The Superweak Theory 35 Years Later

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Abstract

The origins, possible rationale and definition of the superweak theory are reviewed. The observation of direct CP violation in K_L decay provides the first significant evidence against the theory. The much larger direct CP violation expected in B decays should definitively kill it.

When CP violation was discovered in 1964, there existed a very successful theory of weak interactions called the V–A theory. This closely resembled the original Fermi theory, but there were added the helicity projection operators that led to maximal parity (P) violation and particle-antiparticle symmetry (C) violation. However, this theory was CP invariant.

Thus, two possibilities emerged. Either one modified the V–A theory in some way or one assumed there existed a much weaker interaction that led to CP violation. The second possibility existed because the observed effect could be limited to CP violation in $K^0 - \bar{K}^0$ mixing causing a small admixture of the CP-even state K_1 into K_L

$$|K_L\rangle = (|K_2\rangle + \epsilon|K_1\rangle)(1+|\epsilon|^2)^{-1}, \tag{1}$$

where K_2 is the CP-odd state. Because $K^0 - \bar{K}^0$ mixing is a second-order weak effect the CP-violating effect could be limited to a new $\Delta S = 2$ interaction of the form

$$G_{sdsd}(\bar{s}\mathcal{O}d)(\bar{s}\mathcal{O}d) + \text{h.c.}, \tag{2}$$

with G_{sdsd} having a value of order 10^{-10} to 10^{-11} times the standard weak interaction coupling G_F. This came to be called the *superweak theory.*

There are several consequences of the superweak theory:

1. The CP violation would be the same for different decay modes such as $\pi^+\pi^-$ and $\pi^0\pi^0$ since it was determined entirely by the mixing ϵ. In a standard notation, this says ϵ' vanishes.

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2. Assuming CPT invariance the phase of ϵ is given exactly by

$$\theta_\epsilon = \tan^{-1}(2\Delta M/\Delta\Gamma) \approx 43.5°,$$

where $\Delta M = M_L - M_S$ and $\Delta\Gamma = \Gamma_S - \Gamma_L \approx \Gamma_S$. This is referred to as the superweak phase.

3. It would be very unlikely that CP violation would be discovered outside the K^0 system.

All these predictions appeared to be true for the next 25 years.

The superweak theory is really a large class of theories with the common feature that their effect at low energies contains the term (2). The original form I wrote in 1964 [1] had the standard $\Delta Q = \Delta S$ current coupled very weakly to a postulated $\Delta Q = -\Delta S$ current. With the advent of the quark picture, this is no longer a possibility. It is possible that the superweak character arises from the exchange of a superheavy particle, from very weak coupling constants, or from higher-order effects.

In 1973, with the discovery of neutral currents, a new theory of weak interactions came to be accepted, the Weinberg-Salam model. As shown by Glashow, Iliopoulos, and Maiani, this required a fourth quark. This theory was more elegant that the previous theory because it was based on a gauge principle and was renormalizable. We all loved it, but it was CP-invariant, not by design, but because of its basic form. I'm not sure why people didn't worry more about that, but perhaps it was because the one small effect could always be explained by superweak.

However, in 1973, a paper was published in the Progress of Theoretical Physics [2], a paper that, at first, few people noticed. It pointed out in one paragraph that if there were six quarks instead of four (at the time, in fact, only three were known), then CP violation could be accommodated in the Weinberg-Salam model. With the discovery of the b quark system, this became the standard model of CP violation. In this model, the up quarks (u_j) are connected to the down quarks (d_i) in a weak decay by

$$d_i \rightarrow V_{ji} u_j,$$

where V_{ji} is the 3×3 CKM matrix. The one significant phase in the matrix is the origin of CP violation.

At first we thought that the small value of ϵ meant that the phase in the CKM matrix was small. The true picture came with the determination of the relatively long B lifetime. It then became clear that the matrix had a hierarchical structure. It could be expanded in powers of the sine of the Cabibbo angle ($\lambda = \sin\theta_0 = .22$) and the phase entered only at order λ^3 in the elements connecting the first and third generation:

$$\begin{aligned} V_{ub} &= A\lambda^3(\rho - i\eta) \\ V_{td} &= A\lambda^3(1 - \rho - i\eta). \end{aligned} \tag{3}$$

The rate of $b \to u$ decays gives roughly

$$(\rho^2 + \eta^2)^{1/2} \approx 0.4 \pm 0.15,$$

while the value of ϵ requires η greater than 0.2. As a result, the phases of Eq. (3) are not small.

Given the standard model, is there any motivation for thinking about superweak? The most compelling arguments are those given by Steve Barr [3]. One would like a more elegant way to introduce CP violation than by putting a whole set of arbitrary phases in the Yukawa matrices. Also, if one assumes CP violation is allowed everywhere you can put it, then the standard model contains an arbitrary parameter called $\theta_{\rm QCD}$, which is restricted to values less than 10^{-9} by the neutron electric dipole moment. A possible solution to these problems is to assume CP invariance is valid at some higher mass scale, but it is broken softly, either spontaneously or in terms of dimensions less than 4. Then $\theta_{\rm QCD}$ is calculable and in suitable theories is small enough. Similarly, the parameter η in the CKM matrix must be calculated from the same origin and it may turn out [4] to be much too small to explain ϵ. In that case, the observed CP violation must be explained by other effective interactions caused by the new physics at the higher mass scale, which may be effectively superweak.

The major feature of the weak Hamiltonian of the standard CKM theory that distinguishes it from superweak is the presence of the CP-violating terms that change flavor by one unit that are of the order $G_F A\lambda^3$. Thus, besides the CP violation in the mixing, we expect *direct CP violation*; that is, CP violation in the decay amplitude. However, direct CP violation for the K^0 system given by the parameter ϵ' is very small and, as discussed in these pages, the value of ϵ' in the CKM model is very uncertain.

A question arises as to the definition of a superweak model since there exist many examples in the literature. In general, a superweak model is defined by the effective Lagrangian at the electroweak scale or below:

$$L_{\rm eff} = L_0 + L_{\rm SW}$$

where L_0 is the standard model with η much less than that required to explain ϵ and $L_{\rm SW}$ contains various four-quark interaction with coefficients G_{ijkl} including the term (2) needed to explain ϵ.

A theory is superweak if all the G_{ijkl} are much smaller than the usual weak interactions in L_0. An extreme superweak theory would be one in which all G_{ijkl} were as small as G_{sdsd} and involved a change of flavor ΔF of 2 units. However, I would consider theories with larger values of G_{ijkl} as long as they were much smaller than those of the standard model and allow for $\Delta F = 1$ as well as 2. This raises the interesting point of whether the observed value of ϵ'/ϵ could possibly be consistent with some theories that are essentially superweak. The simplest superweak models involve the tree-level exchange of a heavy boson. The $\Delta F = 1$ CP-violating term in $L_{\rm SW}$ then might have G_{sddd} of order G_{sdsd} in which case ϵ' would be of order 10^{-9} to 10^{-10} or $\epsilon'/\epsilon \sim 10^{-6}$ to 10^{-7}. However, it is also possible that the exchange might involve a relatively light Higgs boson in a multi-Higgs scenario with very small couplings. In general, the contribution of the neutral Higgs exchange to ϵ'/ϵ in such models is expected to be small, but it is possible that charged Higgs exchange could give ϵ'/ϵ as large as 10^{-3} [5].

There exists a large class of models that fit our definition of superweak although they are called "milliweak" by Barr [3]. These are models in which ϵ occurs only in a box diagram involving a new boson H and either a quark, a lepton, or a new fermion. A large catalogue of such models is given by Barr. If H is a charged diquark, ϵ'/ϵ can actually arise at tree level and be as large as 3×10^{-3}; in contrast, if H is a leptoquark, ϵ'/ϵ occurs only via a loop (penguin-type) graph and is no larger than 10^{-7} [6]. In another model [7], H is a charged scalar and the fermion is a new "quark" with charge 4/3; here again ϵ'/ϵ occurs at loop level and is found to be of order 10^{-4}. It has been suggested [8] that models that are effectively superweak can be constructed with SUSY particles. The possibility that ϵ'/ϵ could be as large as 2×10^{-3} from loops with SUSY particles with particle mixing has recently been noted [9].

We conclude that while it is reasonable to expect ϵ'/ϵ to be small in superweak models, one cannot rule out models that are essentially superweak and still have ϵ'/ϵ of the order 2×10^{-3} as observed. The reason is that ϵ' is only 5×10^{-6} which corresponds to G_{sdqq} of order 10^{-5} to 10^{-6}.

In fact, one might say that if $L_{\rm eff}$ is defined below the b-mass scale, then the standard model becomes practically superweak: L_0 is then the CP-conserving four-quark model and $L_{\rm SW}$ represents the effective four-fermion interactions due to virtual effects of t quarks.

The most definitive way to distinguish the standard model from superweak is in B physics. The first observation of CP violation should come from the asymmetry in the decay $B^0(\bar{B}^0) \to J/\psi K_S$. Within the standard model this measures $\sin 2\beta$, where β is the phase of $1 - \rho + i\eta$. The standard model predicts a large positive value for $\sin 2\beta$ between 0.3 and 0.9. This could be blamed entirely on CP violation in $B - \bar{B}$ mixing and is thus analagous to the ϵ parameter of the K system. It could be explained by a superweak interaction with the coupling G_{bdbd} of order 10^{-7} times G_F. The final, completely definitive death of any superweak theory will come from the observation of direct CP violation in the B system.

This arises from the phase γ in $b \to u$ decays (where γ is the phase of $\rho + i\eta$). To be more accurate, one is comparing the phase of $b \to u$ decays to that of $b \to c$ (such as $B \to J/\psi K_S$). Evidence for such direct CP violation would be given by the difference between the asymmetry parameter in a decay such as $B^0(\bar{B}^0) \to \pi^+\pi^-$ from that of $J/\psi K_S$. This can be considered [10] the ϵ' experiment for the B system. In the tree approximation, this is given by $\sin 2(\beta + \gamma)$, which is expected to be very different from $\sin 2\beta$. Thus, ϵ' for the B system is of order unity in contrast to the K system where it is 5×10^{-6}. It should be emphasized that "penguin pollution" does not affect the Yes-No question as to the existence of direct CP violation; all that is required is a significant difference in the two asymmetries.

It may well be that $B \to \pi\pi$ will not be the first place to find direct CP violation. It may be found in the difference in partial rates of B^+ and B^- decays if final state interactions are large enough. In thinking about B physics, it is important to recognize the importance of finding evidence for direct CP violation even if the extraction of (β, γ) or (ρ, η) is not directly possible from the data.

In retrospect, it is very surprising that it has taken over 30 years to kill the superweak theory as the origin of CP violation. It will still take a long time to establish whether the CKM phase is the only source of CP violation or whether there are important effects from physics beyond the standard model.

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