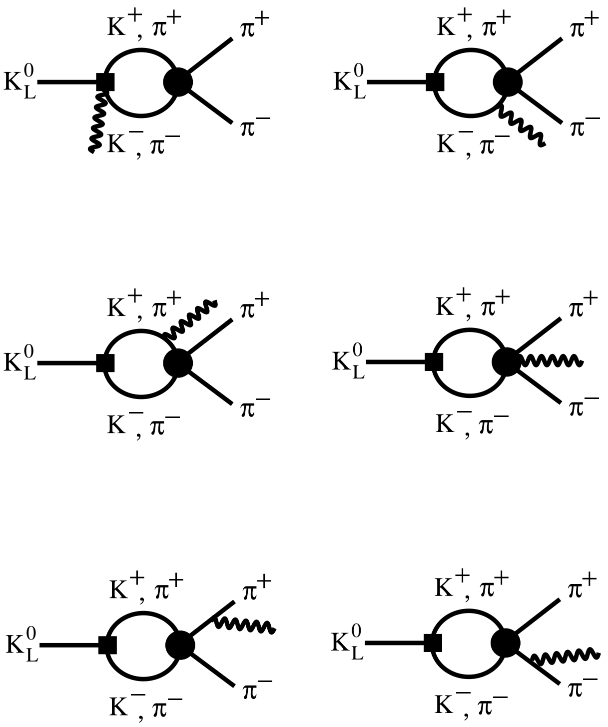


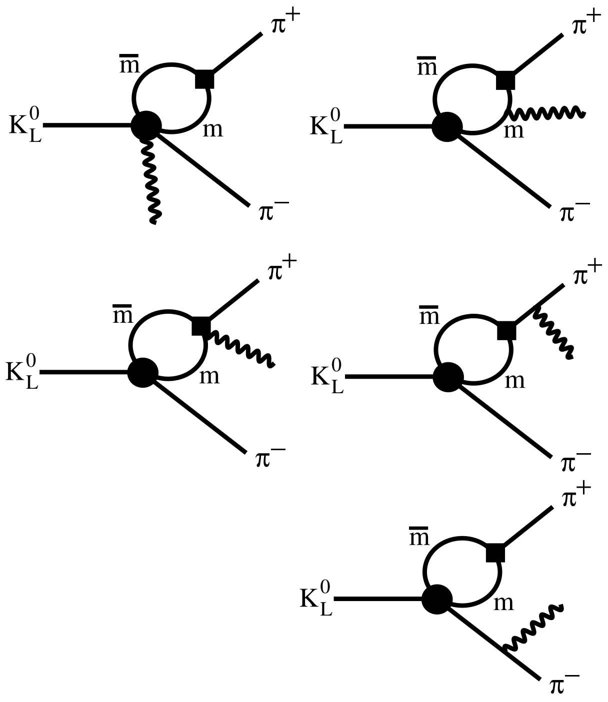


$$\mathcal{L} \;=\; -i\,\lambda_{\mathrm{cr}} \frac{e}{16\pi^2}\, F^{\mu\nu}\; Tr\left[Q\left(D_\mu\Sigma D_\nu\Sigma^\dagger + D_\mu\Sigma^\dagger D_\nu\Sigma\right)\right]$$

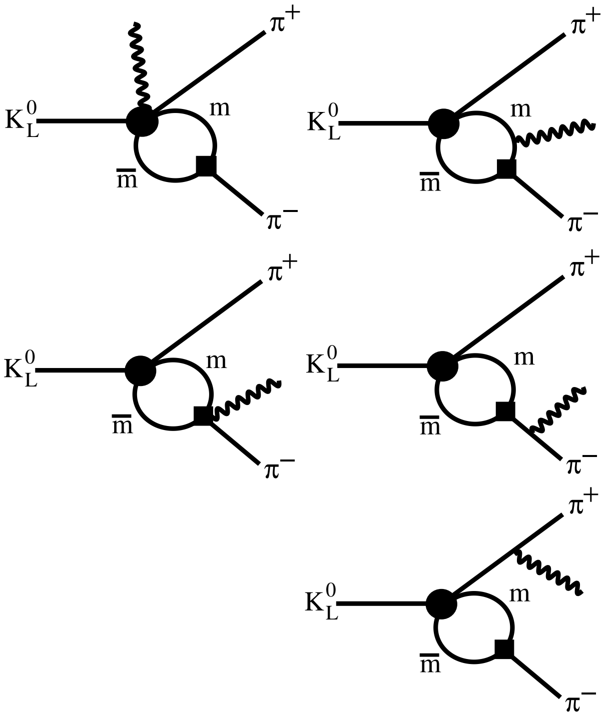
$$\langle r_\pi^2 \rangle \quad\rightarrow\quad \lambda_{\mathrm{cr}} = -0.91 \pm 0.06$$

Next-to-Leading Order in χPT: $F^{(2)}_{+,-}$

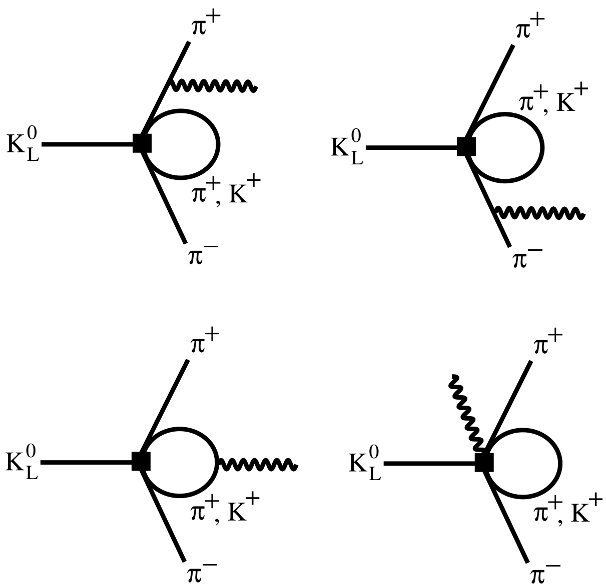




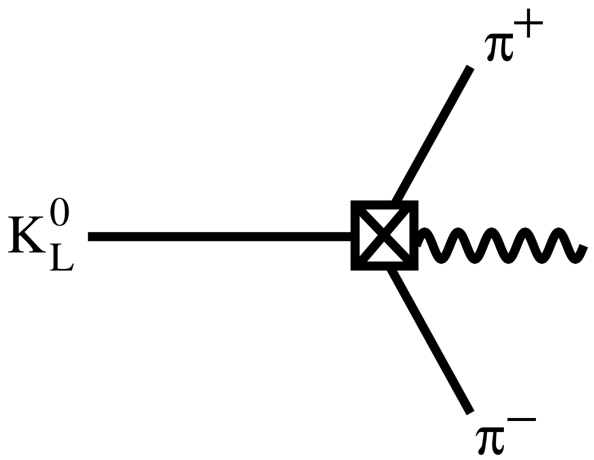
$$(m\,,\bar{m}) \;=\; \begin{array}{ll} (K^+,\,\pi^0) & ,\;(K^+,\,\eta) \\ (\pi^+,\,K^0) & ,\;(\pi^+,\,\bar{K}^0) \end{array}$$



$$(m\,,\bar{m}) \;=\; \begin{array}{ll} (K^-,\,\pi^0) & ,\;(K^-,\,\eta) \\ (\pi^-,\,K^0) & ,\;(\pi^-,\,\bar{K}^0) \end{array}$$



Next-to-Leading Order in χPT: $F^{(2)}_{+,-}$



$$\begin{aligned}\mathcal{L} \;=\; & i\frac{G_F s_1 e f_\pi^2 g_8}{16\sqrt{2}\pi^2}\left[a_1 F^{\mu\nu} Tr[QT(\Sigma D_\mu \Sigma^+)(\Sigma D_\nu \Sigma^\dagger)]\right. \\ & + a_2 F^{\mu\nu} Tr[Q(\Sigma D_\mu \Sigma^\dagger) T(\Sigma D_\nu \Sigma^\dagger)] \\ & + a_3 F^{\mu\nu} Tr[T[Q,\Sigma] D_\mu \Sigma^\dagger \Sigma D_\nu \Sigma^\dagger - T D_\mu \Sigma D_\nu \Sigma^\dagger \Sigma [\Sigma^\dagger, Q]] \\ & \left. + a_4 F^{\mu\nu} Tr[T\Sigma D_\mu \Sigma^\dagger [Q,\Sigma] D_\nu \Sigma^\dagger]\right] + h.c.\end{aligned}$$

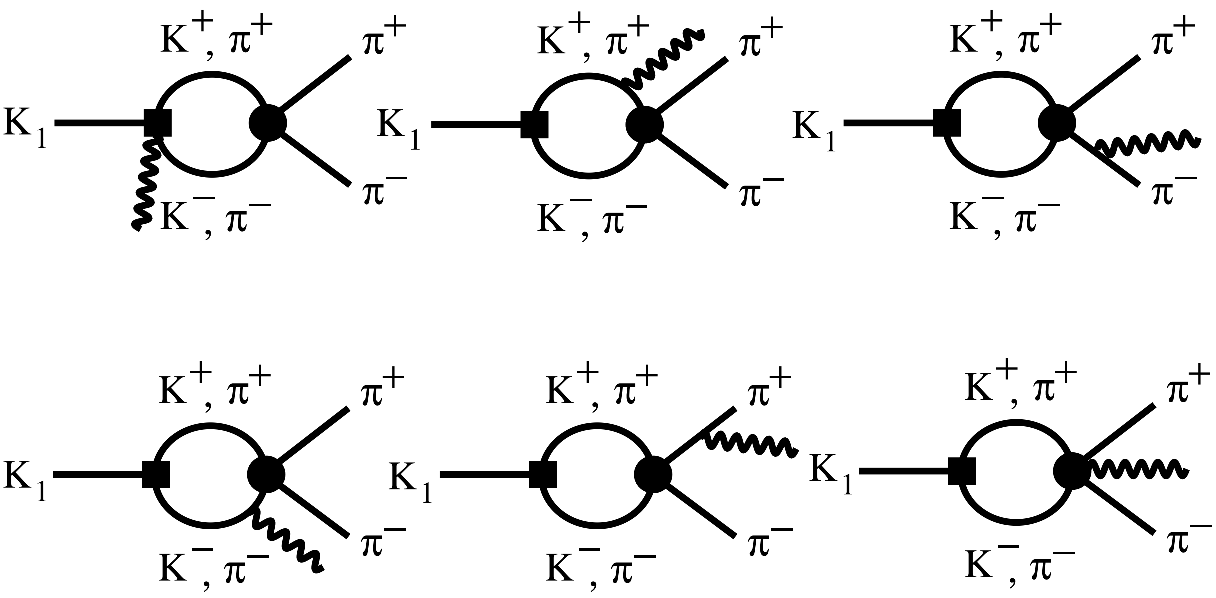
$K^+ \to \pi^+ e^+ e^-$

$$w_+ = \frac{2}{3}(a_1 + 2a_2) - 4\lambda_{\mathrm{cr}} - \frac{1}{6}\log\left(\frac{m_k^2 m_\pi^2}{\mu^4}\right) + \frac{1}{3}$$

$K_L^0 \to \pi^+\pi^- e^+ e^-$

$$\begin{aligned} w &= w_L + \frac{1}{6}(a_1 + 2a_2) + \lambda_{\mathrm{cr}} \\ w_L &= a_3 - a_4 \end{aligned}$$

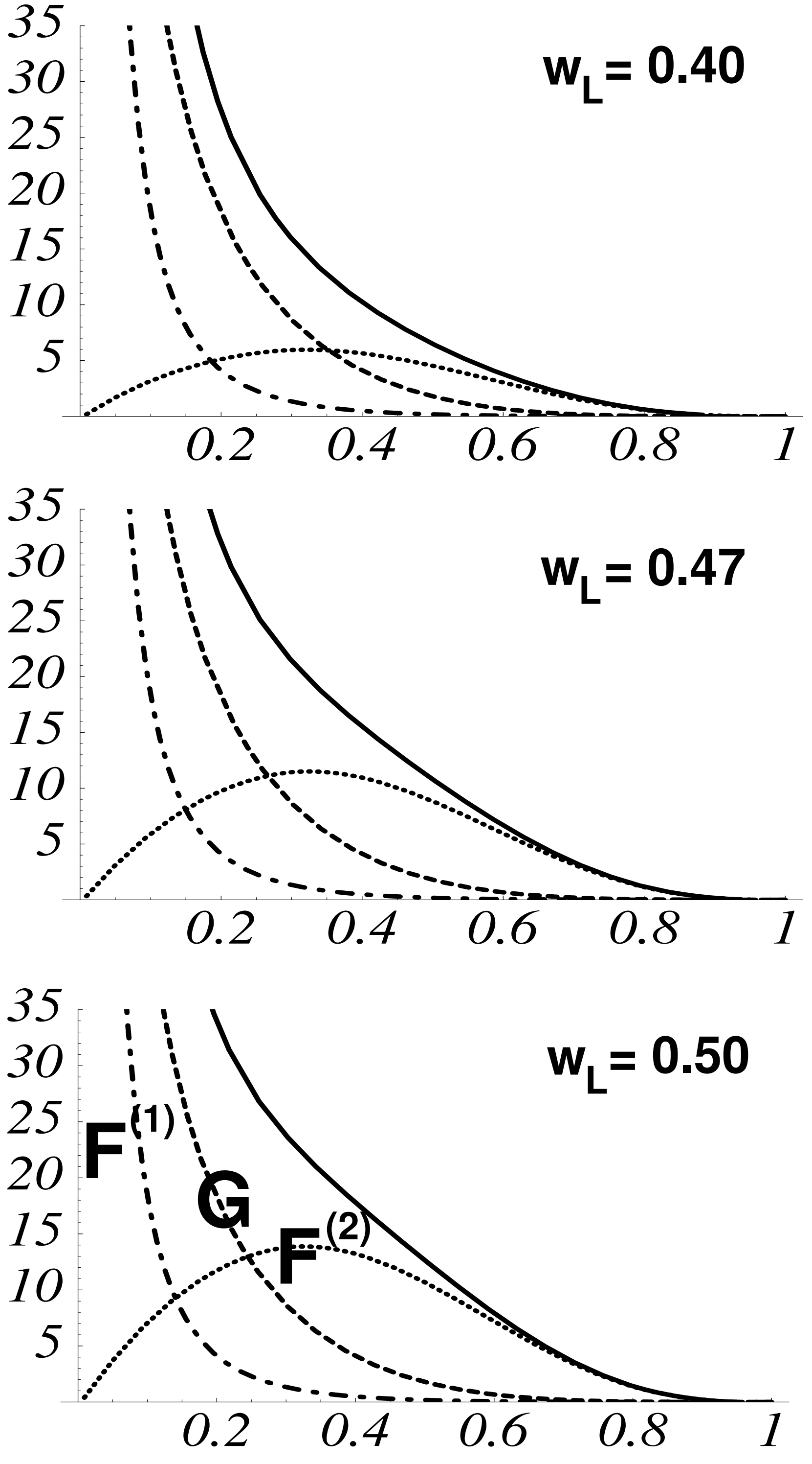
Next-to-Leading Order in χPT: FSI



$$\epsilon \quad \text{AND} \quad p^3 \quad \rightarrow \quad e^{i\delta}$$

Decay Rate

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_G}{dq^2} + \frac{d\Gamma_{F^{(1)}}}{dq^2} + \frac{d\Gamma_{F^{(2)}}}{dq^2}$$



Fixing w_L for $w_+ = 0.89$

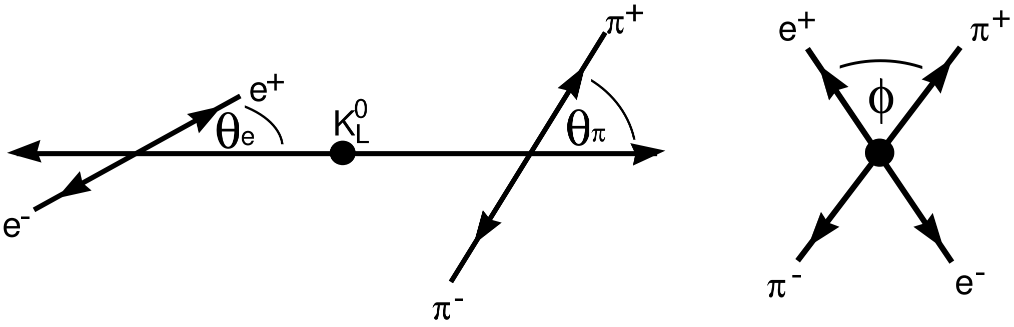
$$\begin{array}{lcccccc} \text{cut} & & 10^8 & [\Gamma_G & \Gamma_{F^{(1)}} & & \Gamma_{F^{(2)}}] \\ (2\ \mathrm{MeV})^2 & : & & 16.1 & + \; 10.7 & + & \left[3.7 - 3.5 w_L + 0.8 w_L^2\right] \\ (80\ \mathrm{MeV})^2 & : & & 0.6 & + \; 0.07 & + & \left[1.9 - 1.8 w_L + 0.4 w_L^2\right] \end{array}$$

$$\begin{aligned} Br(K_L^0 \to \pi^+\pi^- e^+ e^-) &= (3.32 \pm 0.14 \pm 0.28) \times 10^{-7} \\ & \quad \text{for } q^2_{\min} = (2\ \mathrm{MeV})^2 \end{aligned}$$

$$\begin{aligned} w_L &= 4.7 \pm 0.7 \quad \text{or} \quad -0.6 \pm 0.7 \\ \delta G &= \delta F^{(1)} = 0 \end{aligned}$$

CP Violation

$$
\begin{aligned}
B_{\rm CP} &= \langle Sign\,[\sin\phi\cos\phi]\rangle \\
&= \langle Sign\left[(n_e\cdot n_\pi)\,n_e\times n_\pi\cdot\left(\frac{p_+ + p_-}{|p_+ + p_-|}\right)\right]\rangle
\end{aligned}
$$



$$
\begin{aligned}
Im\left[\,G\ \ (F_+ - F_-)^*\right] &= Im\left[\,G^{(2)}\ \ \left(F_+^{(1)} - F_-^{(1)}\right)^*\right] \\
&+ Im\left[\,G^{(2)}\ \ \left(F_+^{(2)} - F_-^{(2)}\right)^*\right] \\
&+ Im\left[\,G^{(3)}\ \ \left(F_+^{(1)} - F_-^{(1)}\right)^*\right]
\end{aligned}
$$

$$
\begin{aligned}
q^2_{\rm min} &= (2\ \mathrm{MeV})^2 \\
B_{\rm CP} &= 9.2\% \ +\ 4.2\ \% \ +\ \ldots\ =\ 13.4\ \%
\end{aligned}
$$

$$
B_{\rm CP}{}^{KTeV} = 13.5 \pm 2.5 \pm 3.0\%
$$

Discussion

(i) $K_L^0 \to \pi^+\pi^-e^+e^-$ computed to second order in χPT

(ii) LO $\propto \epsilon$

(iii) NLO has

(a) ϵ indep. terms important for $q^2 > 0$

(b) ϵ dep. FSI

(iv) w_L contributes to $\frac{d\Gamma}{dq^2}$ at NLO.

(v) $B_{\mathrm{CP}} \sim 13\%$