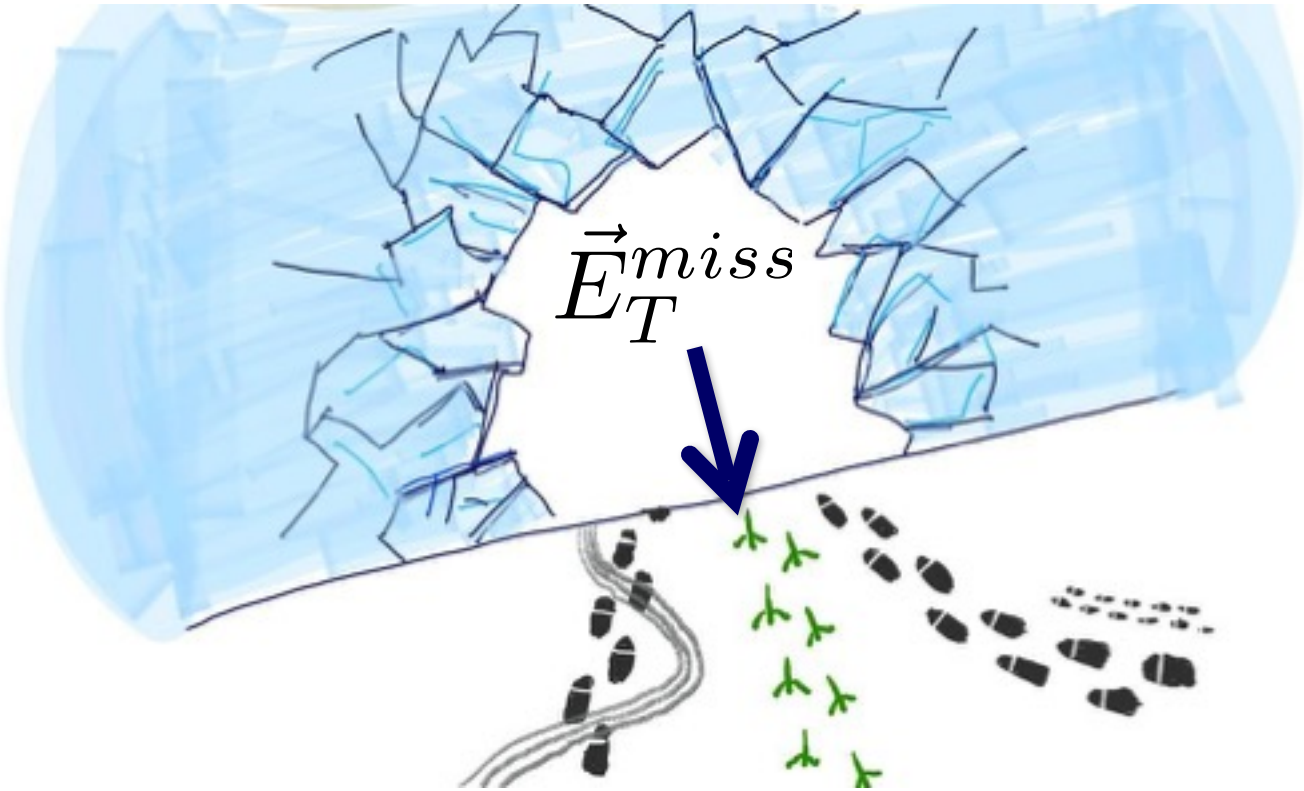


Kinematic variables for Dark Matter at the LHC and Beyond

Christopher Rogan

HARVARD UNIVERSITY

University of Chicago HEP Seminar – March 9, 2014

Talk Outline

- Weakly interacting particles at the LHC
  - Why? How?

- Kinematic handles for studying them
  - MET
  - singularity variables

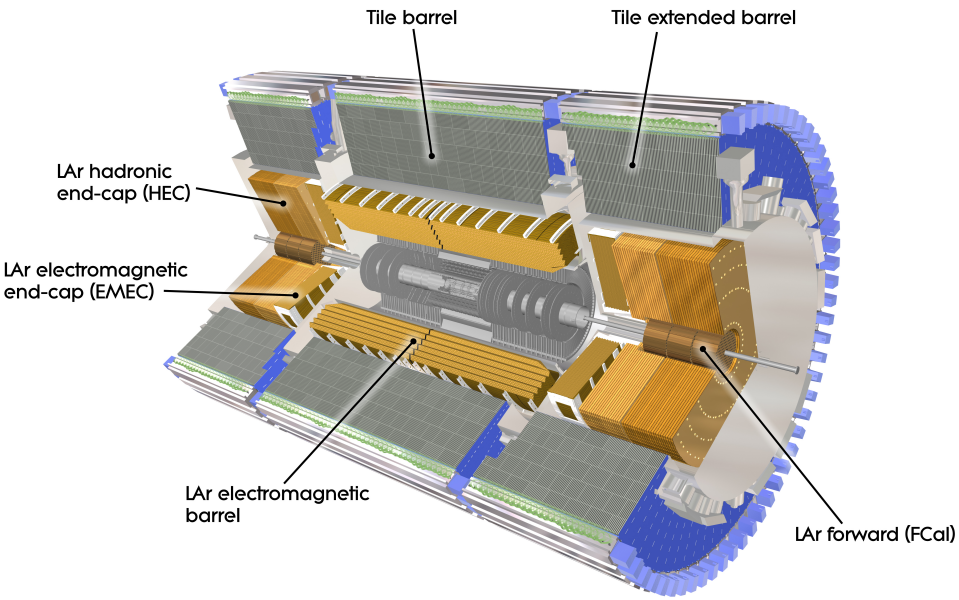
- ***Recursive Jigsaw Reconstruction***
  - ex. di-leptons (i.e. super-razor variables)
  - ex. di-leptonic tops/stops (something sexier)

Weakly interacting particles @ LHC

- **Why are they interesting?**
  - Dark Matter
    - It exists - but what is it? Would like to know if we're producing these particles at the LHC
  - Electroweak bosons
    - Decays of W and Z often produce neutrinos
  - New symmetries
    - Discrete symmetries (ex. R-parity) make lightest new 'charged' particles stable

Weakly interacting particles @ LHC

- **How do we study them?**
  - Can infer their presence through ***missing transverse energy***
  - Hermetic design of LHC experiments allows us to infer ***'what's missing'***



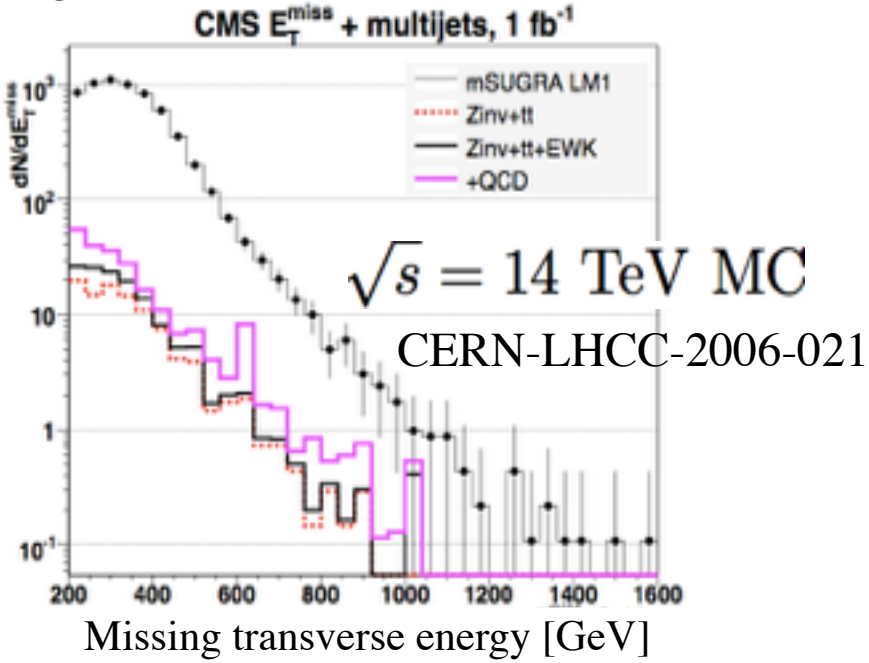
ATLAS Calorimeters

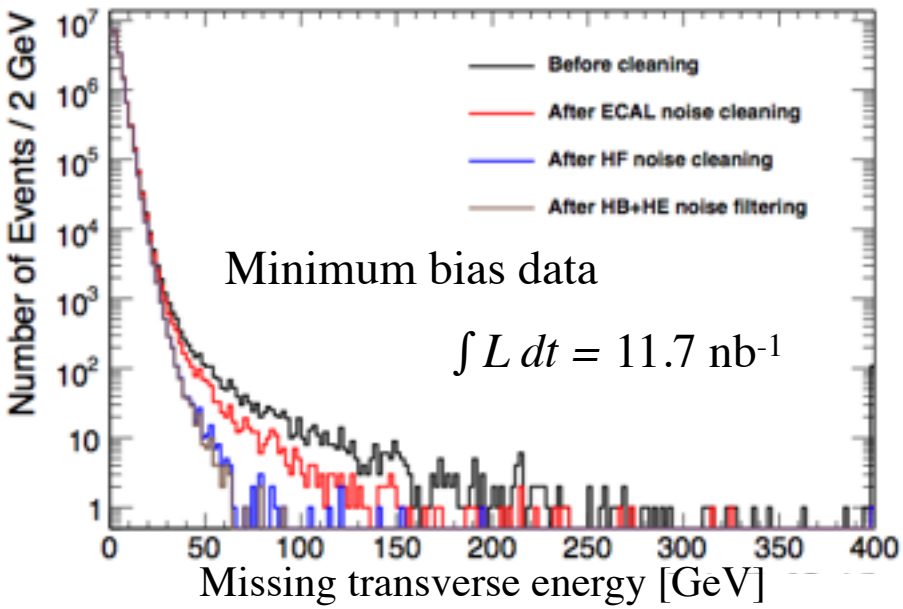
- full azimuthal coverage, up to |η| of ~5
- stopping power of ~12-20 interaction lengths
- ECAL+HCAL components with segmentation comparable to lateral shower sizes

$$\vec{E}_T^{miss} = -\sum_{i}^{cells} \vec{E}_T$$

Missing transverse energy

Figures from SUSY10 conference talk:

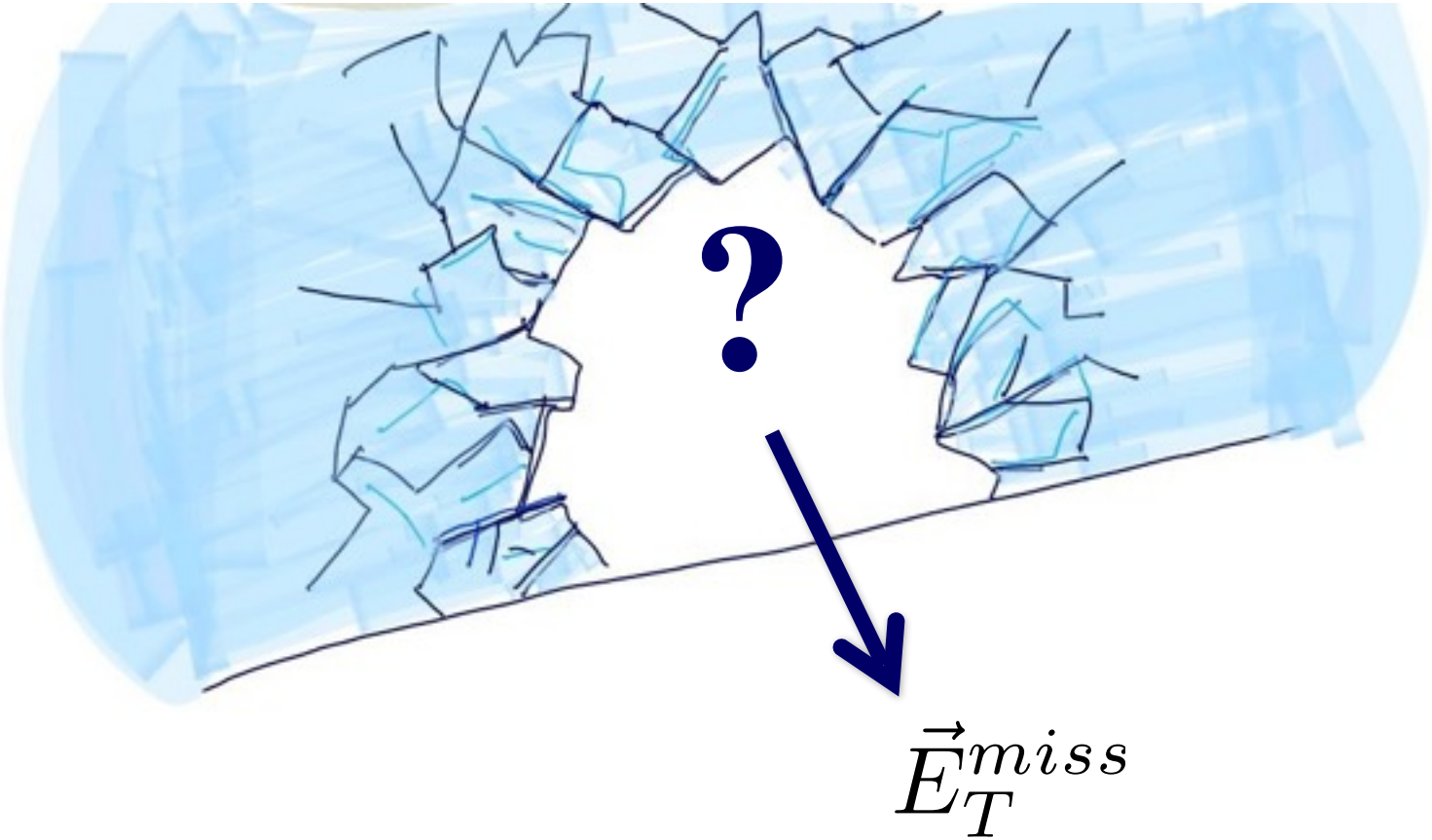




Missing transverse energy is a powerful observable for inferring the presence of weakly interacting particles

But, it only tells us about their transverse momenta – often we can better resolve quantities of interest by using additional information

Missing transverse energy



Missing transverse energy only tells us about the momentum of weakly interacting particles in an event…

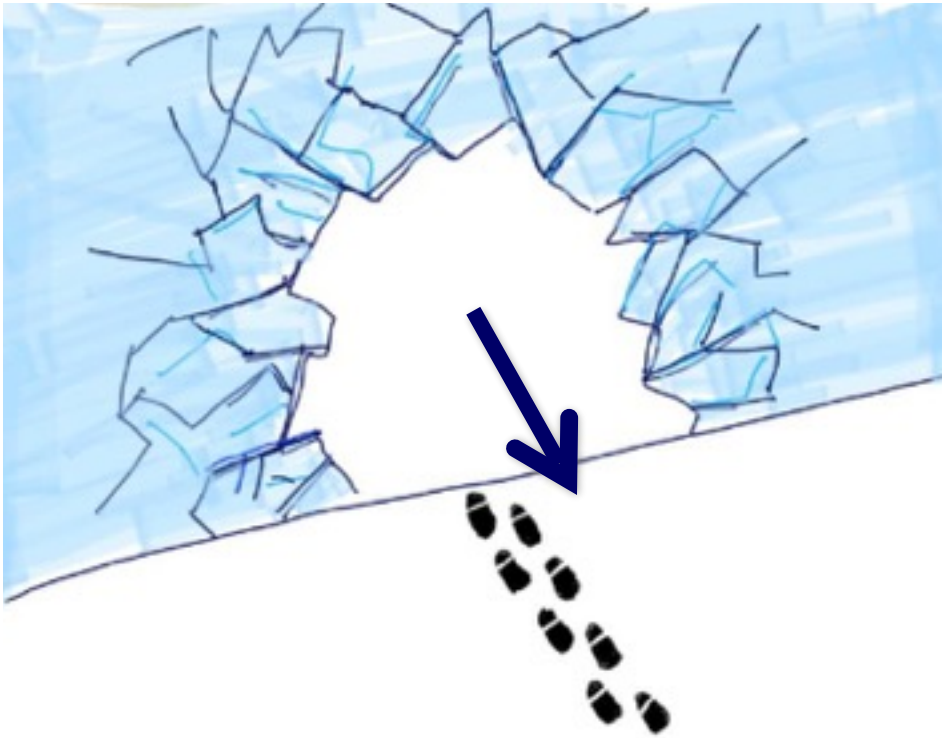
Missing transverse energy

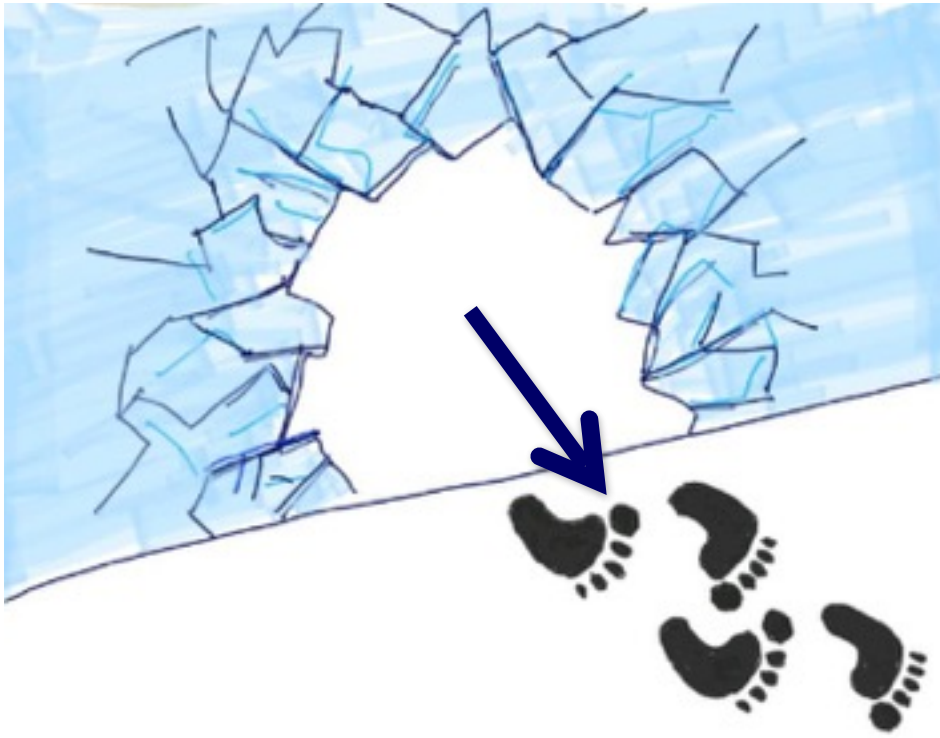
…not about the identity or mass of weakly interacting particles

Missing transverse energy

…not about the identity or mass of weakly interacting particles

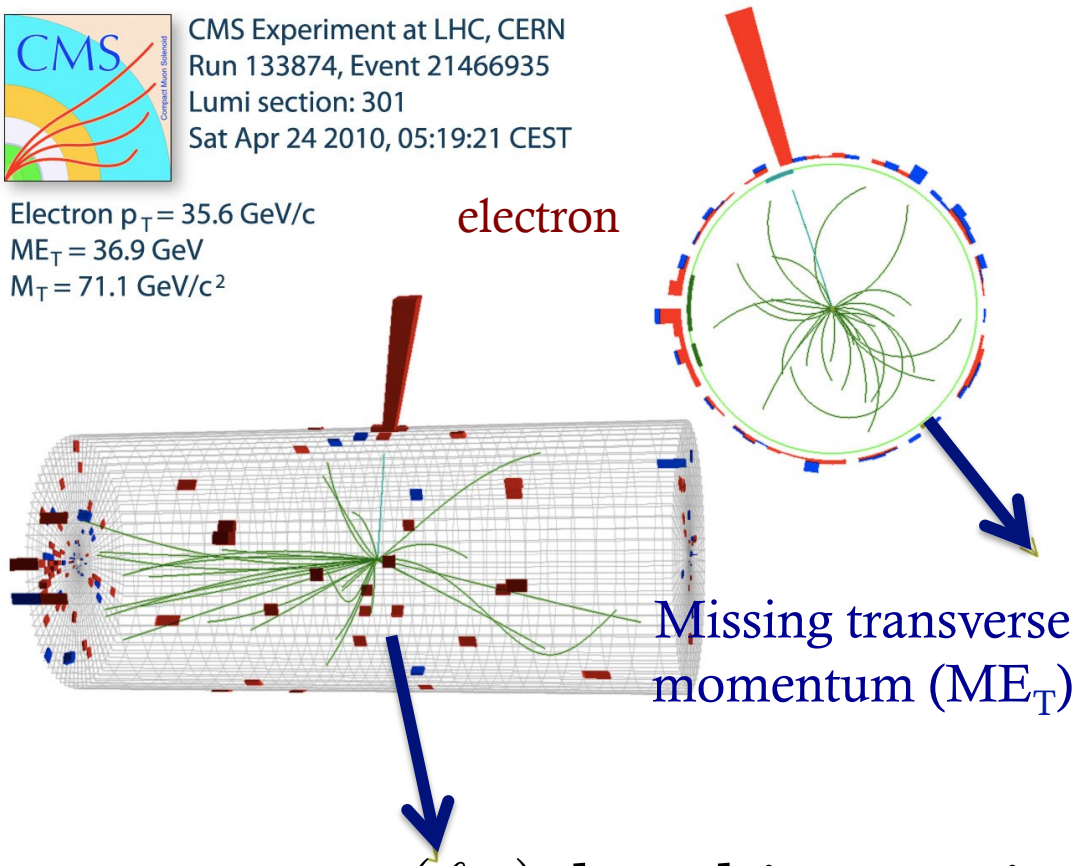
Missing transverse energy

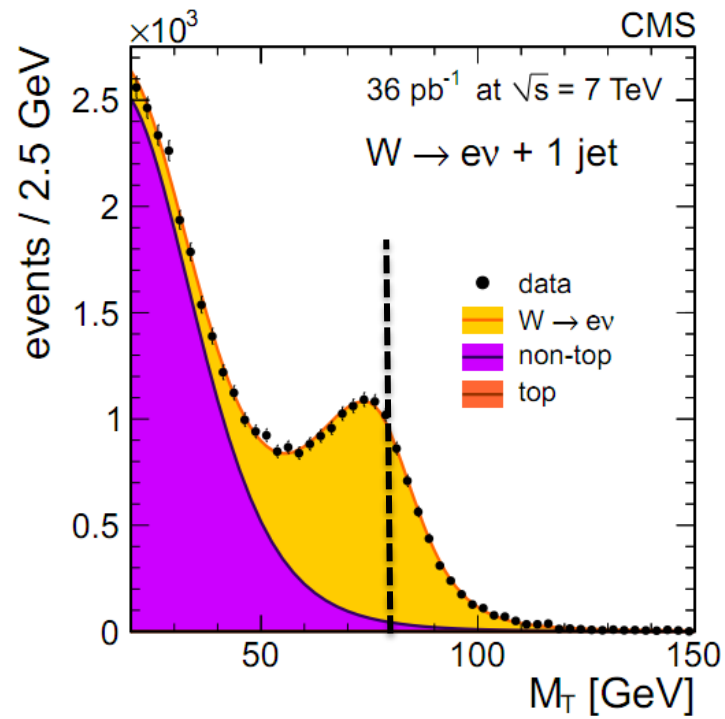




We can learn more by using other information in an event to contextualize the missing transverse energy

Resolving the invisible $W(e\nu)$



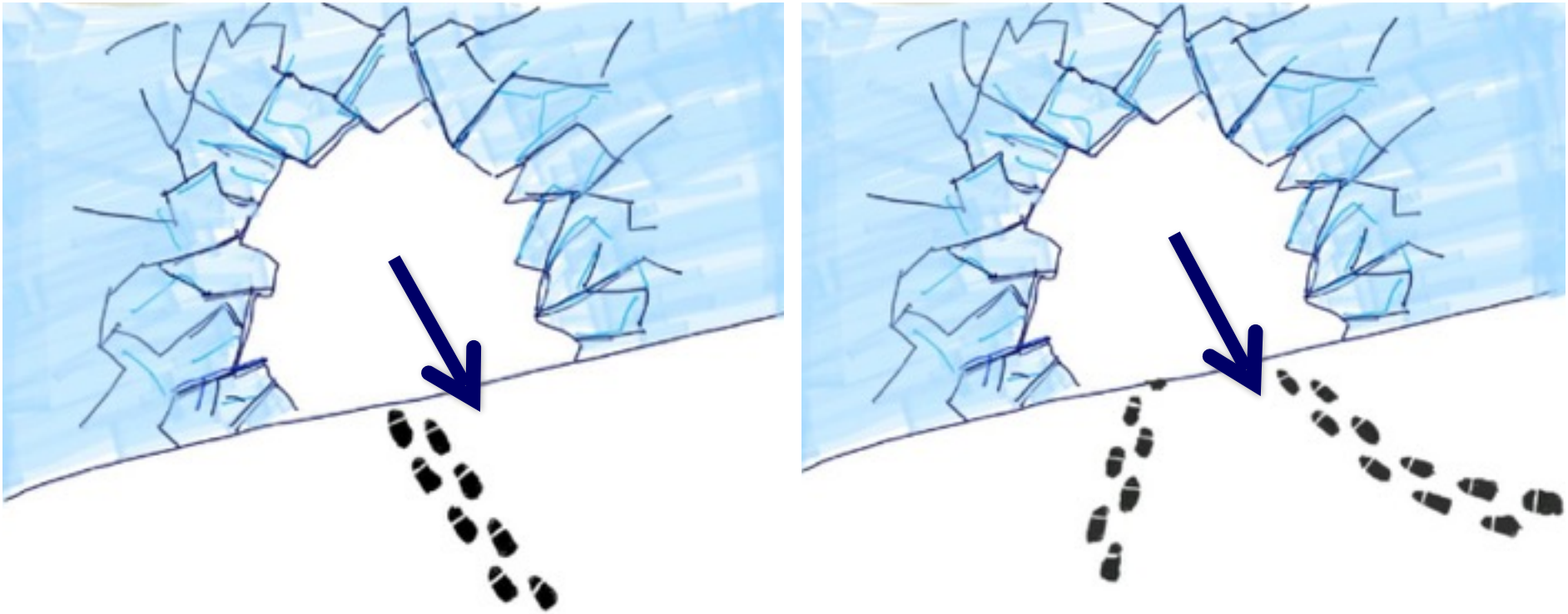


$$m_T = \sqrt{2p_T^e p_T^\nu (1 - \cos\phi)}$$

$m_T(\ell\nu)$ has kinematic *edge* at $m_W \sim 80$ GeV

Can use visible particles in events to contextualize missing transverse energy and better resolve mass scales

Missing transverse energy

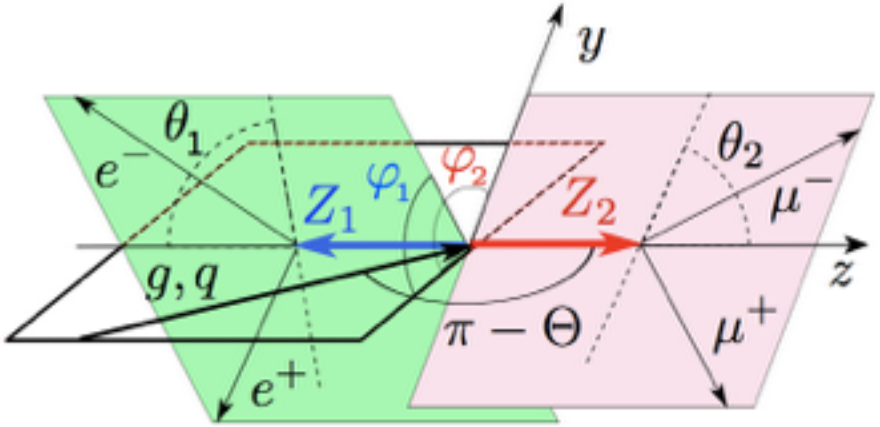


We can learn more by using other information in an event to contextualize the missing transverse energy ⇒ multiple weakly interacting particles?

Open vs. closed final states

CLOSED $H \to Z(\ell\ell)Z(\ell\ell)$

Can calculate all masses, momenta, angles



Can use masses for discovery, can use information to measure spin, CP, etc.

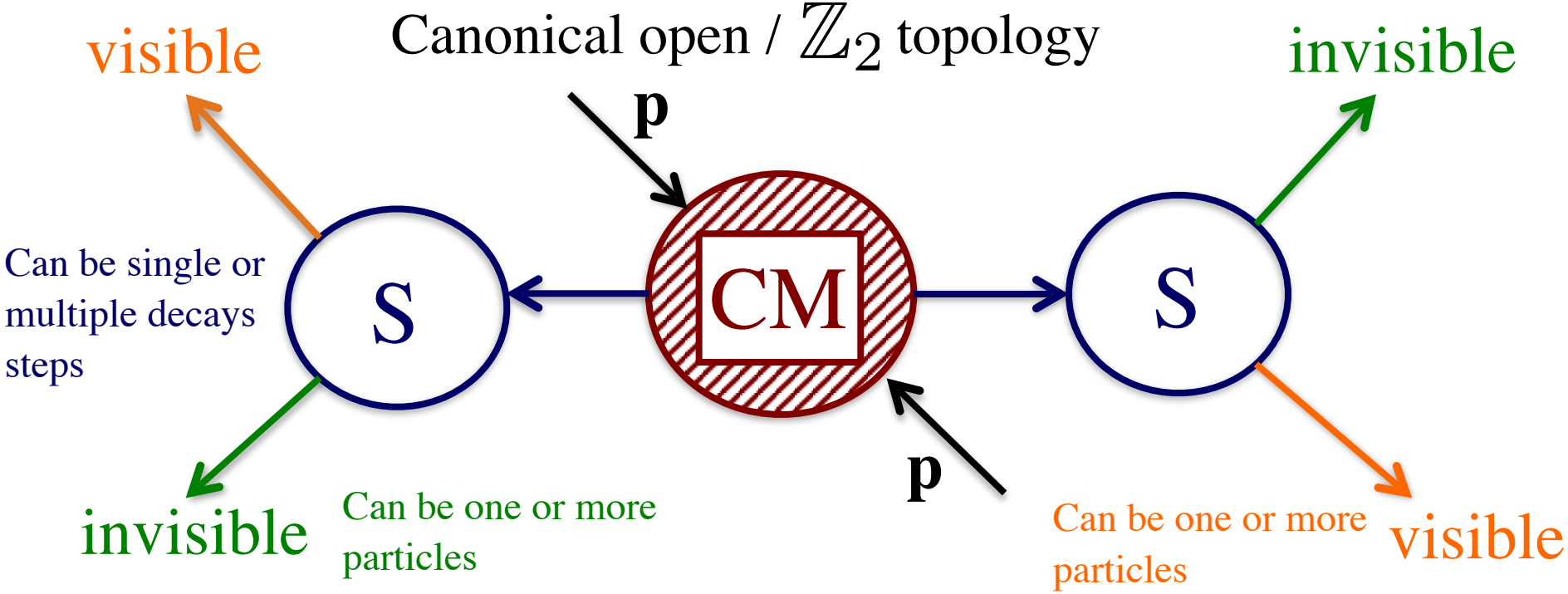
OPEN $H \to W(\ell\nu)W(\ell\nu)$

Under-constrained system with multiple weakly interacting particles – can't calculate all the kinematic information

What useful information can we calculate?

What can we measure?

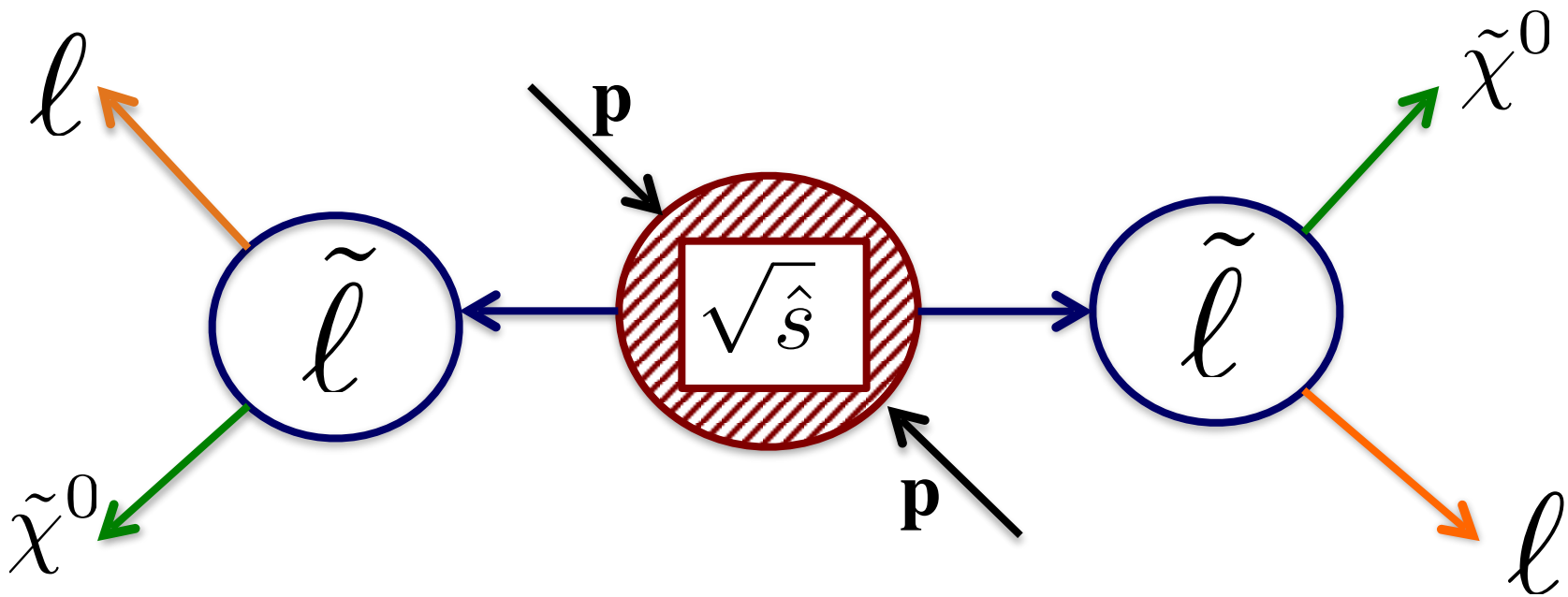
Multiple weakly interacting particles?



- Dark Matter
- Higgs quadratic divergences
- ….

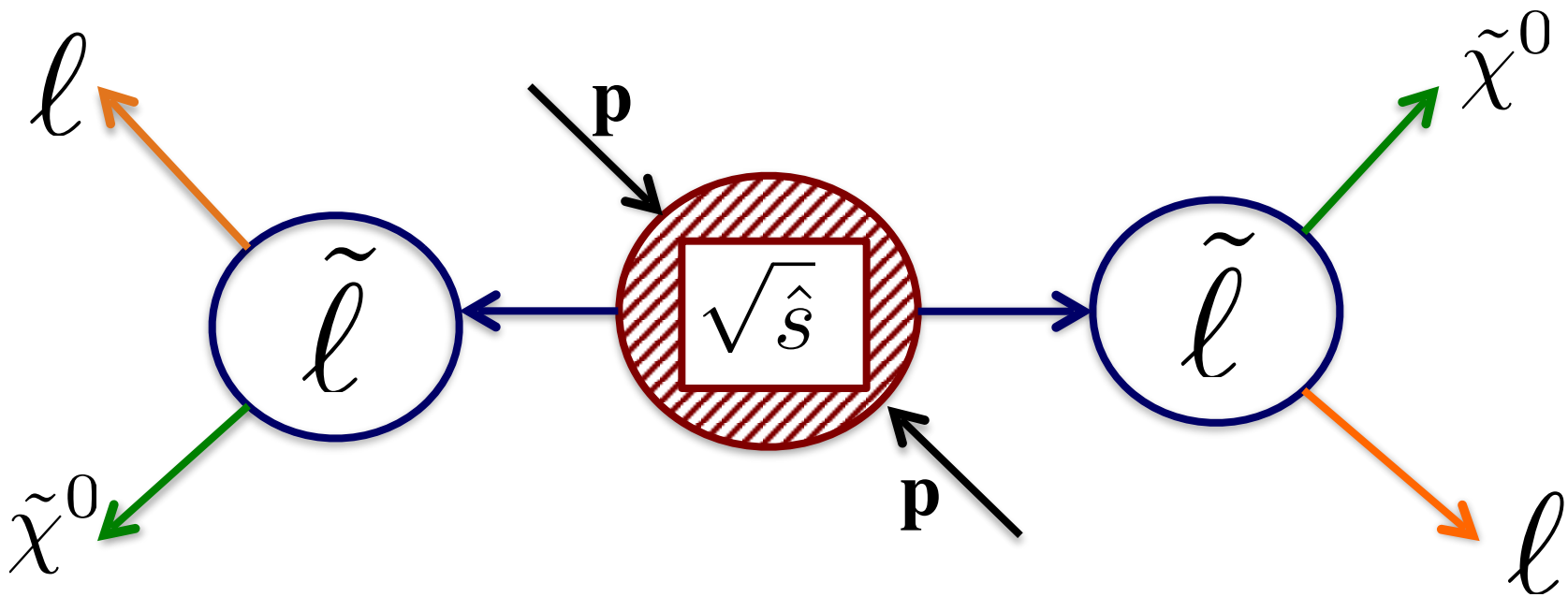
Theory	$\mathbb{Z}_2$
SUSY	R-parity
Little Higgs	T-parity
UED	KK-parity

Example: slepton pair-production

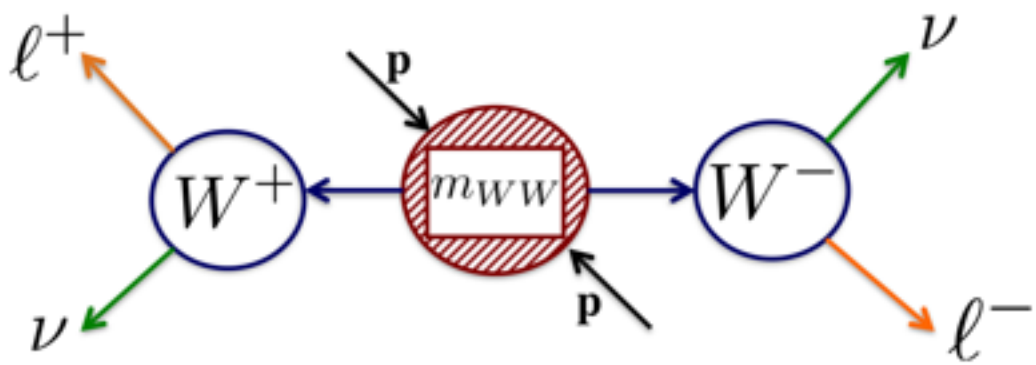


Experimental signature: di-leptons final states with missing transverse momentum

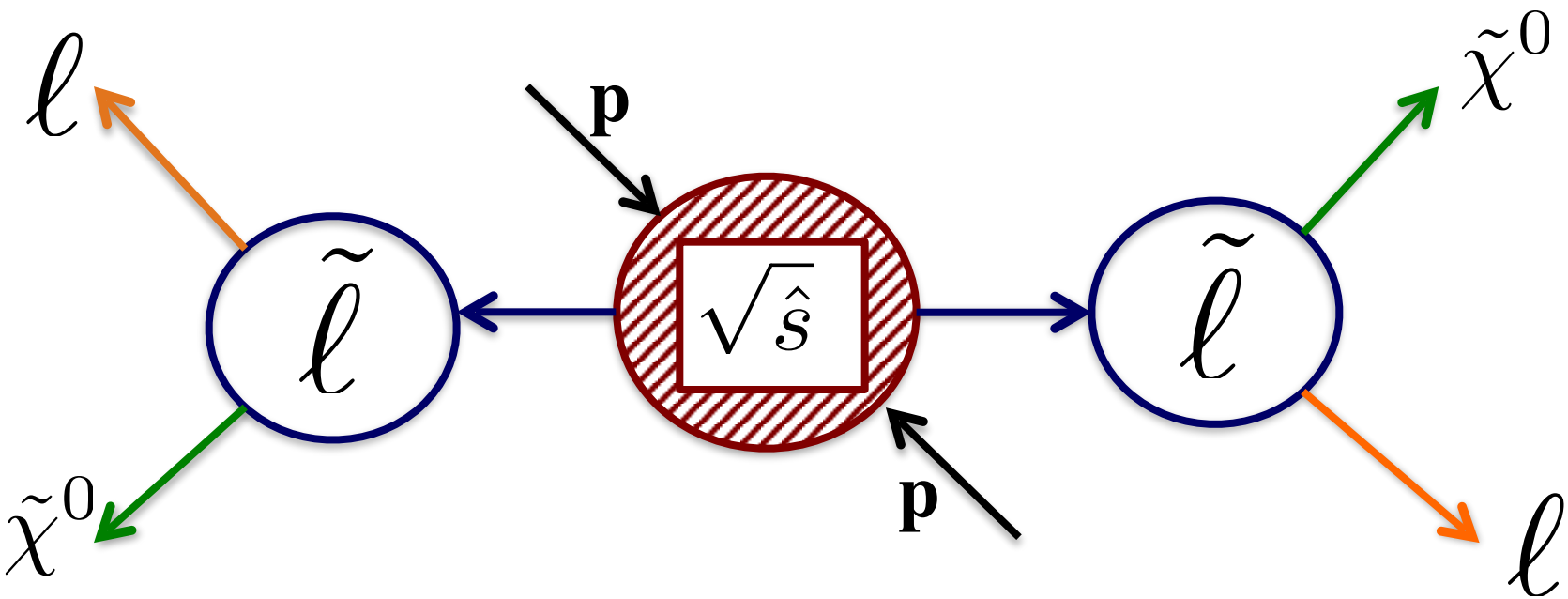
Example: slepton pair-production



Main background:



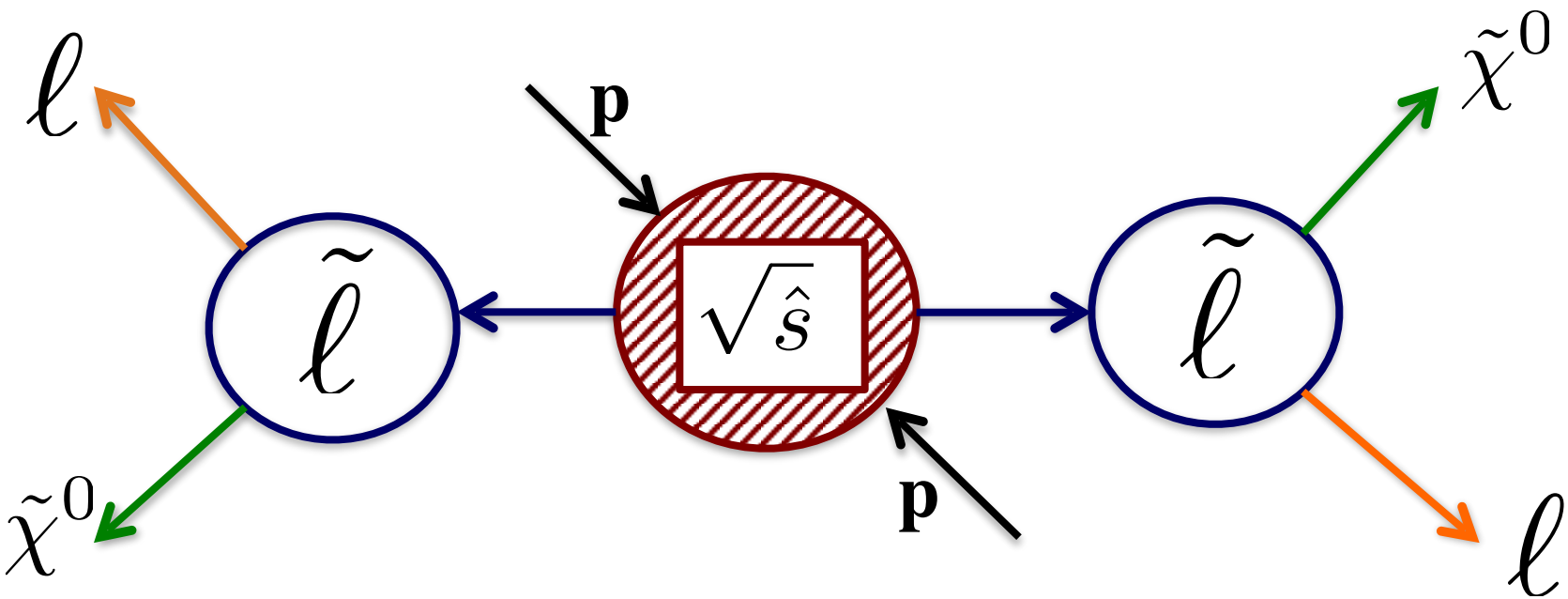
Example: slepton pair-production



What quantities, if we could calculate them, could help us distinguish between signal and background events?

$$\sqrt{\hat{s}} = 2\gamma^{decay} m_{\tilde{\ell}} \qquad M_\Delta \equiv \frac{m_{\tilde{l}}^2 - m_{\tilde{\chi}^0}^2}{m_{\tilde{l}}}$$

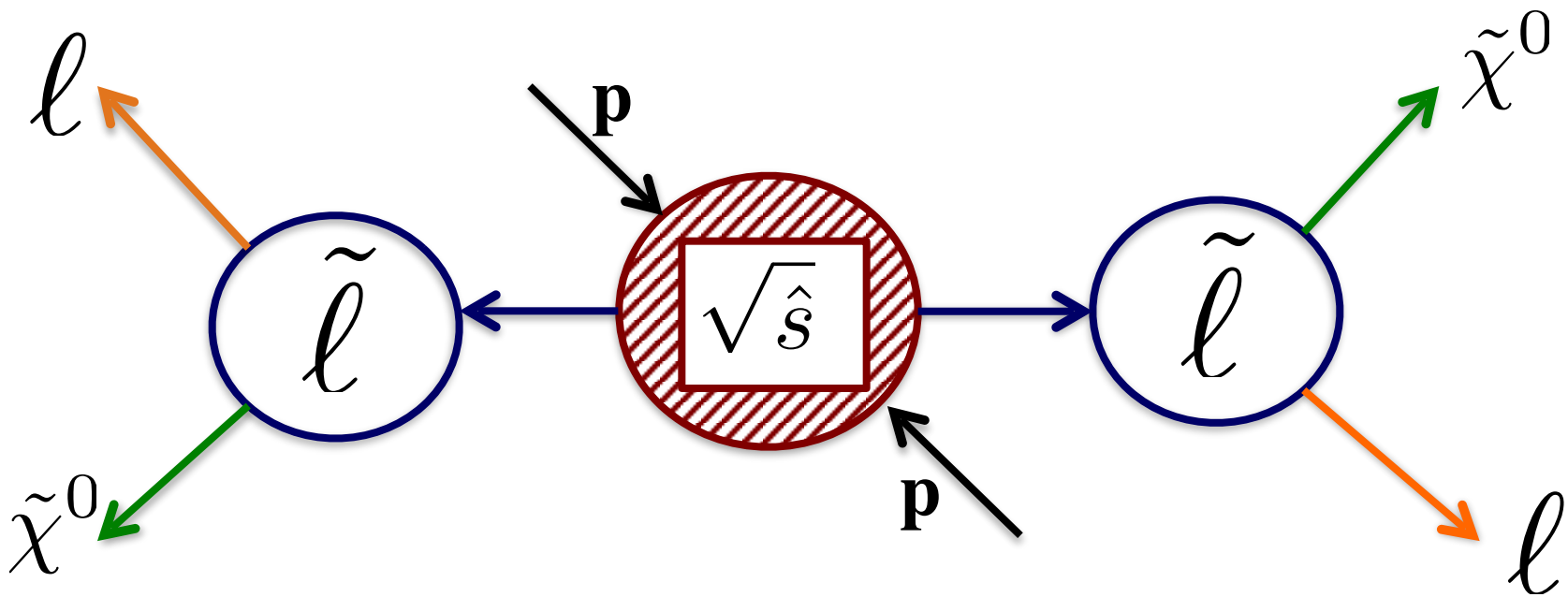
Example: slepton pair-production



What information are we missing?

We don't observe the weakly interacting particles in the event. We can't measure their momentum or masses.

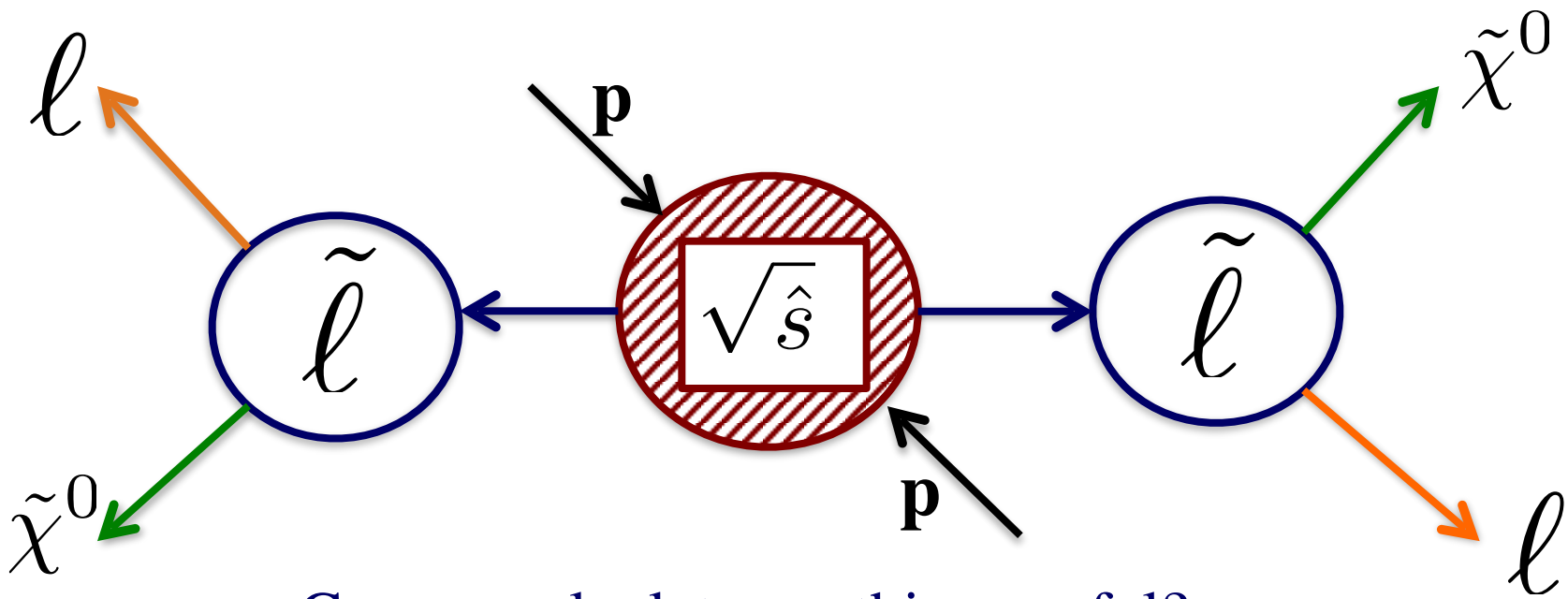
Example: slepton pair-production



What do we know?

We can reconstruct the 4-vectors of the two leptons and the transverse momentum in the event

Example: slepton pair-production



Can we calculate anything useful?

With a number of simplifying assumptions…

$$\vec{E}_T^{miss} = \sum \vec{p}_T^{\tilde{\chi}^0} \qquad m_{\tilde{\chi}^0} = 0$$

…we are still 4 d.o.f. short of reconstructing any masses of interest

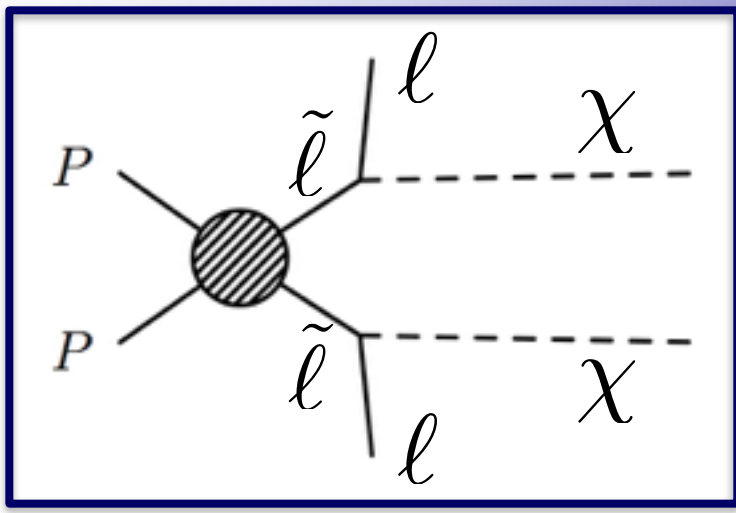
Singularity Variables

- State-of-the-art for LHC Run I was to use singularity variables as observables in searches

- Derive observables that bound a mass or mass-splitting of interest by
  - Assuming knowledge of event decay topology
  - Extremizing over under-constrained kinematic degrees of freedom associated with weakly interacting particles

Singularity Variable Example: M_{T2}

Generalization of transverse mass to two weakly interacting particle events



$$M_{T2}^2(m_\chi) = \min_{\vec{p}_T^{\,\chi_1} + \vec{p}_T^{\,\chi_2} = \vec{E}_T^{miss}} \max\left[m_T^2(\vec{p}_T^{\,\ell_1}, \vec{p}_T^{\,\chi_1}, m_\chi), m_T^2(\vec{p}_T^{\,\ell_2}, \vec{p}_T^{\,\chi_2}, m_\chi)\right]$$

with: $m_T^2(\vec{p}_T^{\,\ell_i}, \vec{p}_T^{\,\chi_i}, m_\chi) = m_\chi^2 + 2\left(E_T^{\ell_i} E_T^{\chi_i} - \vec{p}_T^{\,\ell_i} \cdot \vec{p}_T^{\,\chi_i}\right)$

From:

C.G. Lester and D.J. Summers. Measuring masses of semiinvisibly decaying particles pair produced at hadron colliders. *Phys.Lett.*, B463:99–103, 1999.

Singularity Variable Example: M_{T2}

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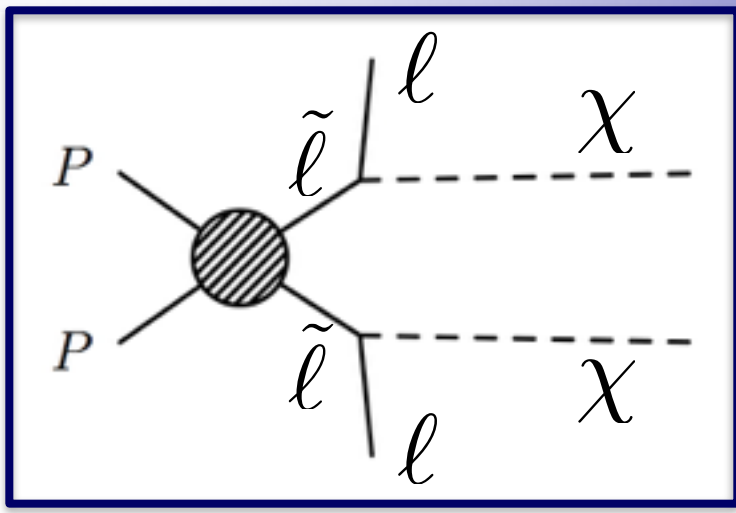

LSP 'test mass'

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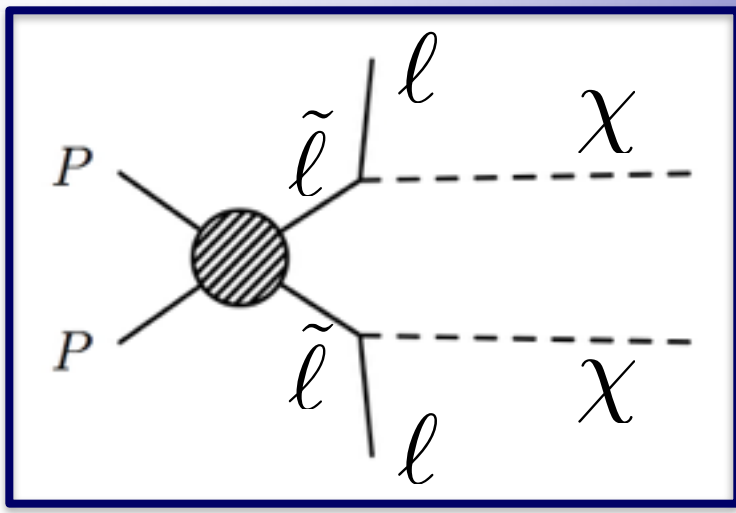

Extremization of unknown degrees of freedom

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Subject to constraints

with: $m_T^2(\vec{p}_T^{\,\ell_i}, \vec{p}_T^{\,\chi_i}, m_\chi) = m_\chi^2 + 2\left(E_T^{\ell_i} E_T^{\chi_i} - \vec{p}_T^{\,\ell_i} \cdot \vec{p}_T^{\,\chi_i}\right)$

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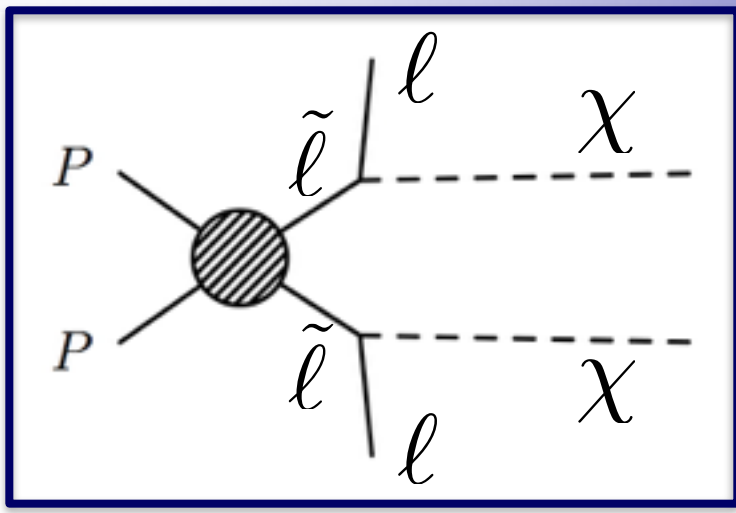

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Constructed to have a kinematic endpoint (with the right test mass) at: $M_{T2}^{\max}(m_\chi) = m_{\tilde{\ell}}$ $\quad M_{T2}^{\max}(0) = M_\Delta \equiv \dfrac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}}^2}{m_{\tilde{\ell}}}$

From:

C.G. Lester and D.J. Summers. Measuring masses of semiinvisibly decaying particles pair produced at hadron colliders. *Phys.Lett.*, B463:99–103, 1999.

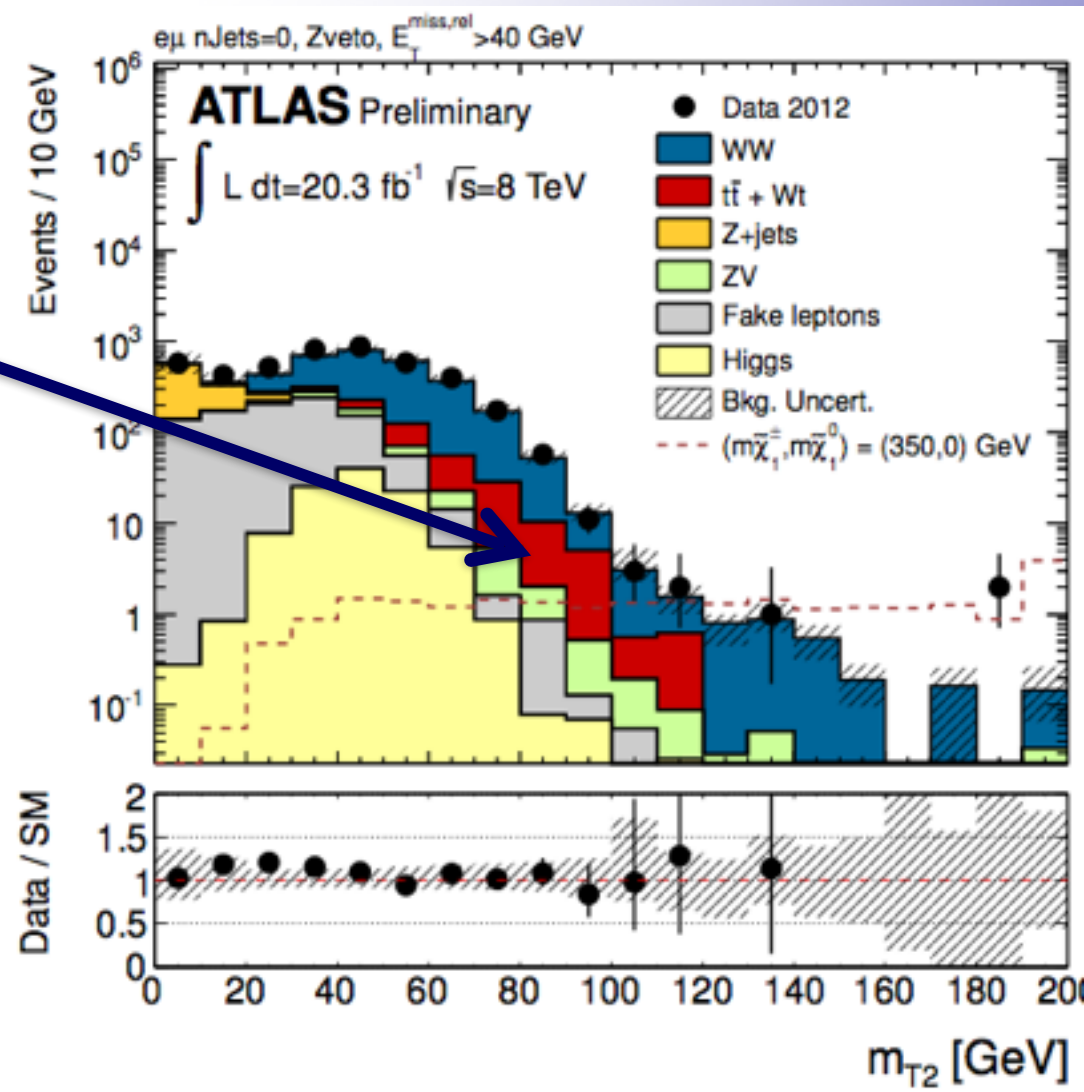
M_{T2} in practice

From:

ATLAS-CONF-2013-049

Backgrounds with leptonic W decays fall steeply once M_{T2} exceeds the W mass

Searches based on singularity variables have sensitivity to new physics signatures with mass splittings larger than the analogous SM ones



Recursive Jigsaw Reconstruction

New approach to reconstructing final states with weakly interacting particles: ***Recursive Jigsaw Reconstruction***

- The strategy is to transform observable momenta iteratively ***reference-frame to reference-frame***, traveling through each of the reference frames relevant to the topology
- At each step, ***extremize only the relevant d.o.f. related to that transformation***
- Repeat procedure recursively, using only the momenta encountered in each reference frame

- **Rather than obtaining one observable, get a *complete basis* of useful observables for each event**

Recursive rest-frame reconstruction

For two lepton case, these are the **'super-razor variables'**:

M. Buckley, J. Lykken, CR, M. Spiropulu, PRD 89, 055020 (2014)

$\ell_1^{lab}, \ell_2^{lab}$ Begin with reconstructed lepton 4-vectors in lab frame

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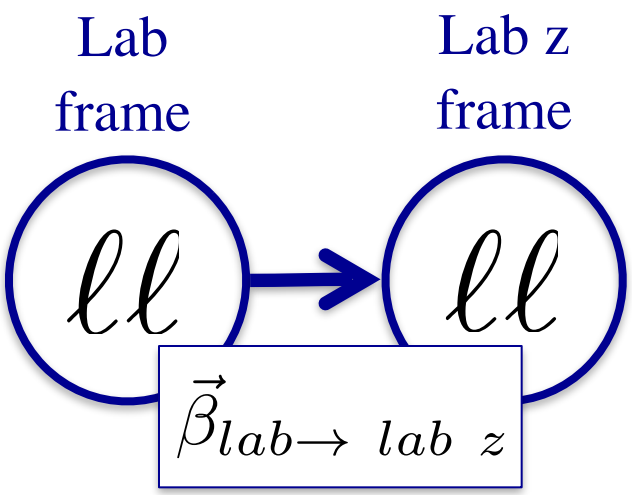
$$\frac{\partial(E_{\ell_1}^{lab\ z} + E_{\ell_2}^{lab\ z})}{\partial \beta_z} = 0 \rightarrow \beta_z$$

Remove dependence on unknown longitudinal boost by moving from 'lab' to 'lab z' frames



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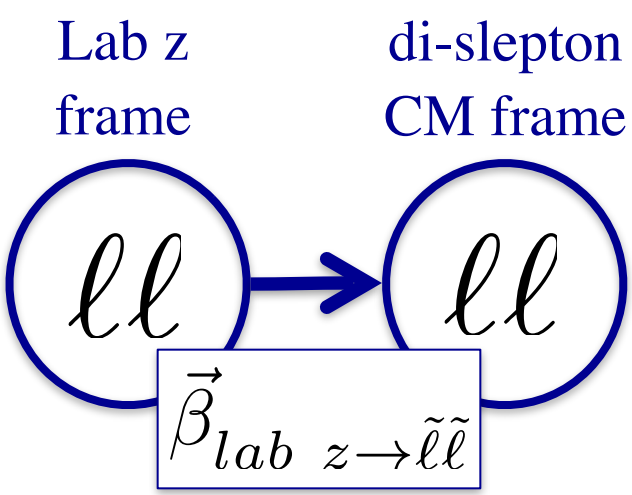
$$(\tilde{\chi}_1 + \tilde{\chi}_2)^2 = (\ell_1 + \ell_2)^2$$

Determine boost from 'lab z' to 'CM ($\tilde{\ell}\tilde{\ell}$)' frame by specifying Lorentz-invariant choice for invisible system mass



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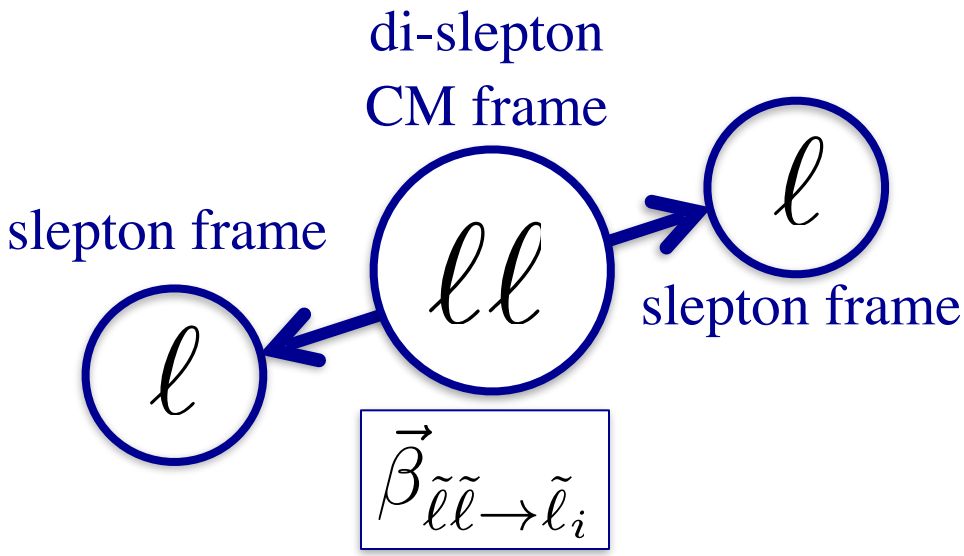

$$\frac{\partial(E_{\ell_1}^{\tilde{\ell}_1} + E_{\ell_2}^{\tilde{\ell}_2})}{\partial\vec{\beta}_{\tilde{\ell}\tilde{\ell}\to\tilde{\ell}_i}} = 0 \to \vec{\beta}_{\tilde{\ell}\tilde{\ell}\to\tilde{\ell}_i}$$

Determine asymmetric boost from CM to slepton rest frames by minimizing lepton energies in those frames

Recursive rest-frame reconstruction

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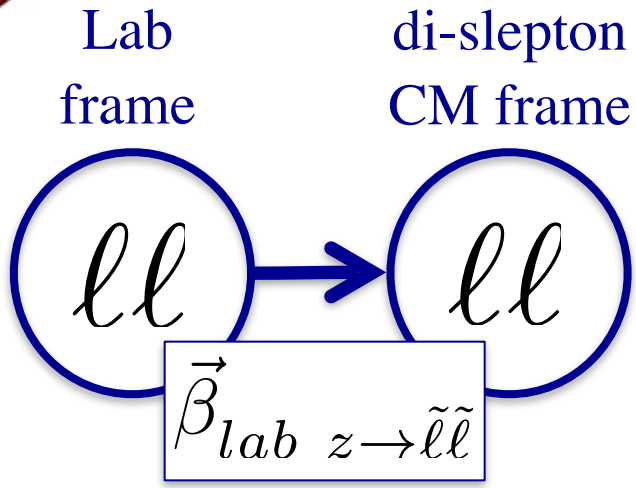
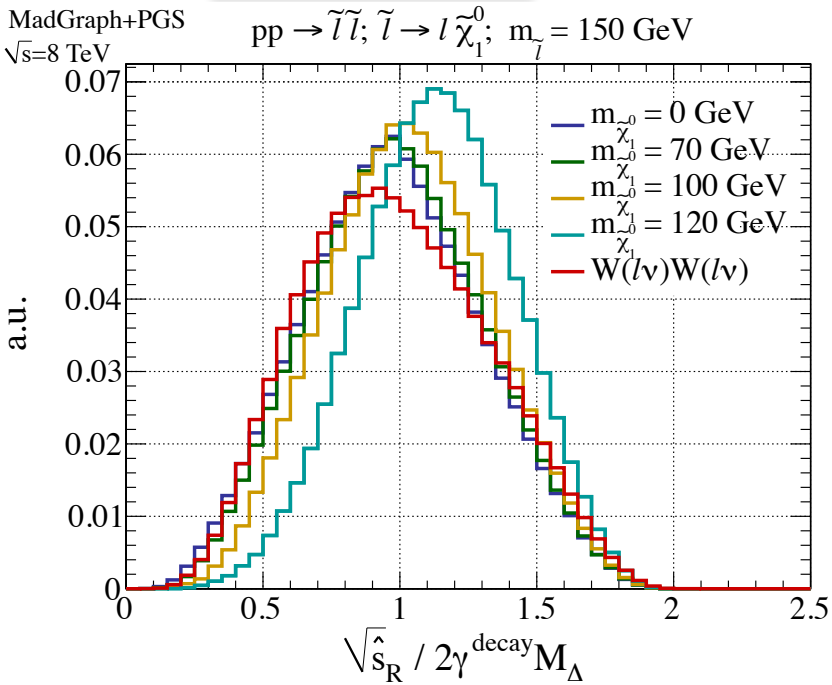
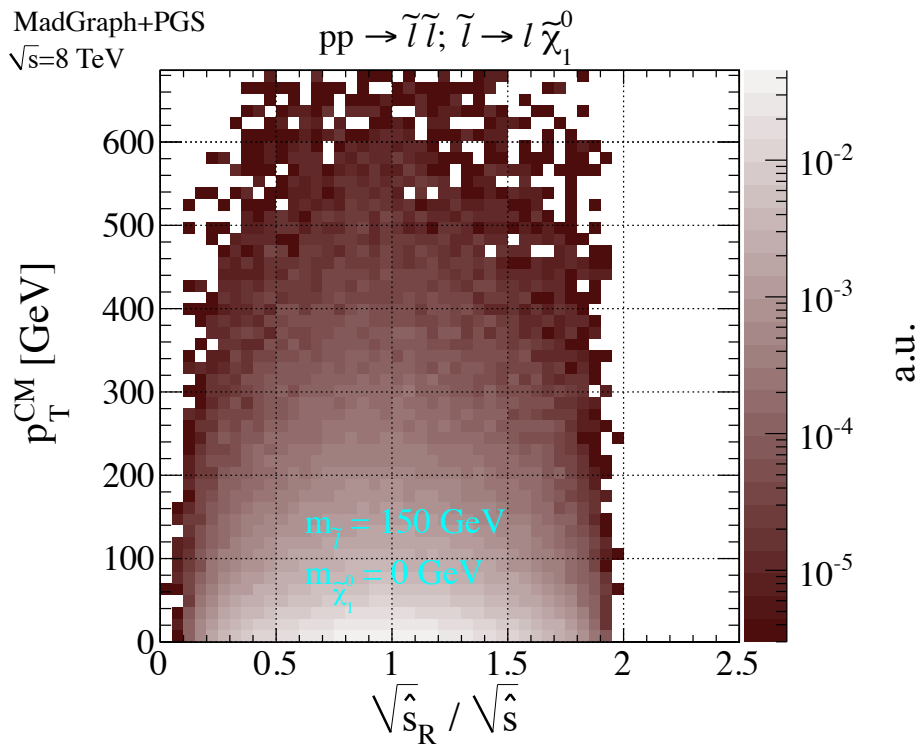


1st transformation: extract variable sensitive to invariant mass of total event: $\sqrt{\hat{s}_R}$

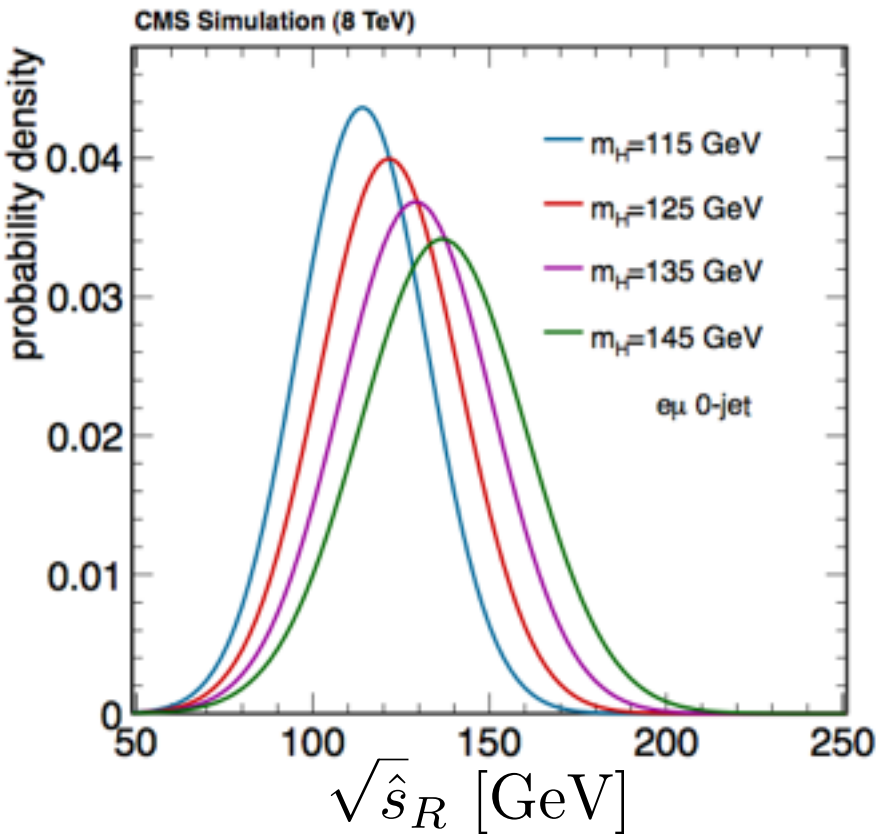
Resulting variable is ~invariant under p_T of di-slepton system





Resonant Higgs production

$$H \to WW^* \to 2\ell 2\nu$$

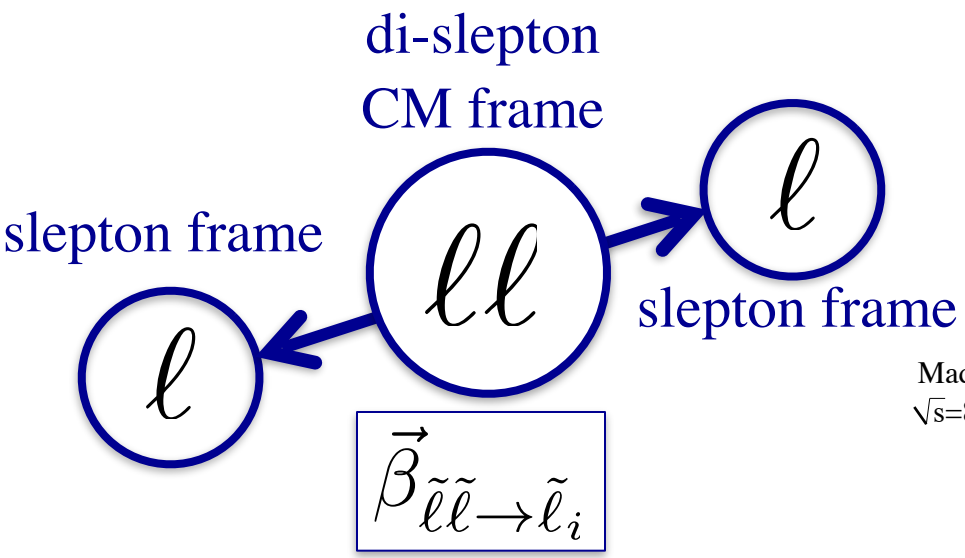


Using information from the two leptons, and the missing transverse momentum, the observable $\sqrt{\hat{s}_R}$ is directly sensitive to the Higgs mass

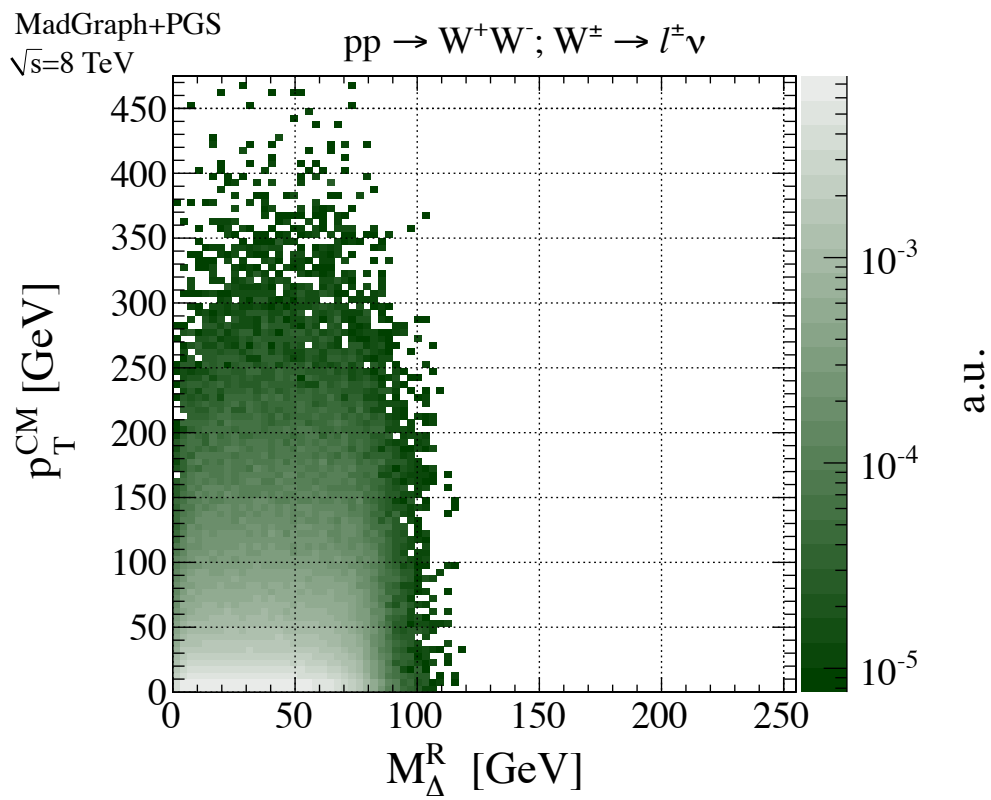
From:

CMS Collaboration, *Measurement of Higgs boson production and properties in the WW decay channel with leptonic final states*, arXiv:1312.1129v1 [hep-ex]

Recursive rest-frame reconstruction



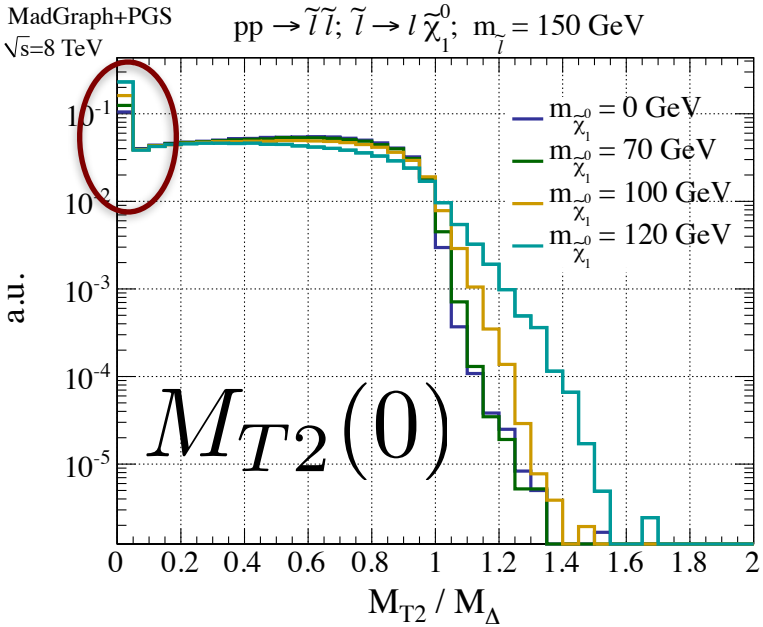
2nd transformation(s): extract variable sensitive to invariant mass of squark: M_Δ^R

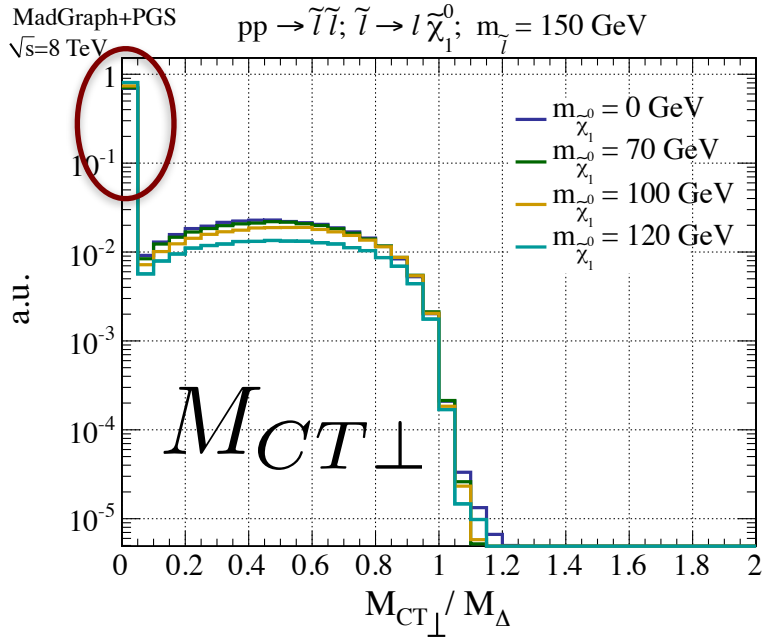


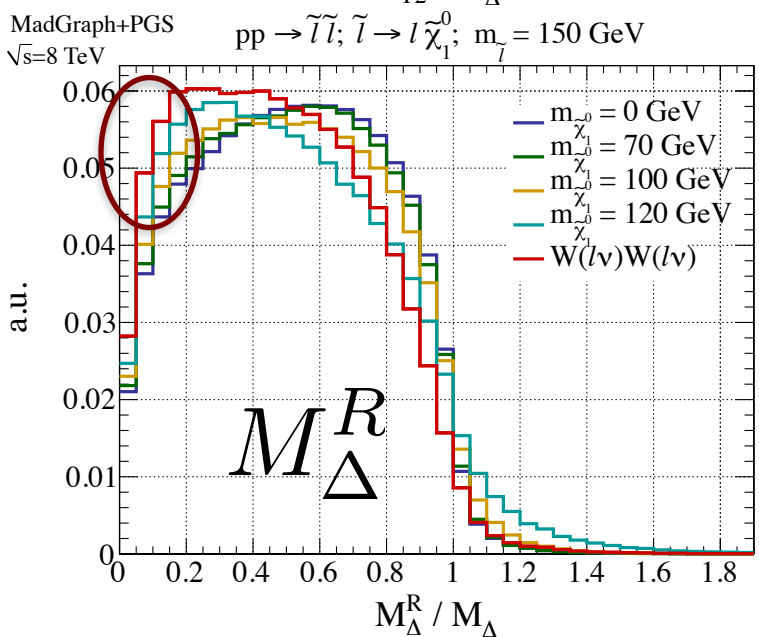
Resulting variable has kinematic endpoint at:

$$M_\Delta \equiv \frac{m_{\tilde{l}}^2 - m_{\tilde{\chi}^0}^2}{m_{\tilde{l}}}$$

Variable comparison







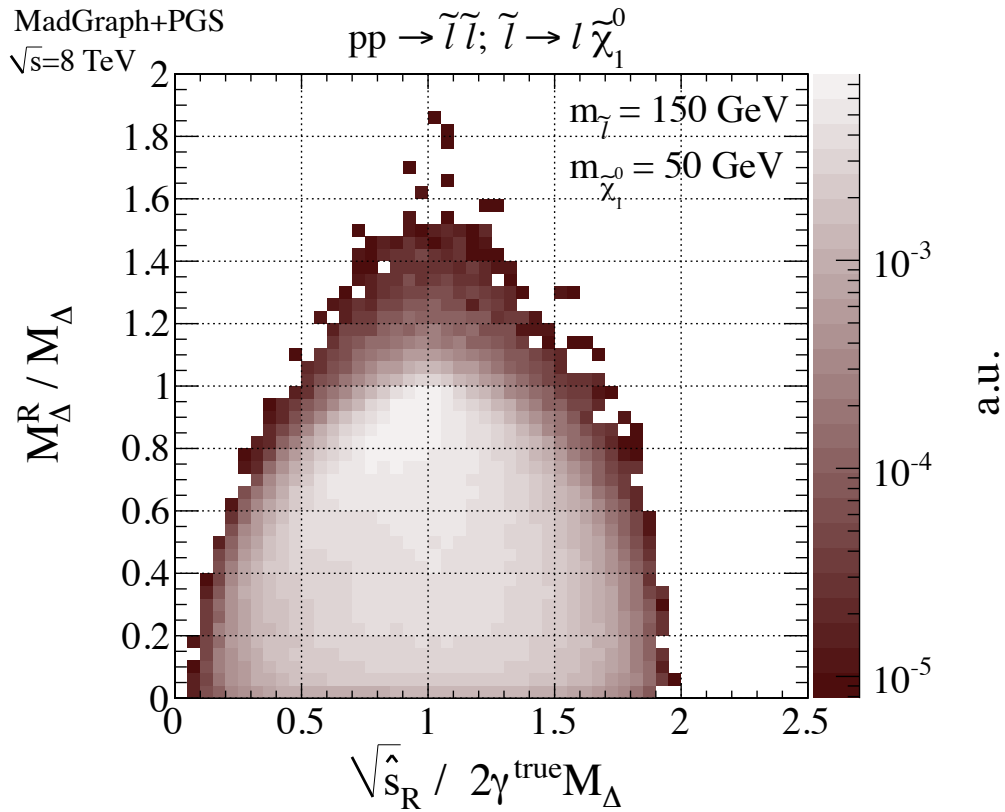
Three different singularity variables, all attempting to measure the same thing

$$M_\Delta^R \geq M_{T2}(0) \geq M_{CT\perp}$$

More details about variable comparisons in PRD 89, 055020 (arXiv:1310.4827) and backup slides

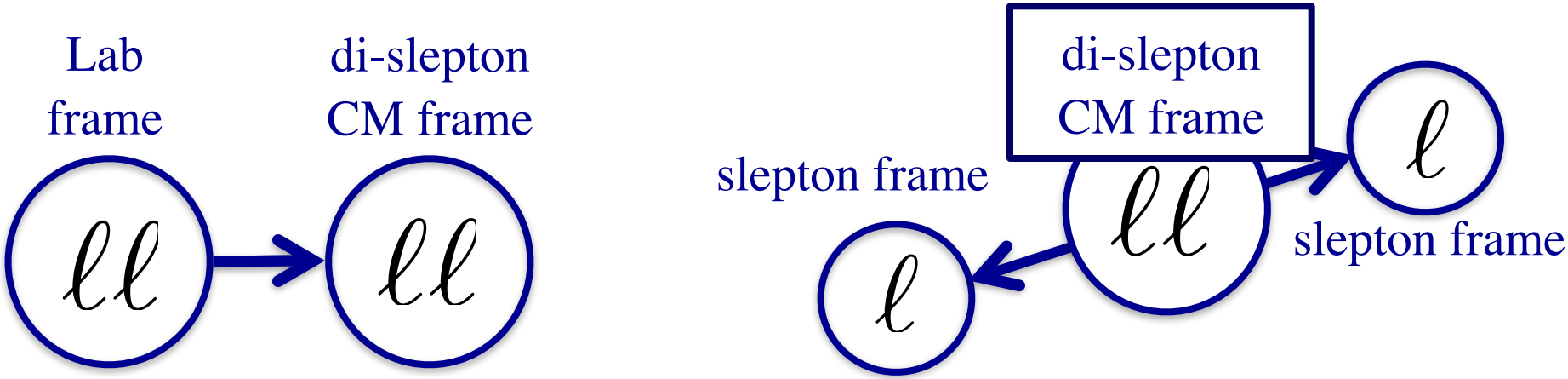
But what else can we calculate?

With recursive scheme can extract the two mass scales $\sqrt{\hat{s}}_R$ and M_Δ^R almost completely independently



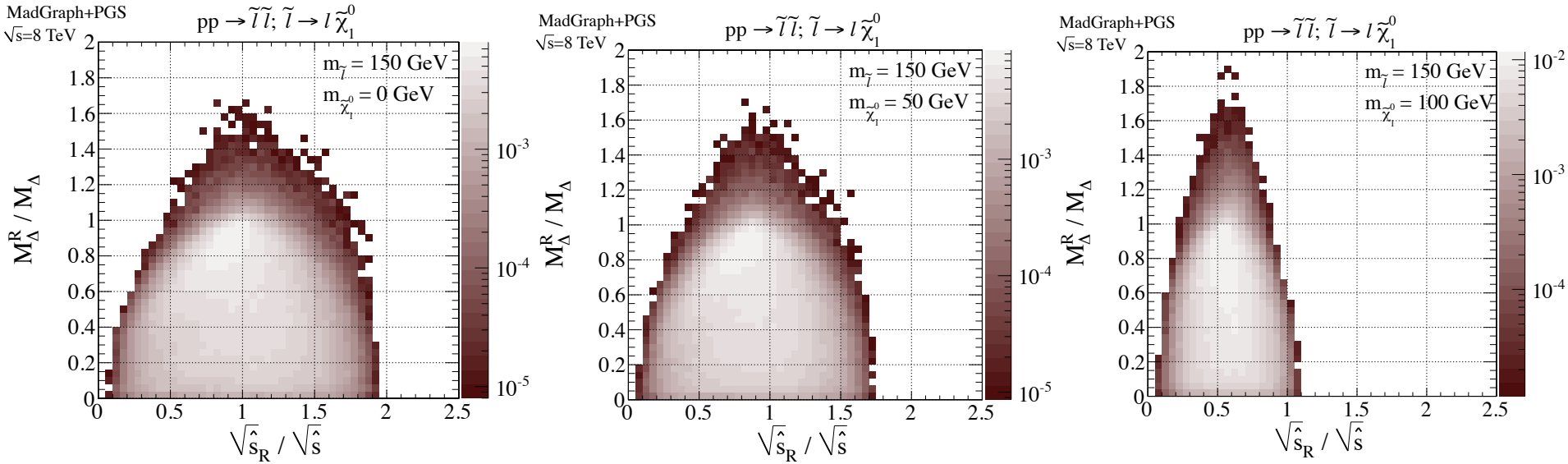
Angles, angles, angles…

Recursive scheme fully specifies approximate event decay chain, also yielding angular observables



Two transformations mean at least two independent angles of interest (essentially the decay angle of the state whose rest-frame you are in)

Towards a kinematic basis



but $\sqrt{\hat{s}}_R \sim 2\gamma^{decay} M_\Delta$

while $\sqrt{\hat{s}} = 2\gamma^{decay} m_{\tilde{\ell}}$

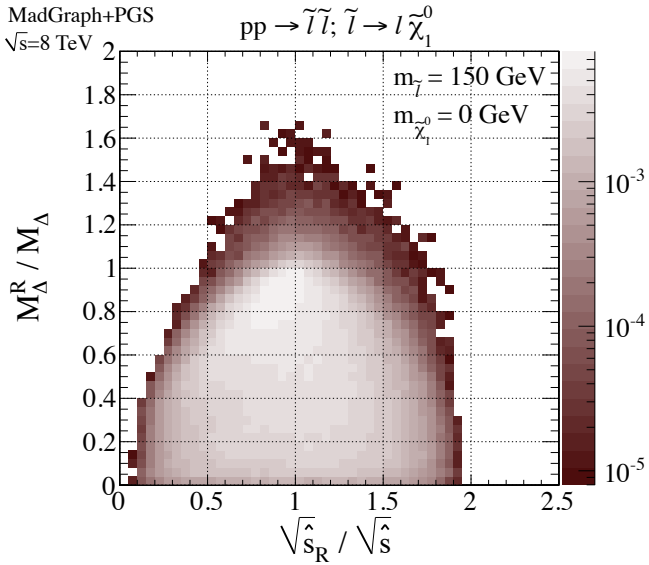
$$\vec{p}_T^{\,CM} = \vec{p}_T^{\,\ell_1} + \vec{p}_T^{\,\ell_2} + \vec{E}_T^{\,\mathrm{miss}}$$

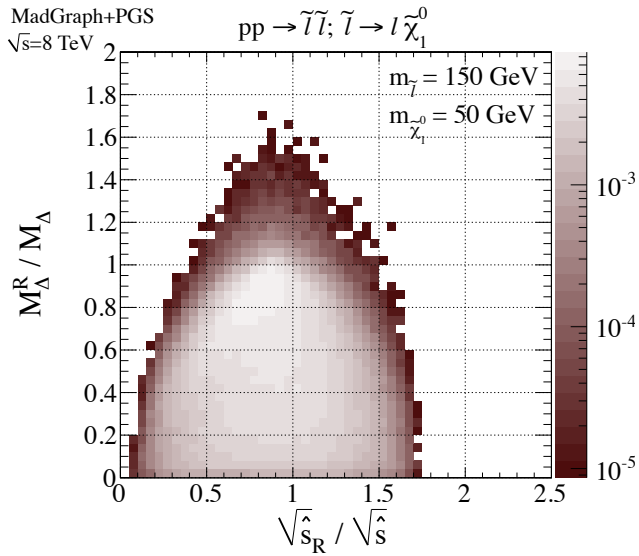
Underestimating the real mass means over-estimating the boost magnitude:

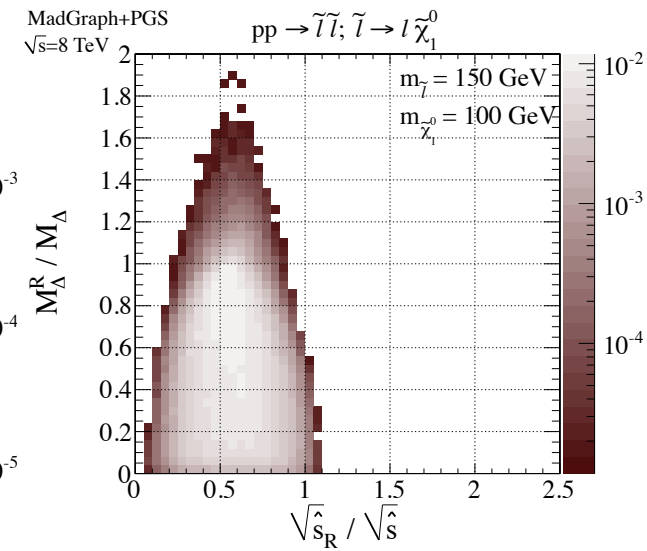
$$\vec{\beta}_{lab\to CM} = \frac{\vec{p}_T^{\,CM}}{\sqrt{|\vec{p}_T^{\,CM}|^2 + \hat{s}}}$$

From PRD 89, 055020 (2014)

Towards a kinematic basis







but $\sqrt{\hat{s}}_R \sim 2\gamma^{decay} M_\Delta$

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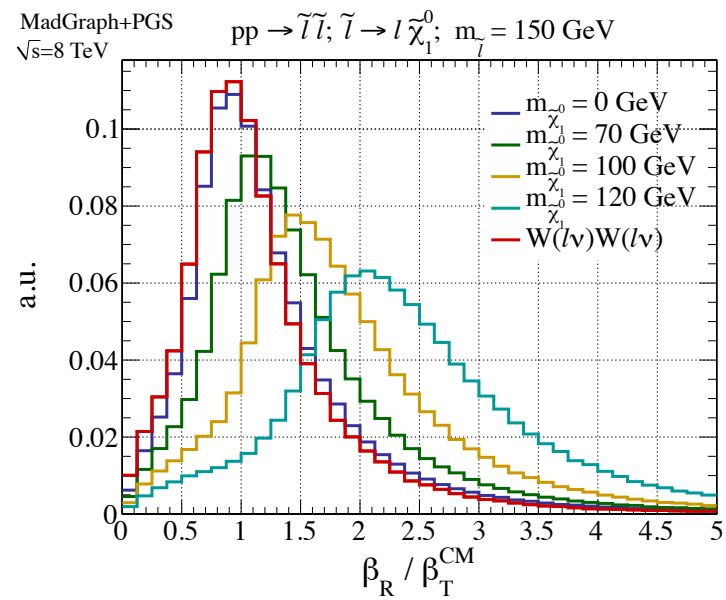
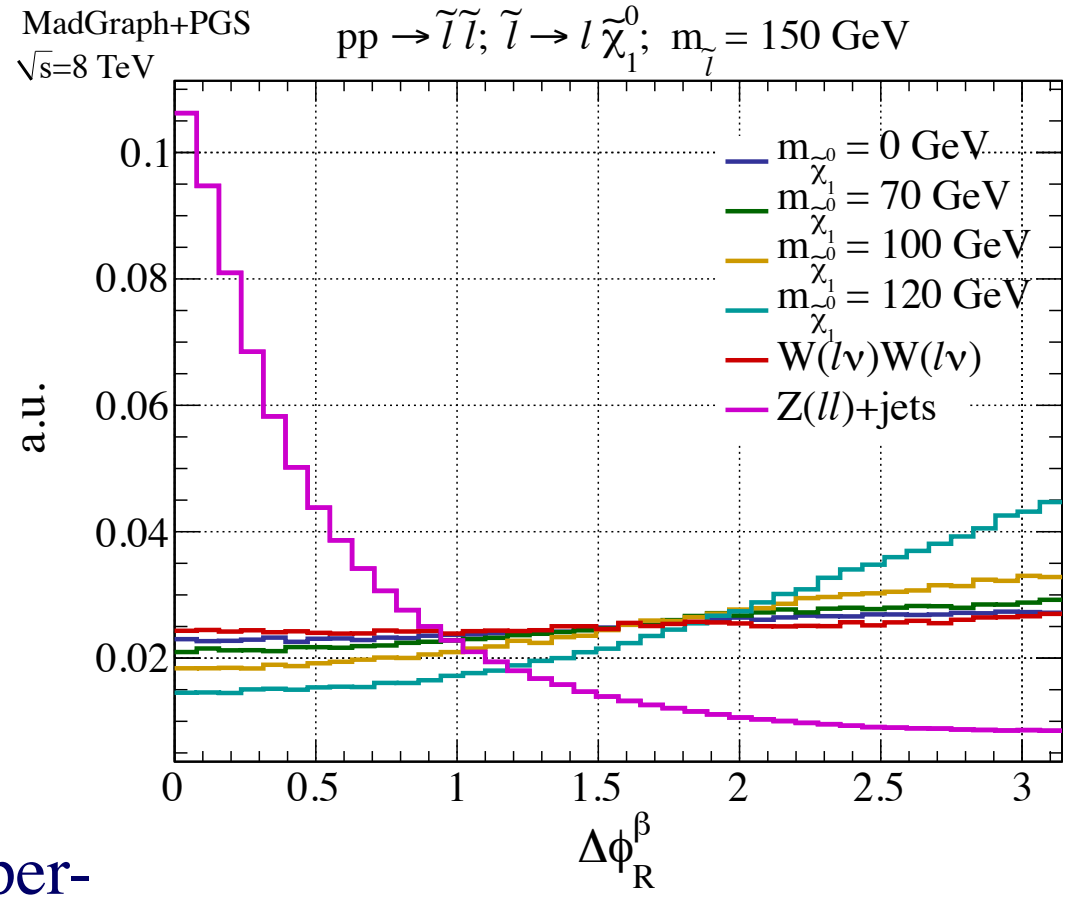


From PRD 89, 055020 (2014)

Angular Variables

Incorrect boost magnitude
induces correlation

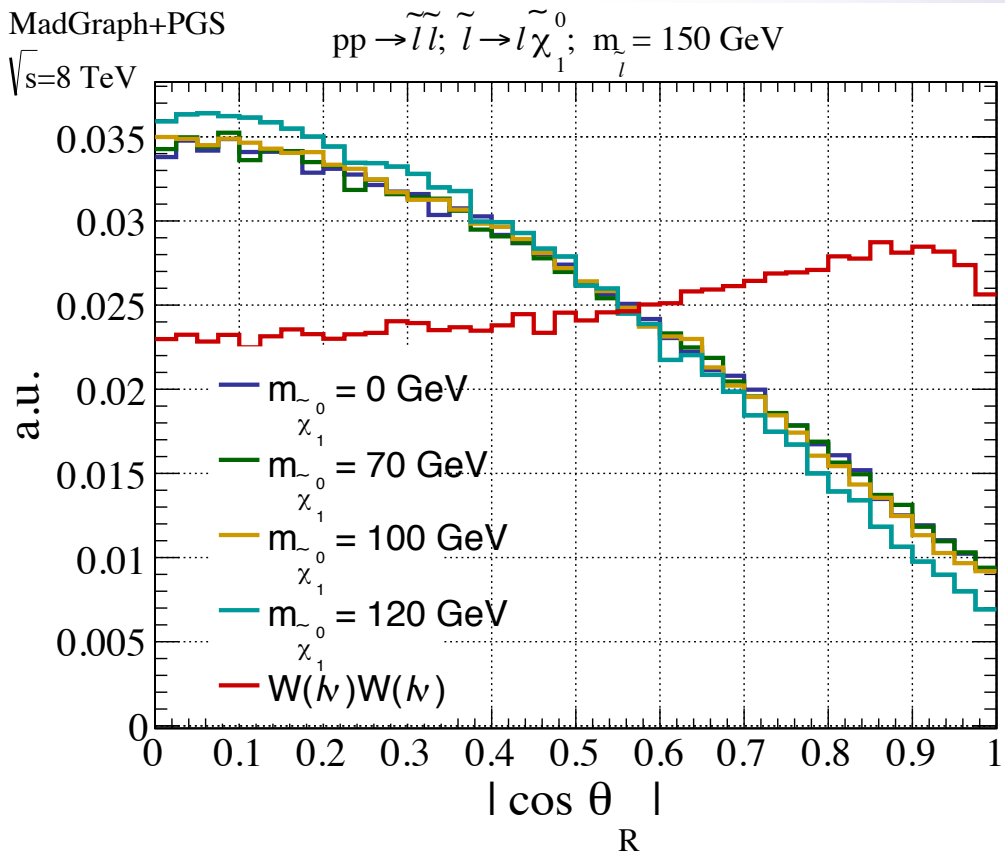


Angle between
lab ➔ CM frame boost and
di-leptons in CM frame is
sensitive to

$$\frac{m_\chi}{m_{\tilde{\ell}}} \text{ rather than } M_\Delta$$

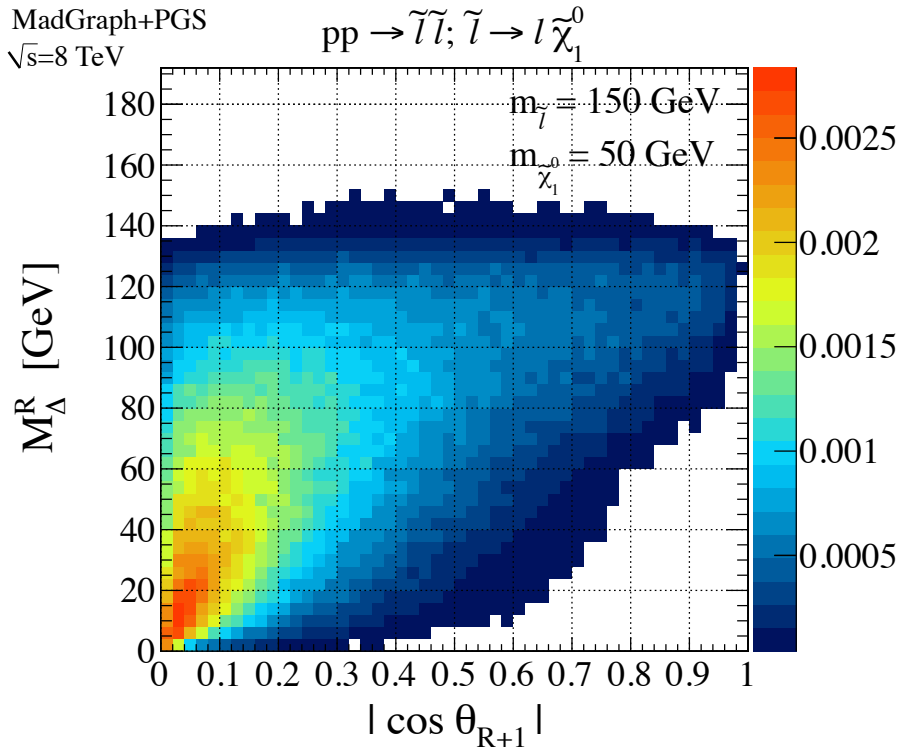
~Uncorrelated with other super-razor variables

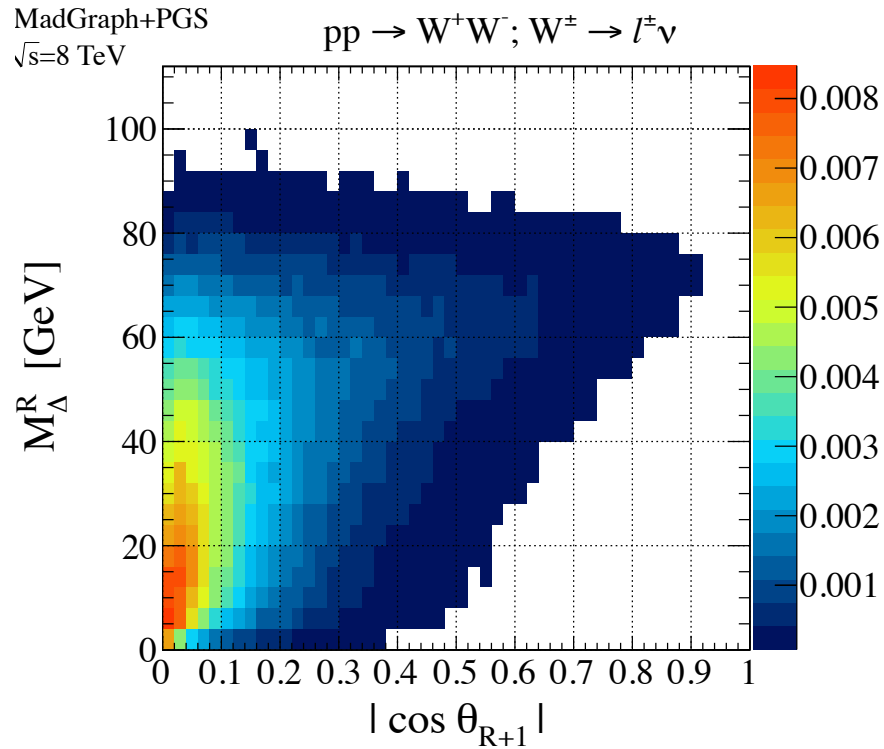
Angular Variables



In the approximate di-slepton rest frame, reconstructed decay angle sensitive to particle spin and production

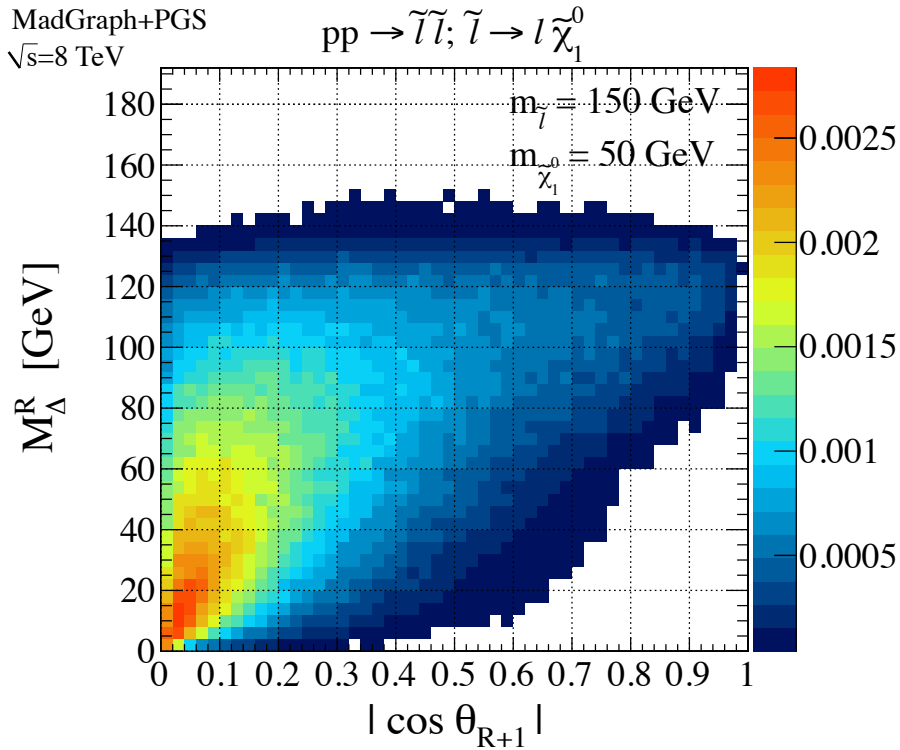
Angular Variables

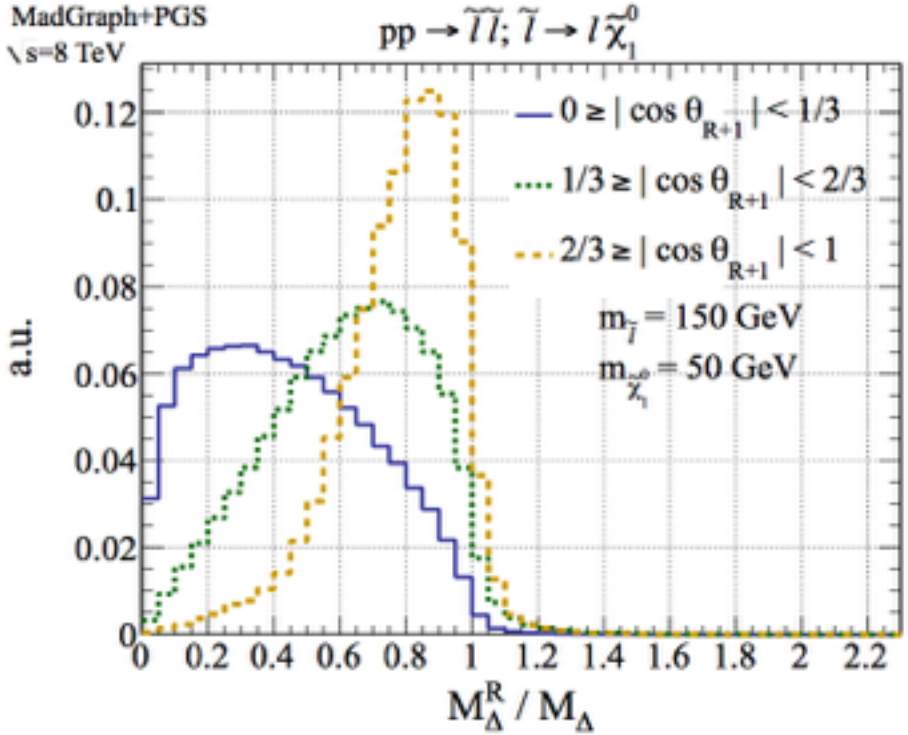




In the approximate slepton rest frames, reconstructed slepton decay angle sensitive to particle spin correlations

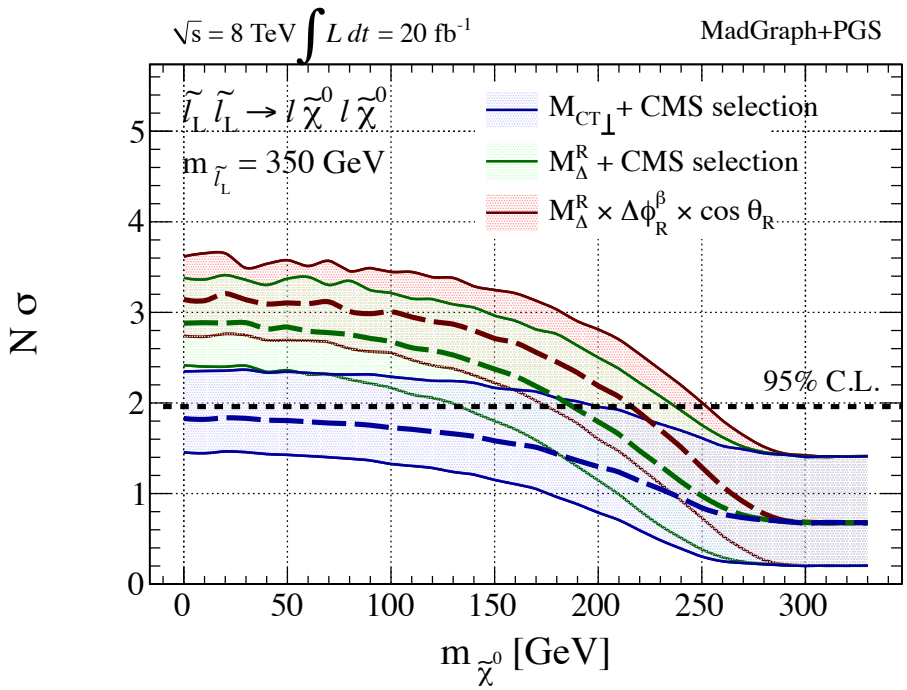
Angular Variables





Also allows us to better resolve the kinematic endpoint of interest

Super-razor variable basis



Can re-imagine a di-lepton analysis in new basis of variables

Can improve sensitivity while removing MET cuts!

$\Delta\phi^\beta_R$ Sensitive to ratio of invisible and visible masses

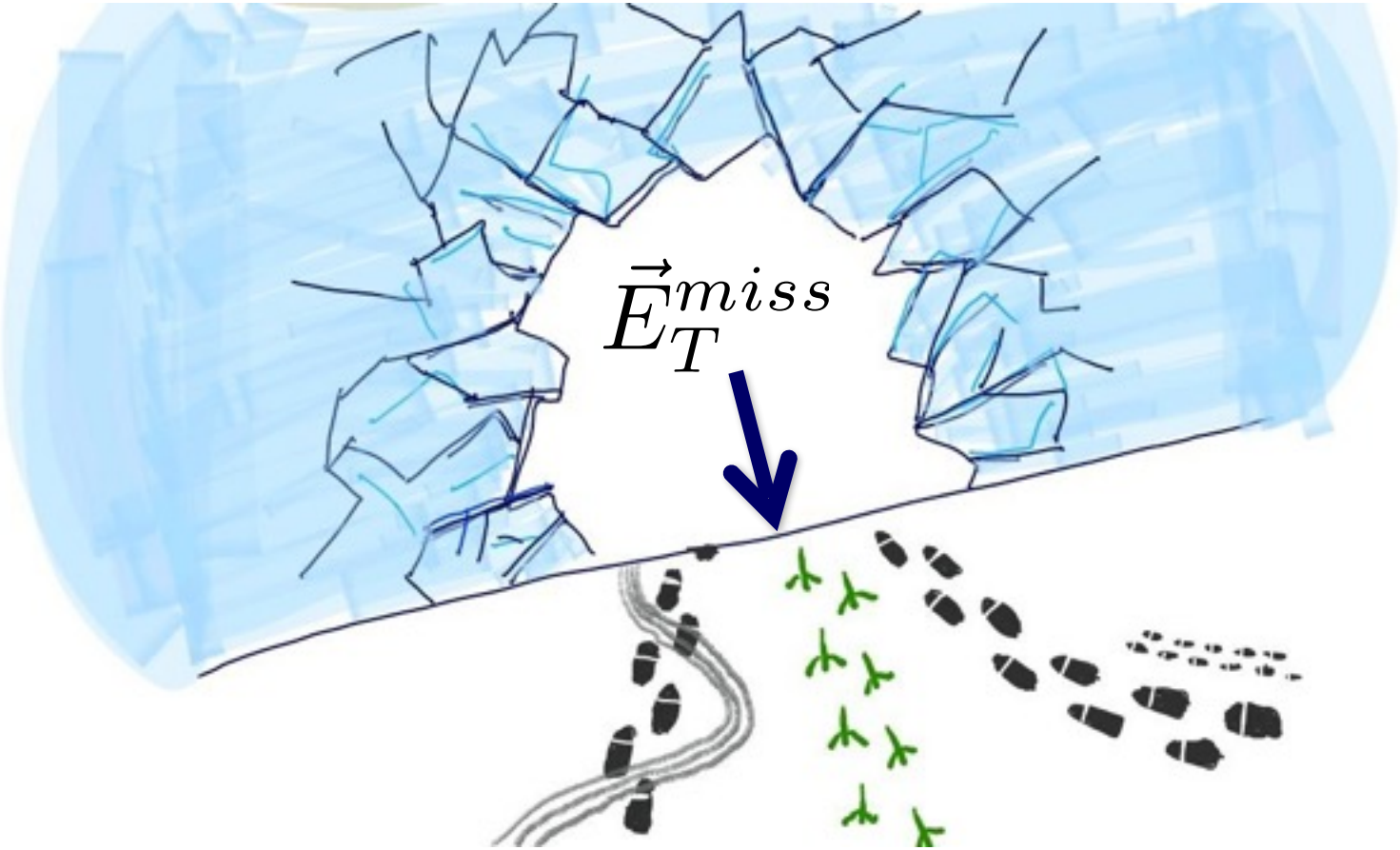
$\sqrt{\hat{s}}_R$ Sensitive to mass of CM Good for resonant production of heavy parents

$|\cos\theta_R|$ Spin and production

M^R_Δ Mass-squared difference resonant/non-resonant prod.

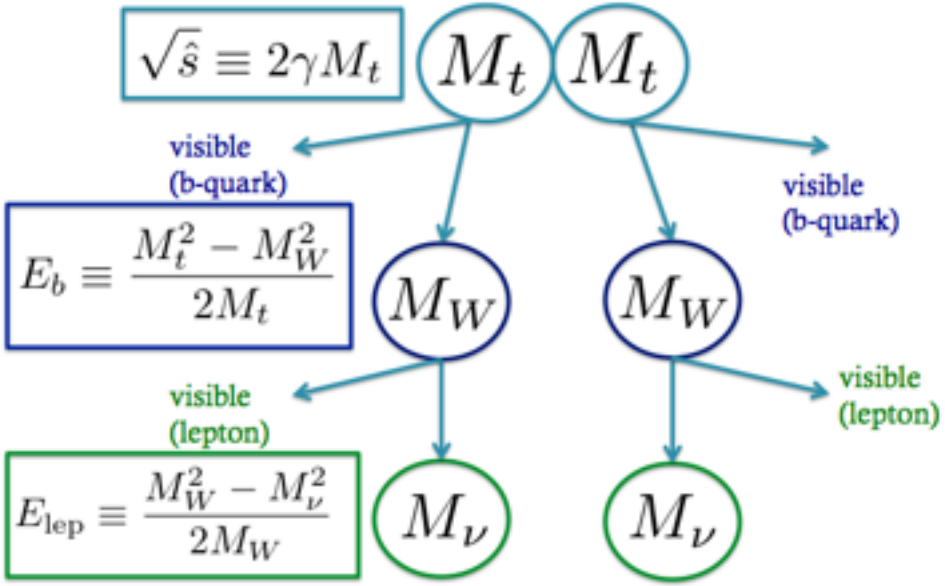
$|\cos\theta_{R+1}|$ Spin correlations, better resolution of mass edge

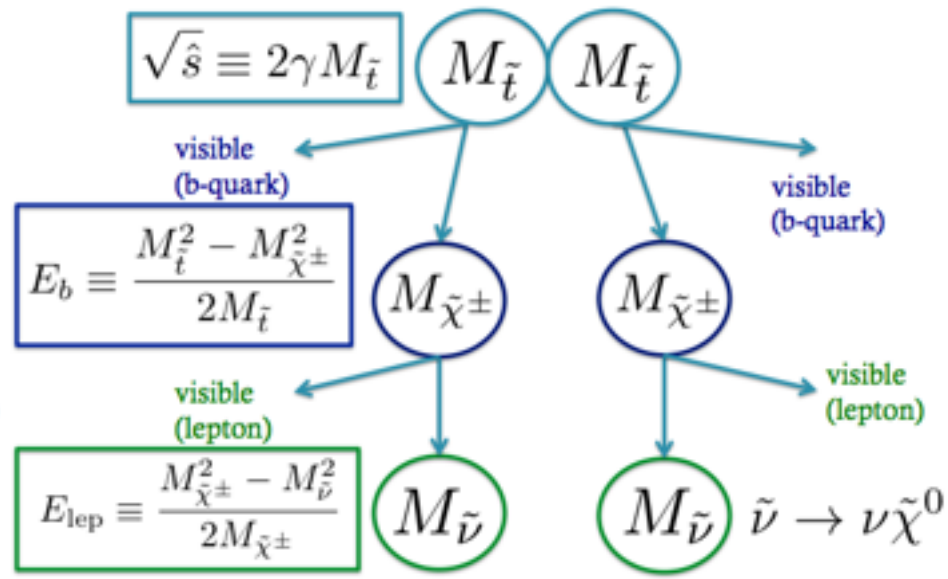
Generalizing further



Recursive Jigsaw approach can be generalized to arbitrarily complex final states with weakly interacting particles

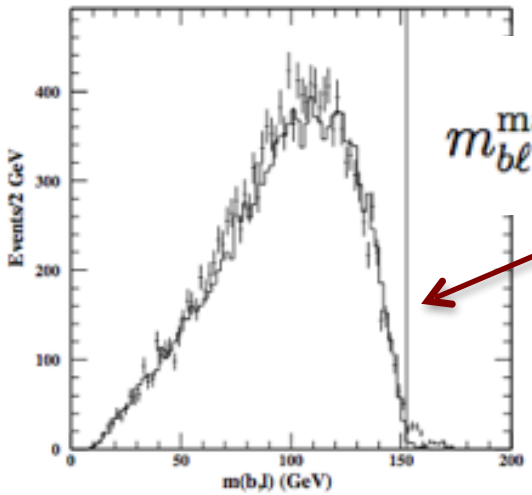
Example: the di-leptonic top basis



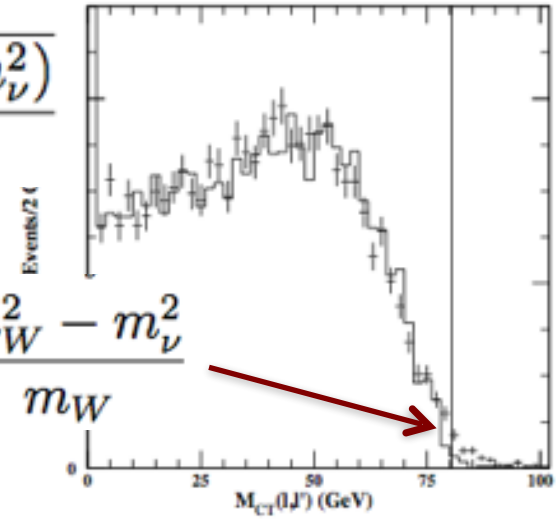


In more complicated decay topologies there can be many masses/mass-splittings, spin-sensitive angles and other observables of interest that can be used to distinguish between the SM and SUSY signals

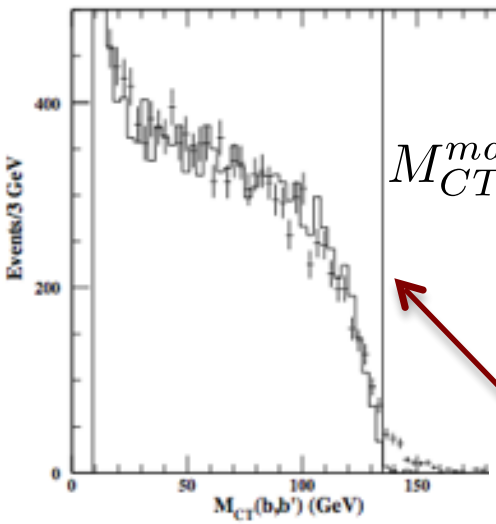
Singularity Variables for Tops



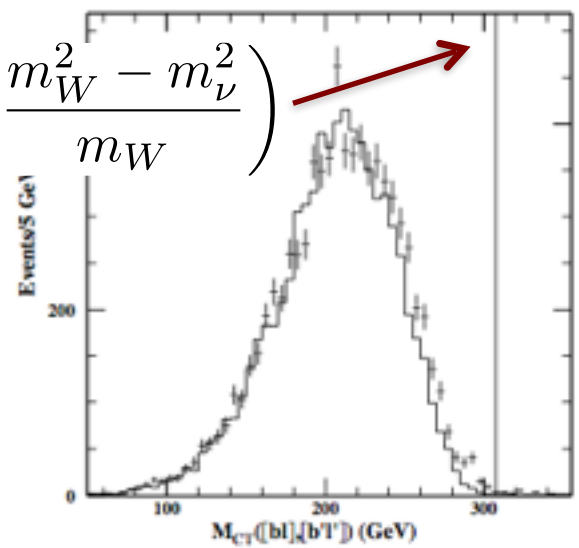
$$m_{b\ell}^{\max} = \frac{\sqrt{(m_t^2 - m_W^2)(m_W^2 - m_\nu^2)}}{m_W}$$



$$M_{CT}^{\max}(\ell_1, \ell_2) = \frac{m_W^2 - m_\nu^2}{m_W}$$



$$M_{CT}^{max}(b_1\ell_1, b_2\ell_2) = \frac{m_t^2 - m_W^2}{m_t} + \frac{m_t}{m_W}\left(\frac{m_W^2 - m_\nu^2}{m_W}\right)$$



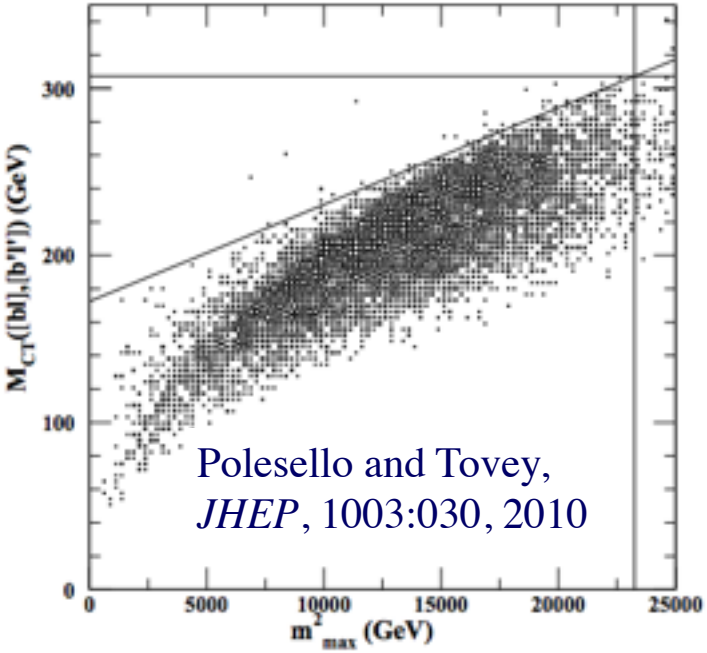
$$M_{CT}^{\max}(b_1, b_2) = \frac{m_t^2 - m_W^2}{m_t}$$

Giacomo Polesello and Daniel R. Tovey. Supersymmetric particle mass measurement with the boost-corrected contransverse mass. *JHEP*, 1003:030, 2010.

Singularity Variables for Tops

If there are multiple masses or mass splittings of interest in the event, appearing at different points in the decay chains, then singularity variables correspond to different extremizations of missing d.o.f.

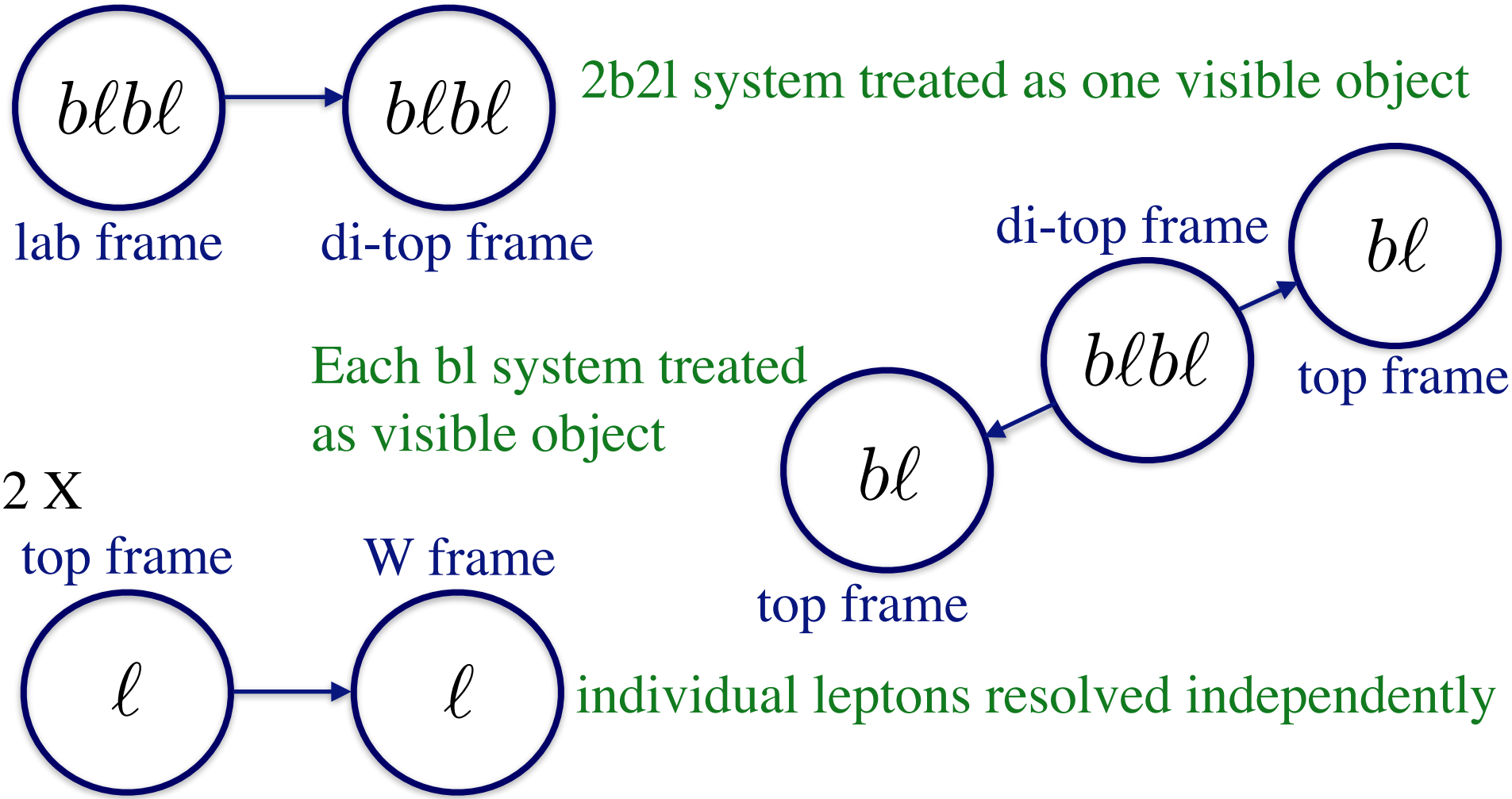
What are the correlations between the different singularity variables estimating different mass splittings? Can be large/ complicated when using M_{CT} or M_{T2} in this case



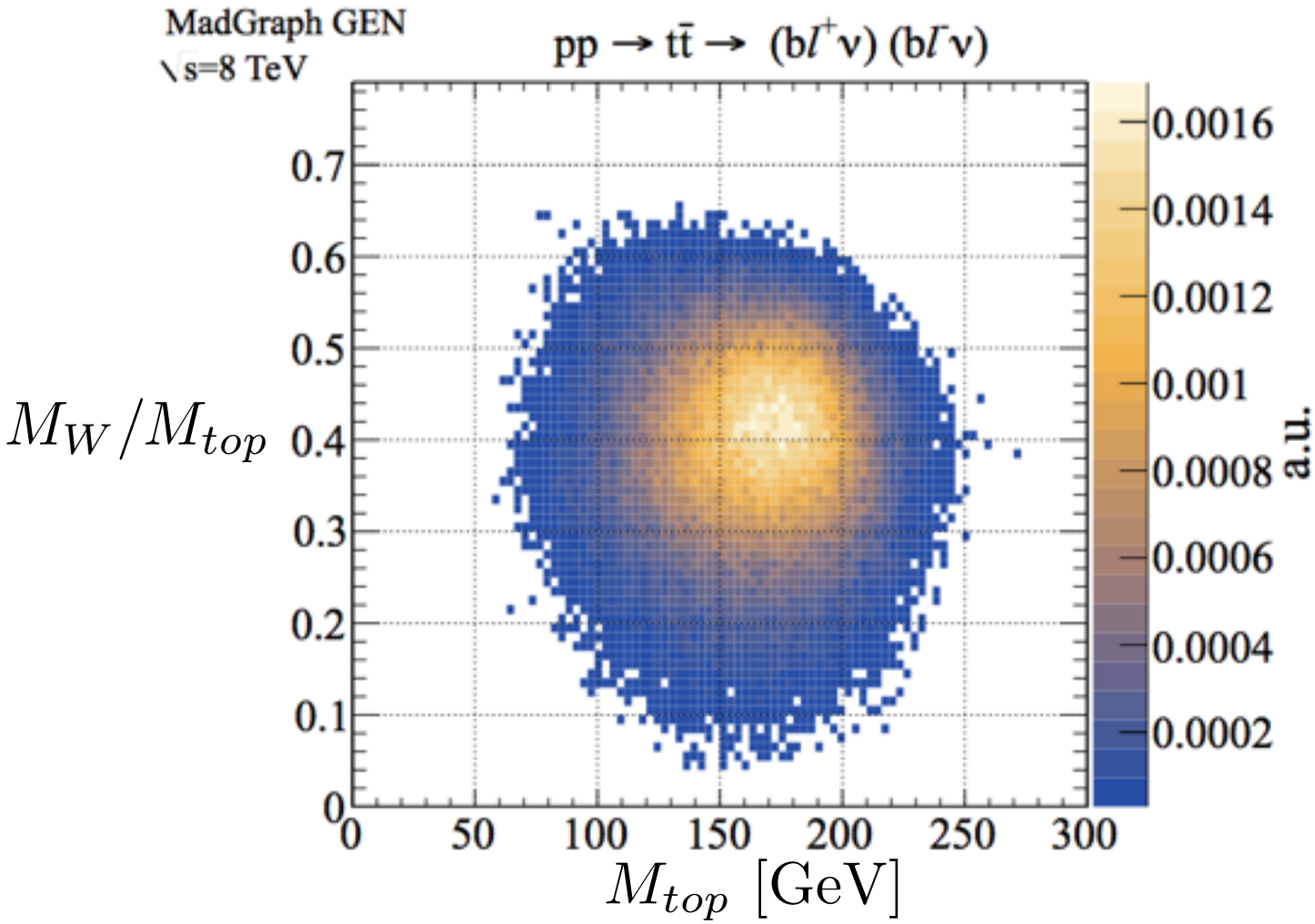
What if the two tops/stops are decaying differently through different particles?

Recursive Jigsaw reconstruction

Move through each reference frame of interest in the event, specifying only d.o.f. relevant to each transformation:

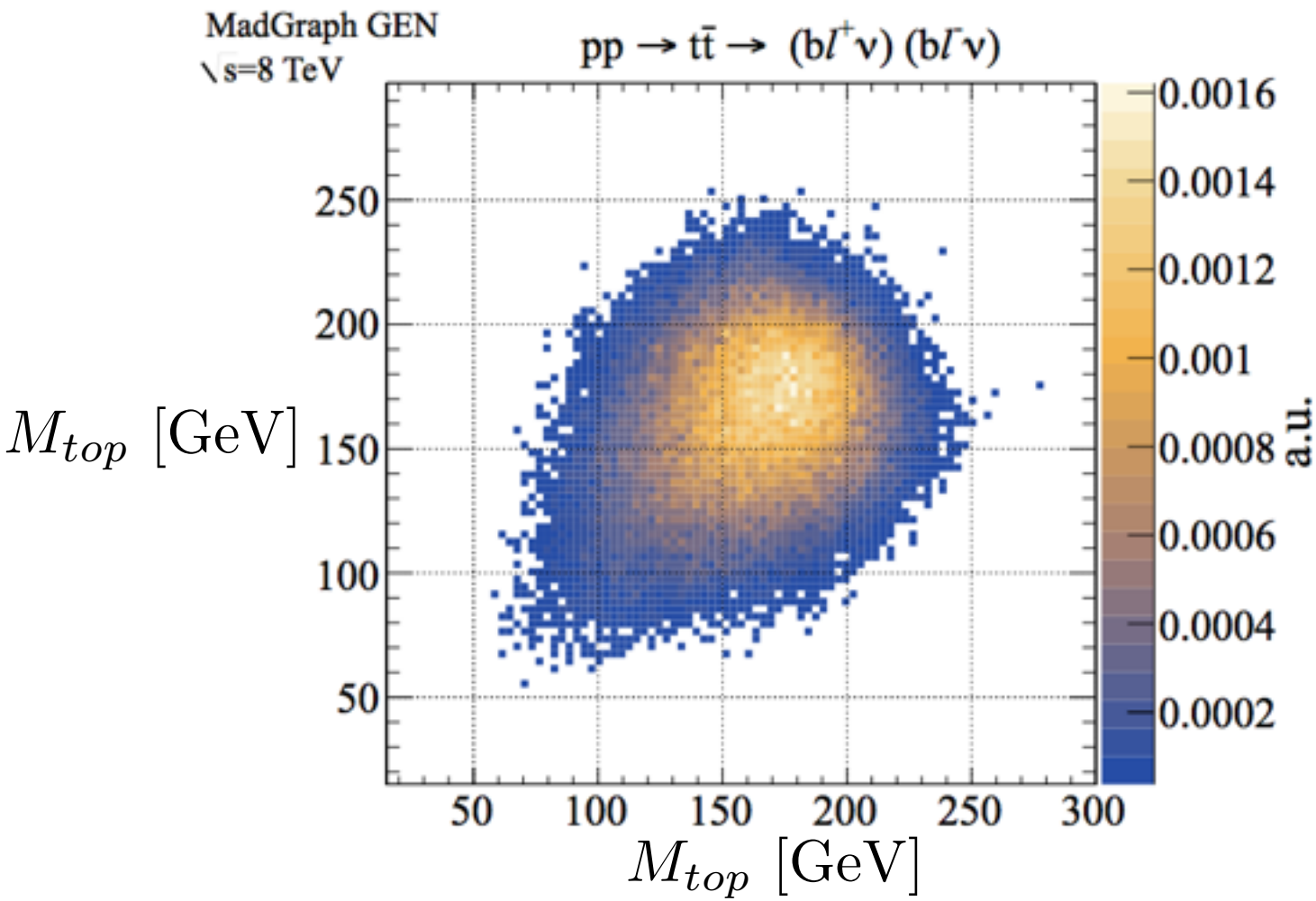


Recursive Jigsaw reconstruction



The scales can be extracted independently

Recursive Jigsaw reconstruction



In fact the scales can be extracted independently for each top – the reconstruction chains are ***decoupled***

The di-leptonic top basis

$$M_{t\bar{t}},\ \vec{p}_{t\bar{t}}, \cos\theta_{TT}$$

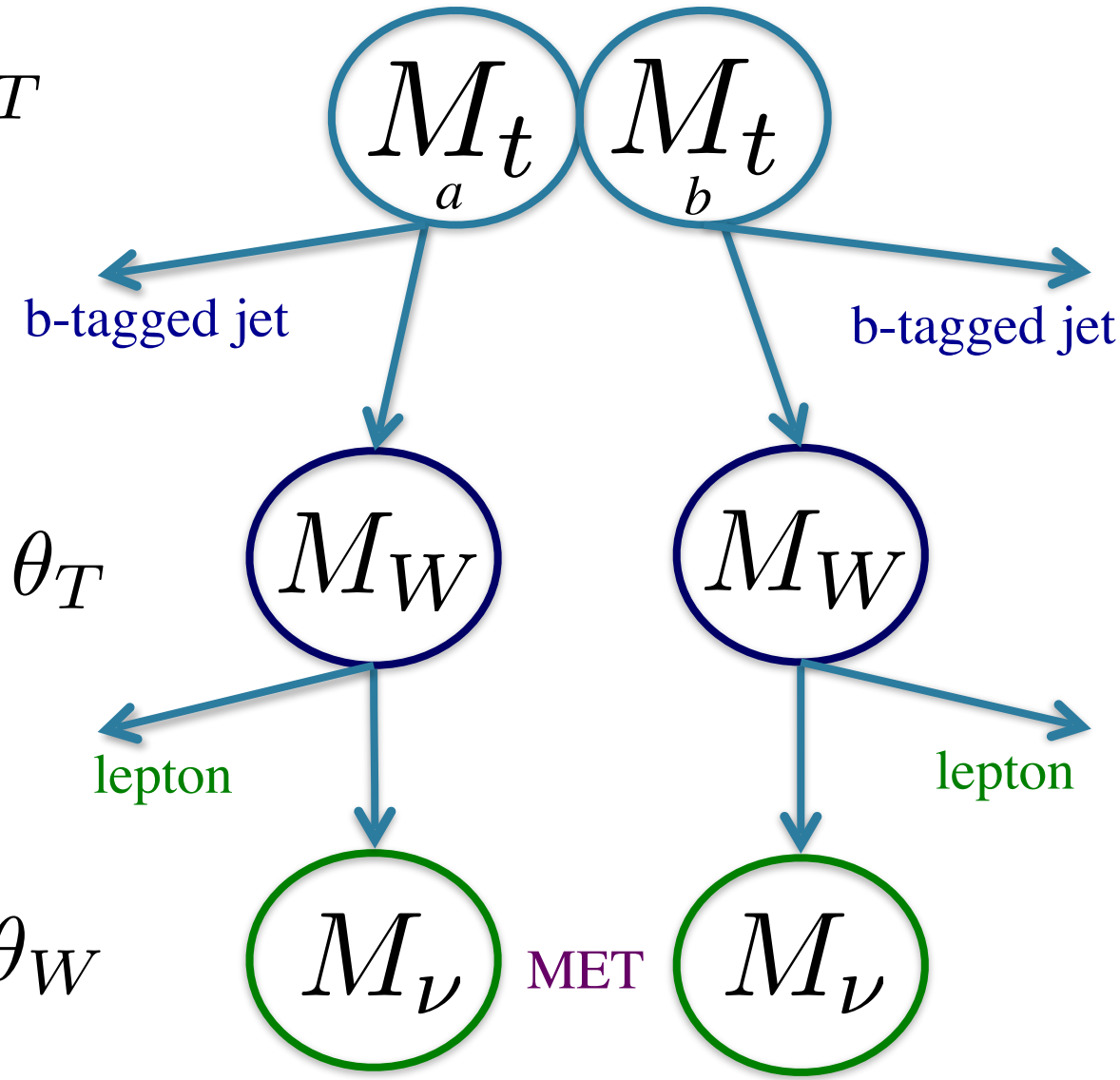
$$\Delta\phi_{T1,T2}$$

2 X

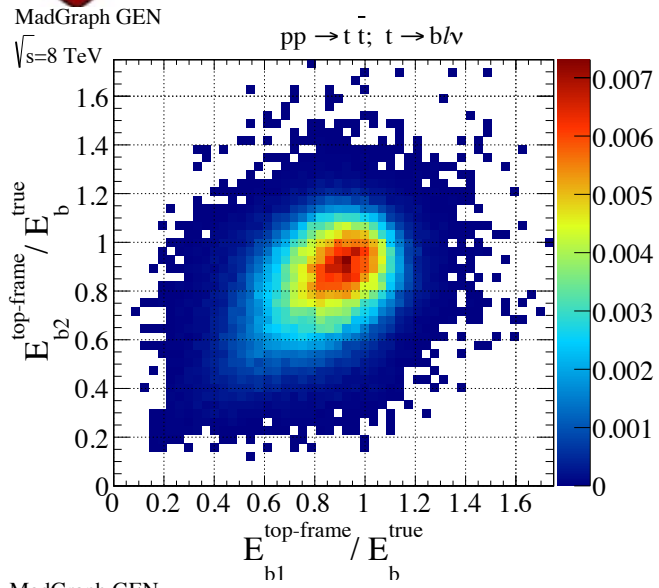
$$E_b^{\mathrm{top-frame}}, \cos\theta_T$$

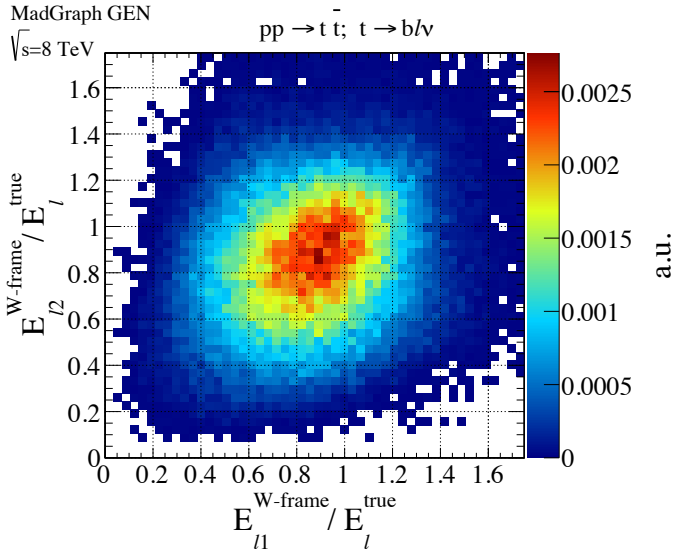
$$\Delta\phi_{T,W}$$

$$E_\ell^{\mathrm{W-frame}}, \cos\theta_W$$

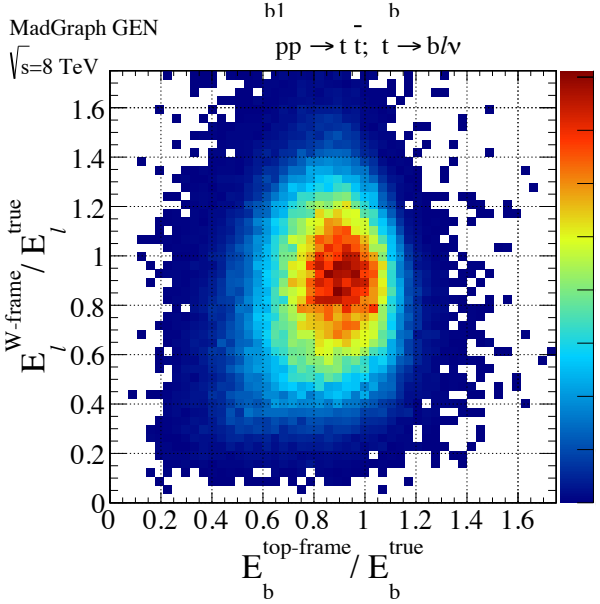


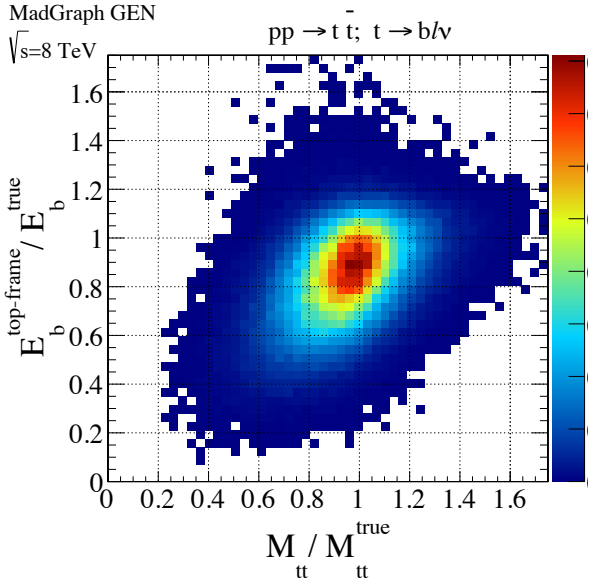
The di-leptonic top basis

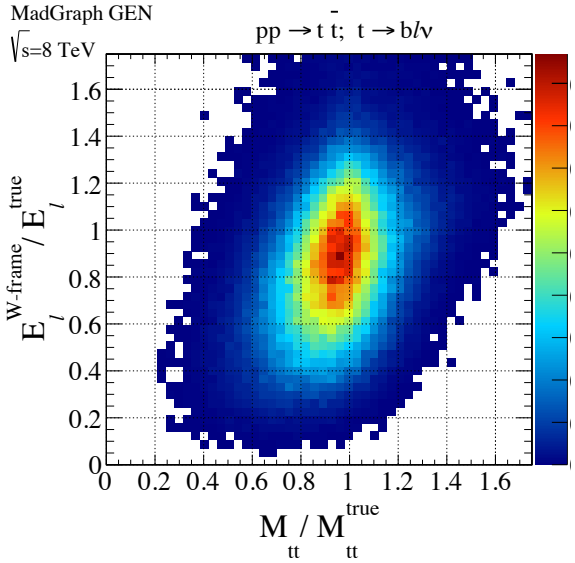




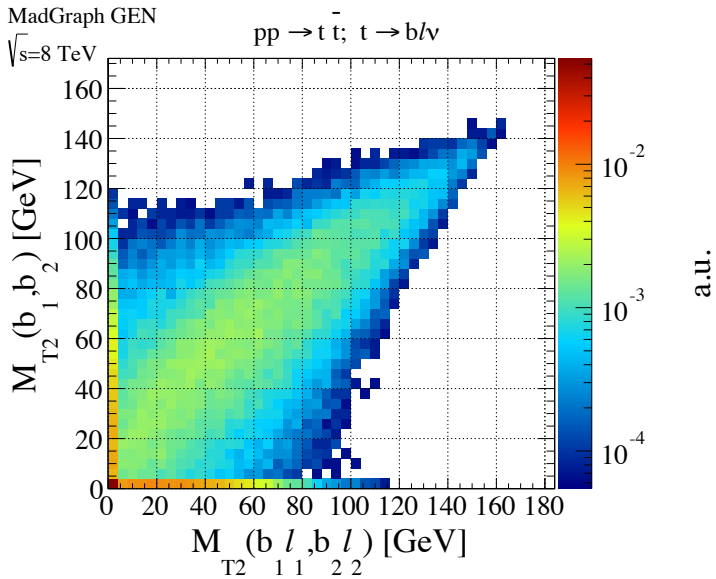
largely independent information about five different masses



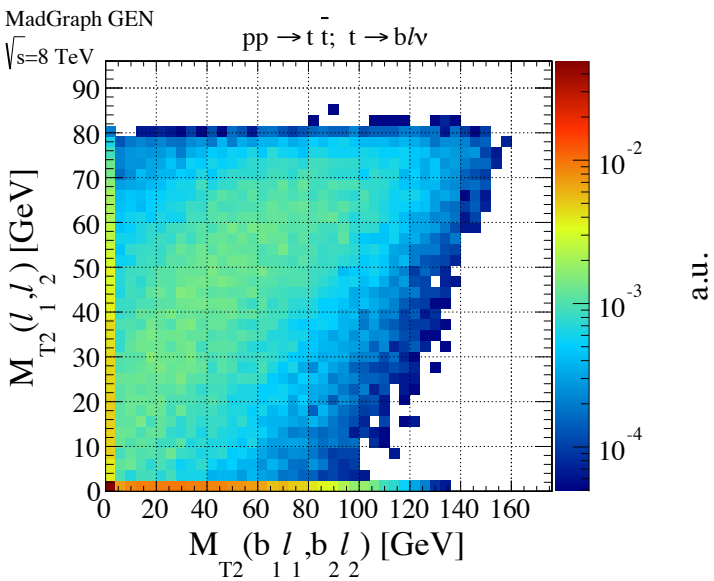


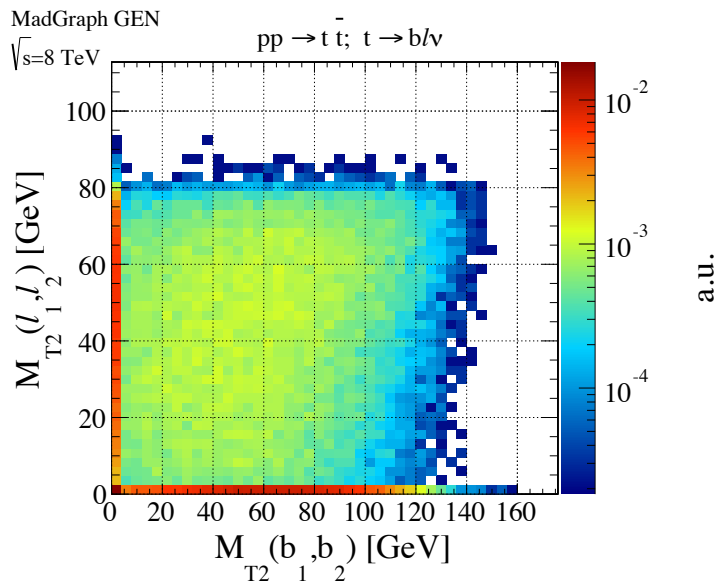


Previous state-of-the-art



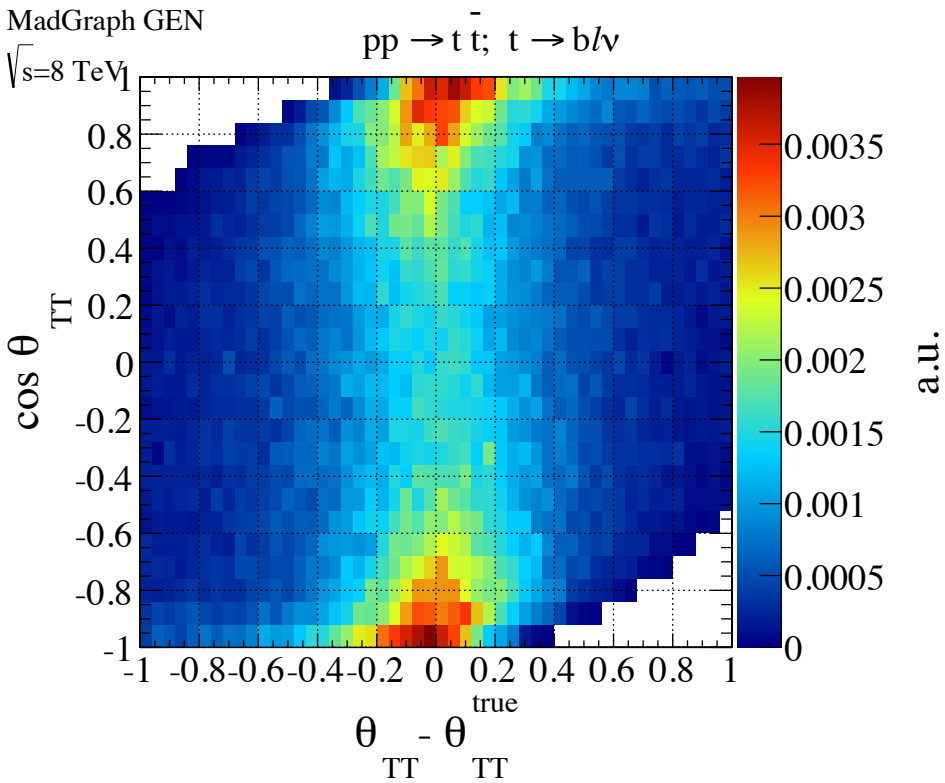
Mass-sensitive singularity variables are not necessarily independent

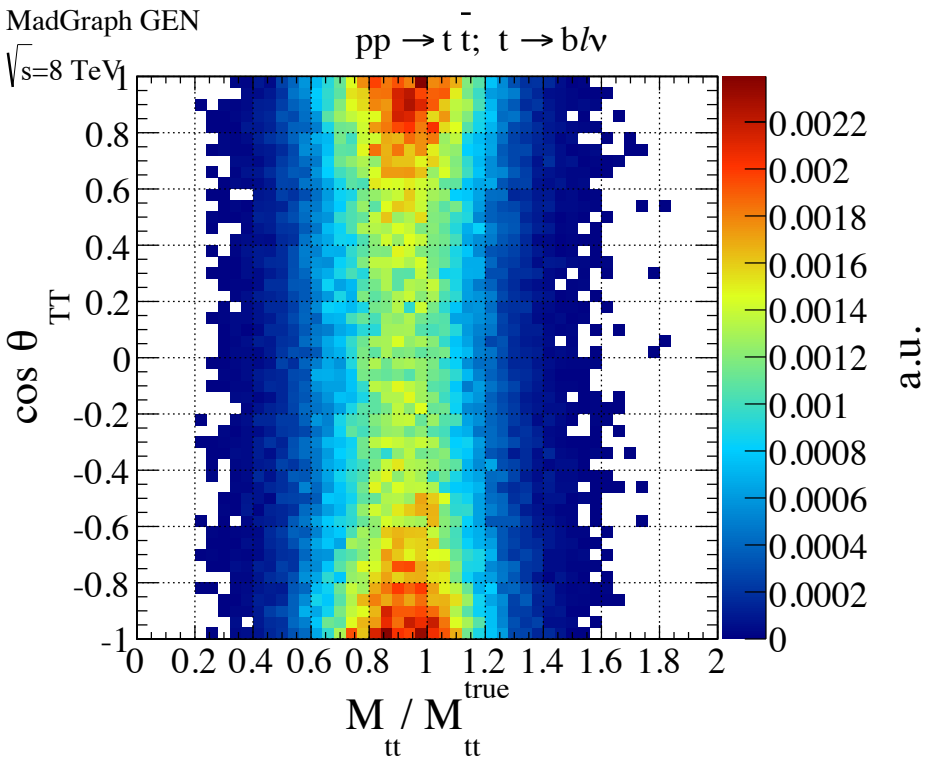




The di-leptonic top basis

largely independent information about decay angles

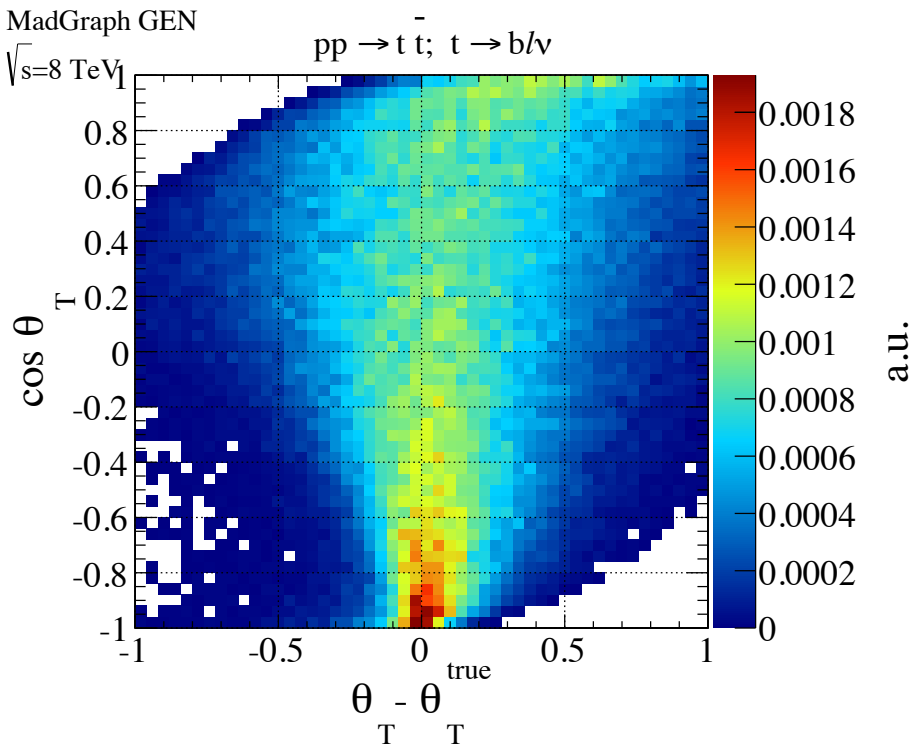


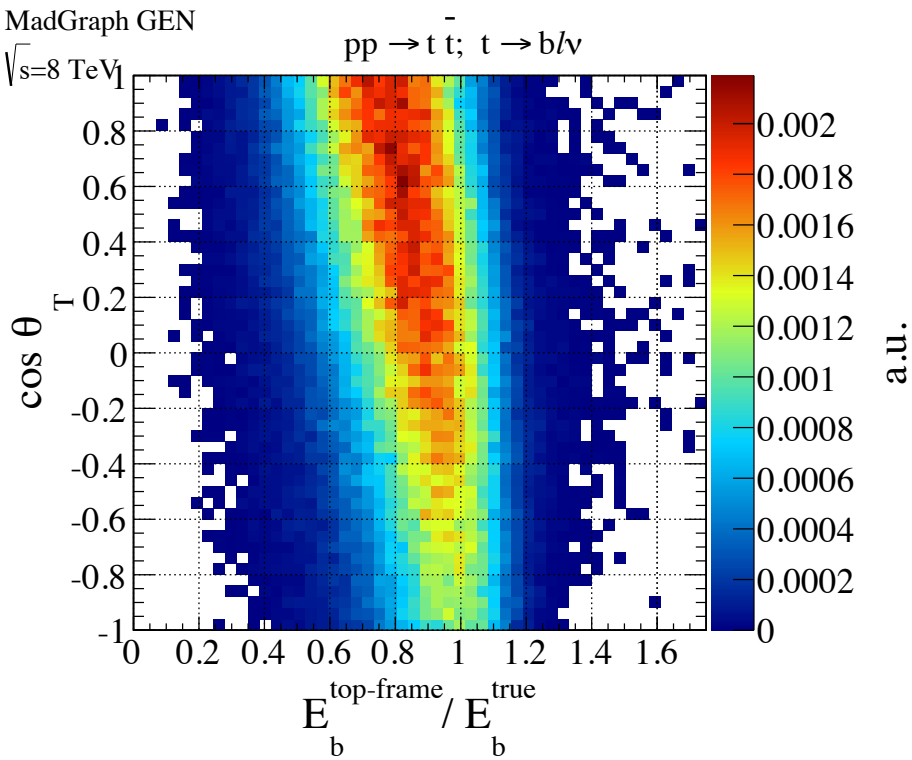


Here, the decay angle of the top/anti-top system

The di-leptonic top basis

largely independent information about decay angles

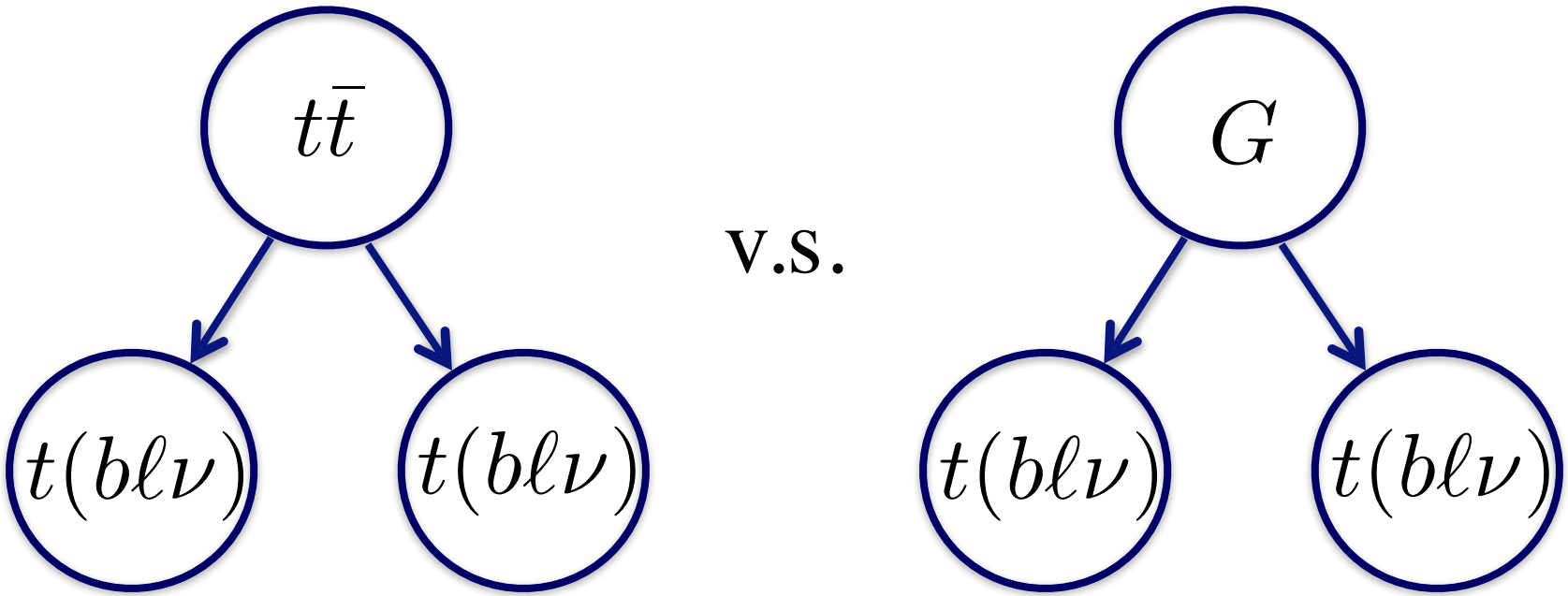




Here, the decay angle of one of the top quarks

The di-leptonic top basis vs. signals

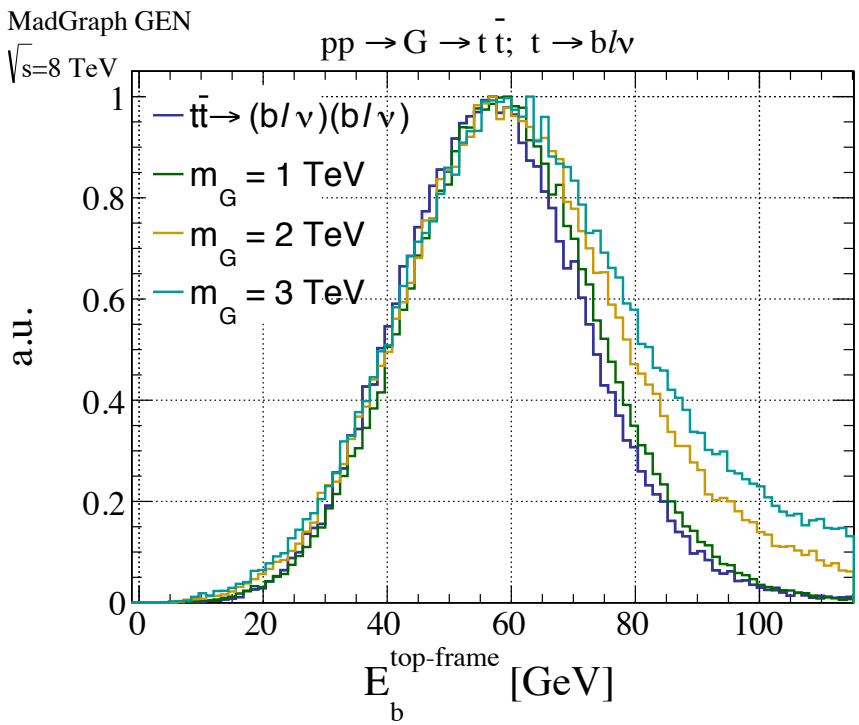
Different variables in the basis are useful for different signals

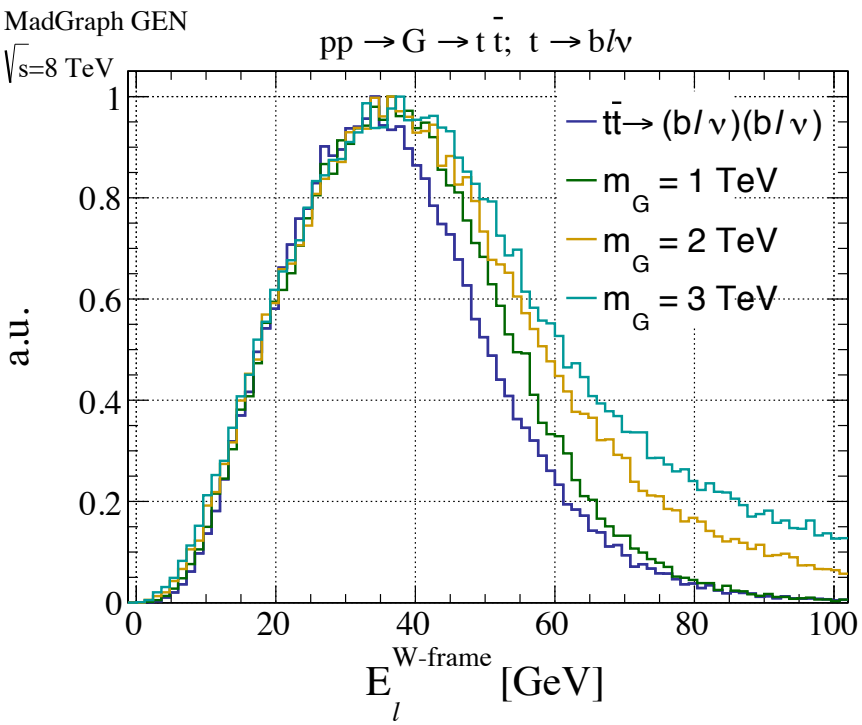


First, we consider resonant ttbar production through a graviton

The di-leptonic top basis vs. gravitons

Different variables in the basis are useful for different signals

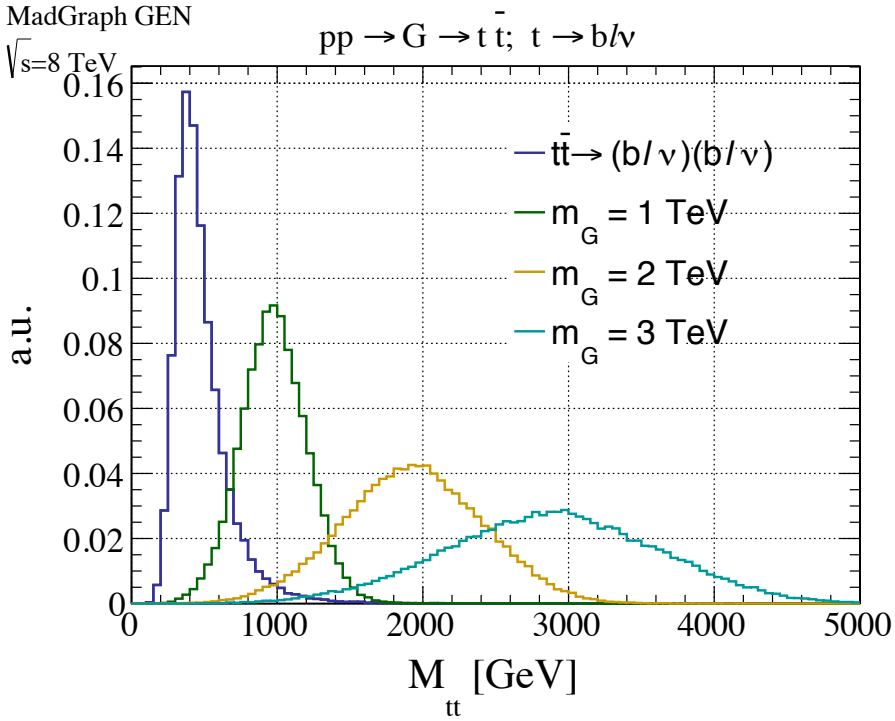


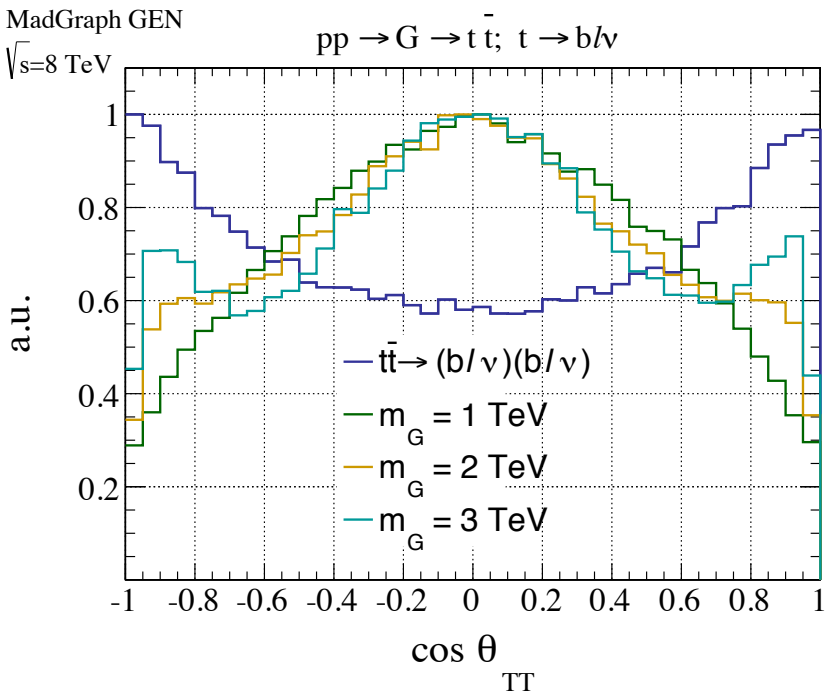


Distributions of top/W/neutrino mass-splitting-sensitive observables are nearly identical since graviton signal and non-resonant background both contain on-shell tops

The di-leptonic top basis vs. gravitons

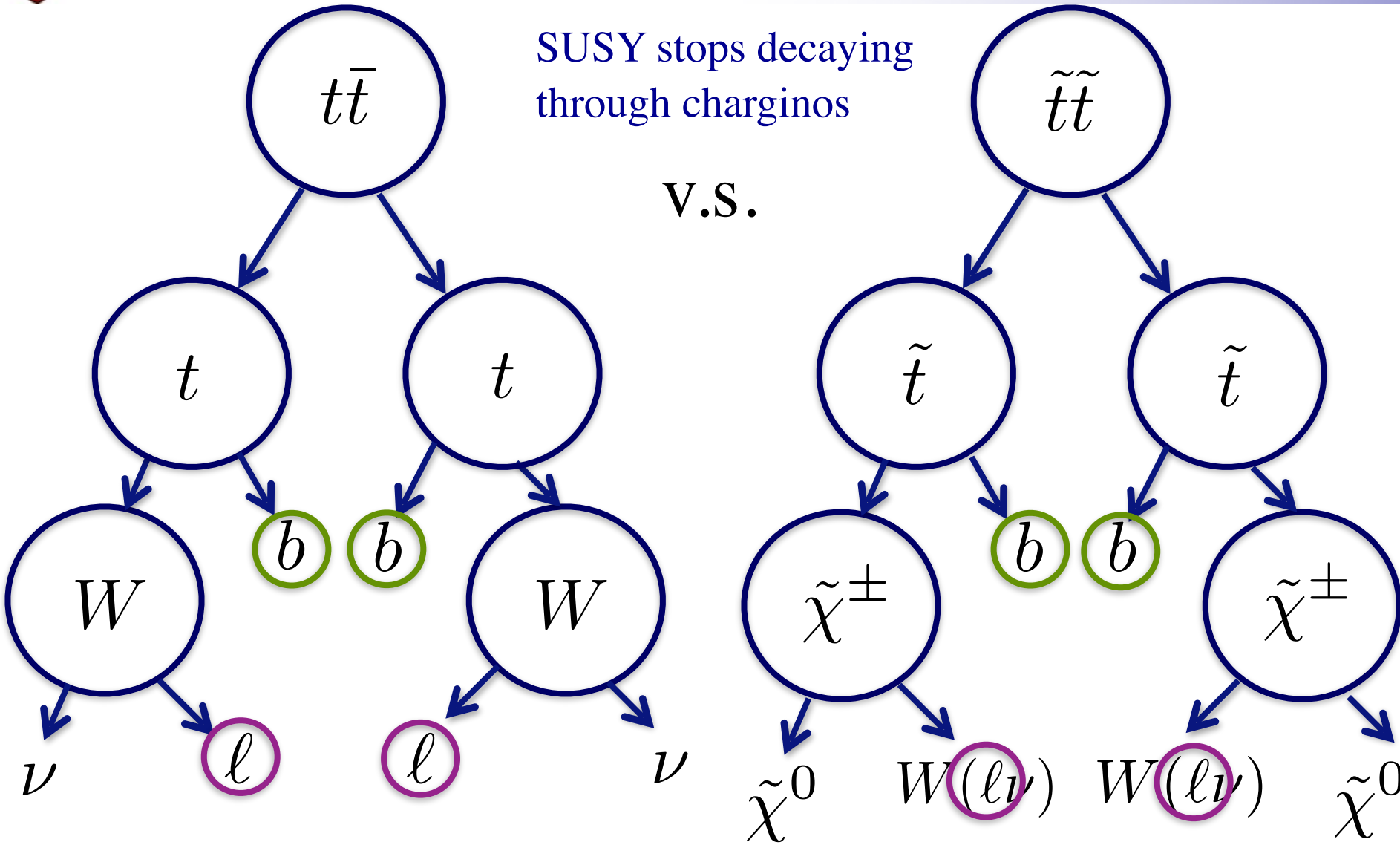
Different variables in the basis are useful for different signals



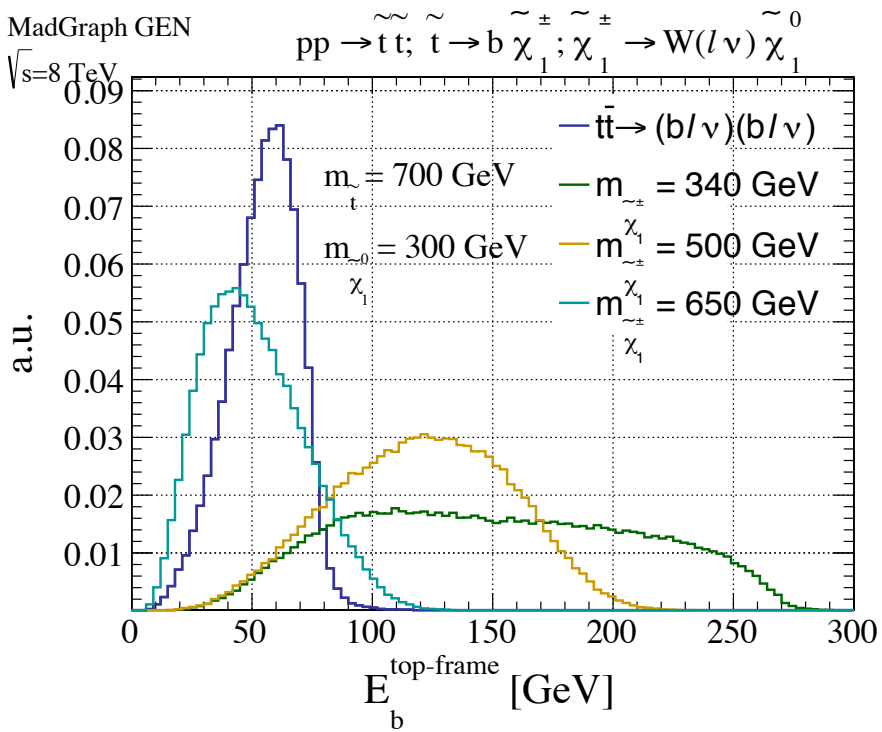


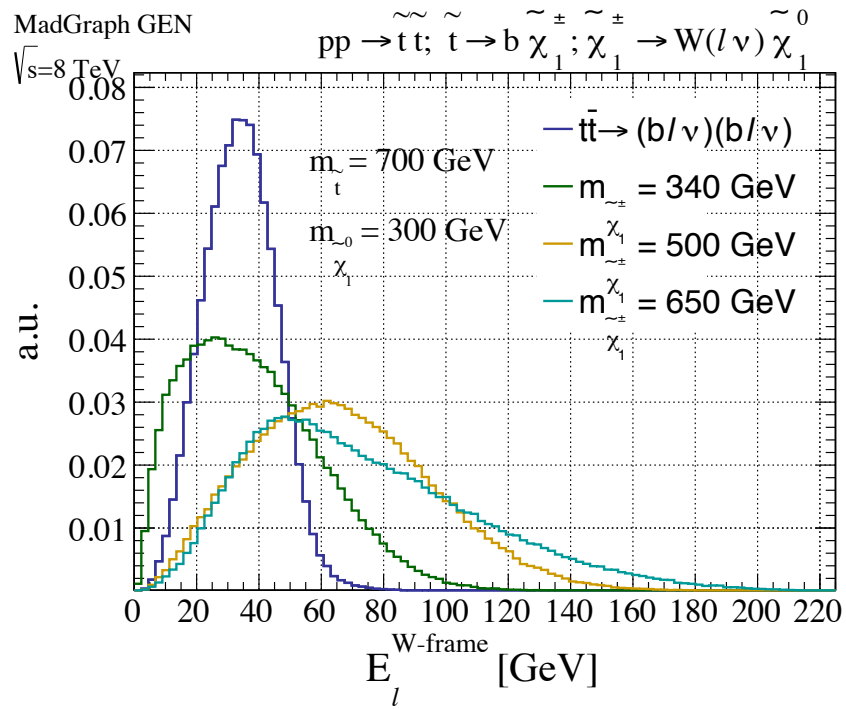
Instead, observables related to the production of the two tops are sensitive to the intermediate resonance

The di-leptonic top basis vs. stops



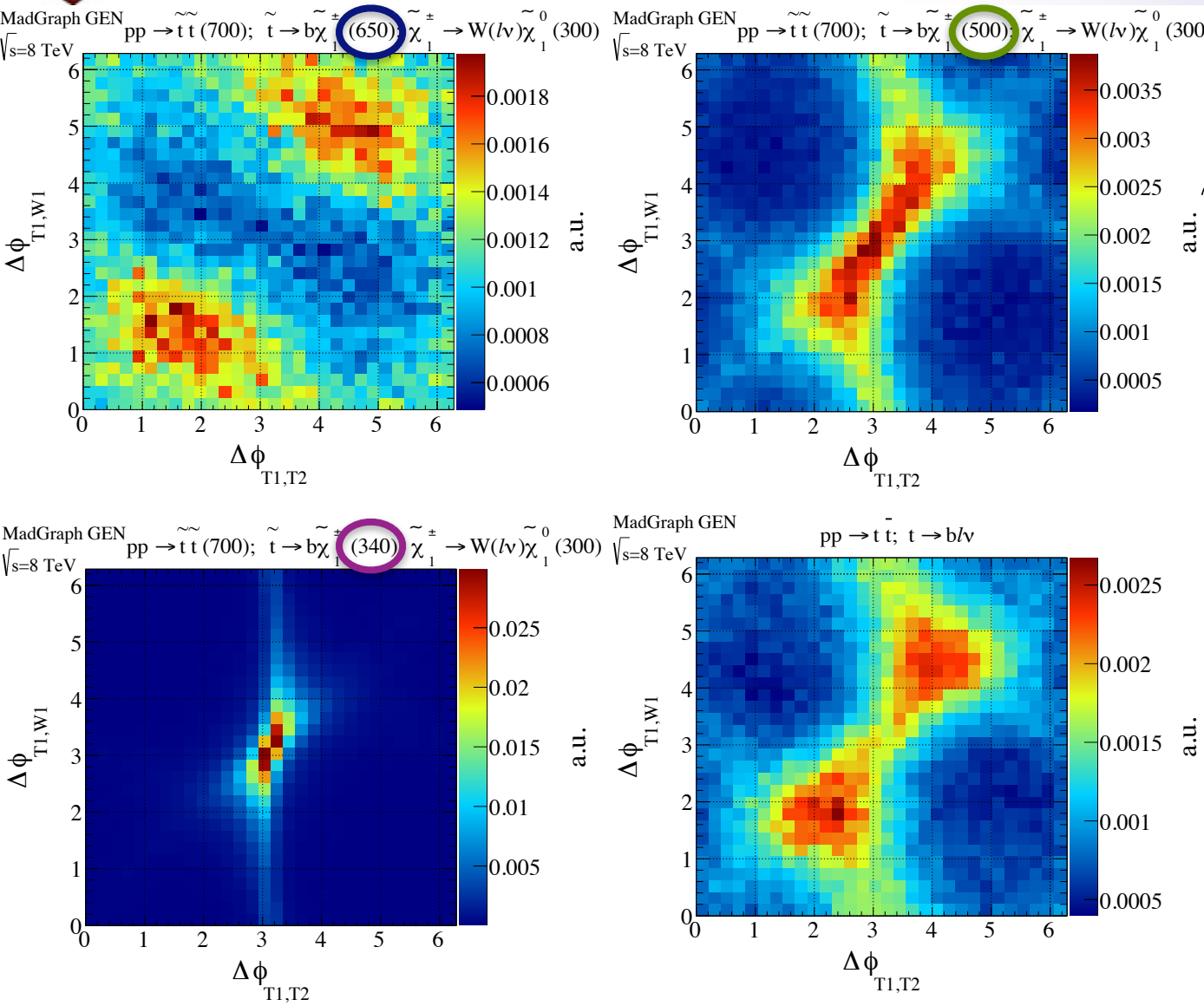
The di-leptonic top basis vs. stops





Mass-splitting-sensitive observables can be used to distinguish presence of signals

The di-leptonic top basis vs. stops



$m_{\tilde{t}} = 700$ GeV

$m_{\tilde{\chi}^{\pm}} =$ 650 / 500 / 340 GeV

$m_{\tilde{\chi}^{0}} = 300$ GeV

Here, the azimuthal angle between the the top and W decay planes $\Delta\phi_{T1,W1}$ and the angle between the two top decay planes $\Delta\phi_{T1,T2}$

Summary

- The strategy of ***Recursive Jigsaw Reconstruction*** is to not only develop 'good' mass estimator variables, but to decompose each event into a ***basis of kinematic variables***
- Through the recursive procedure, each variable is (as much as possible***) independent of the others***
- The interpretation of variables is straightforward; they each correspond to an ***actual, well-defined, quantity in the event***
- Can be generalized to arbitrarily complex final states with ***many weakly interacting particles***
- Code package to be released imminently ***(RestFrames -*** see back-up***)***
- First papers nearing completion (***Recursive Jigsaw Reconstruction***, ***The Di-leptonic ttbar basis*** - with Paul Jackson)

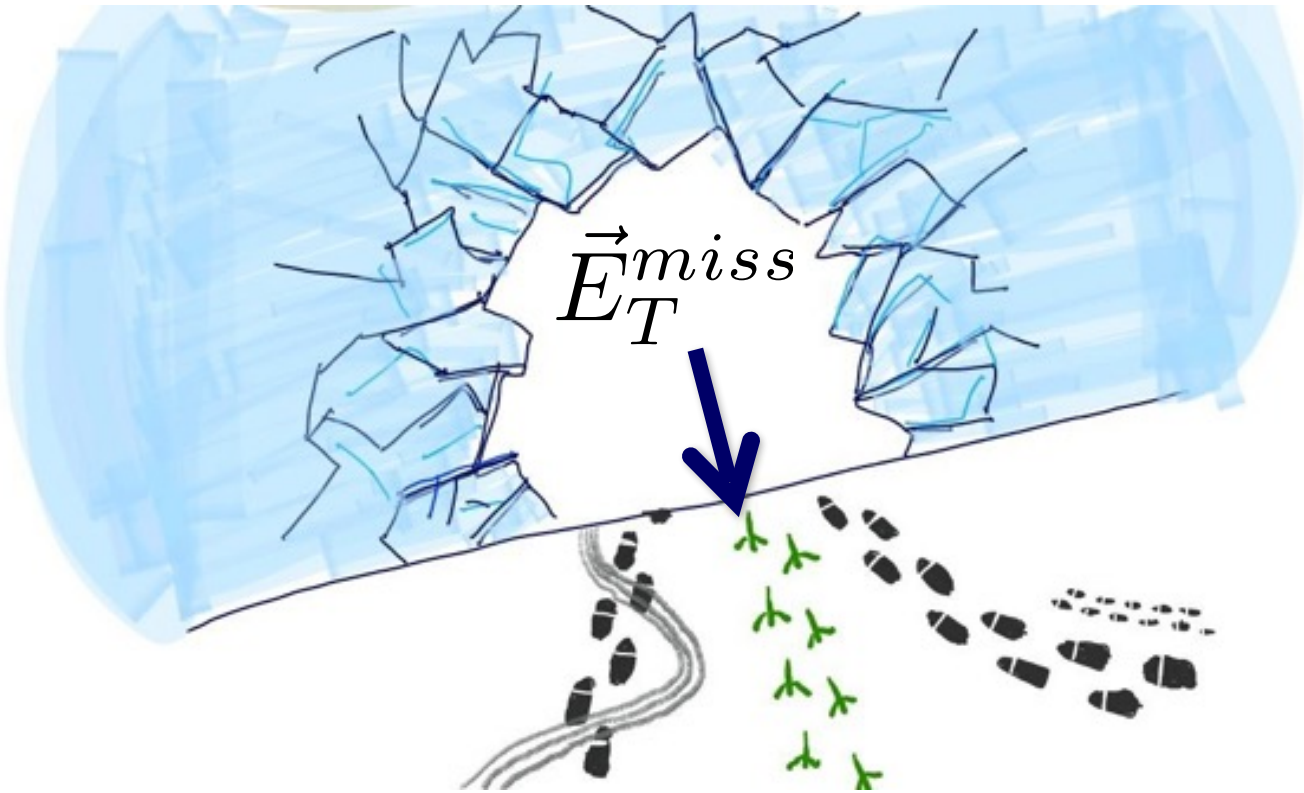
Outlook

- ***Recursive Jigsaw Reconstruction*** is a systematic recipe for deriving a kinematic basis for any open final state
- Mixed decay cases can now be treated in a sensible way

$$\tilde{t} \to t\tilde{\chi}^0 \text{ vs. } \tilde{t} \to b\tilde{\chi}^\pm$$

$$\tilde{\chi}^\pm \to W^\pm\tilde{\chi}^0 \text{ vs.}$$
$$\tilde{\chi}^\pm \to \ell^\pm\tilde{\nu} \text{ vs.}$$
$$\tilde{\chi}^\pm \to \nu\tilde{\ell}^\pm$$

- Lots of potential applications – some I'm currently thinking about:
  - Di-lepton searches and differential measurements
  - stop / top-quark partner and resonant ttbar searches
  - other $\mathbb{Z}_2$ signatures (SUSY, exotica)
  - $H \to WW^* \to \ell\nu\ell\nu$ spin measurements
  - …



BACKUP SLIDES

RestFrames software library

RestFrames: Soon-to-be-public code that can be used to calculate kinematic variables associated with any decay chain and can implement "Recursive Jigsaw" rules

Example: Di-leptonic ttbar decays - how to use the code to initialize a decay tree, implement a jigsaw rule for determining the neutrino four-momenta, and analyze events

www.RestFrames.com

RestFrames software library

```
RLabFrame LAB("LAB","lab");
RDecayFrame TT("TT","t #bar{t}");
RDecayFrame T1("T1","t_{a}");
RDecayFrame T2("T2","t_{b}");
RVisibleFrame Bjet1("B1","b_{a}");
RVisibleFrame Bjet2("B2","b_{b}");
RDecayFrame W1("W1","W_{a}");
RDecayFrame W2("W2","W_{b}");
RVisibleFrame Lep1("L1","#it{l}_{a}");
RVisibleFrame Lep2("L2","#it{l}_{b}");
RInvisibleFrame Neu1("NU1","#nu_{a}");
RInvisibleFrame Neu2("NU2","#nu_{b}");
```

initialize all your reference frames of interest

```
LAB.SetChildFrame(TT);

TT.AddChildFrame(T1);
TT.AddChildFrame(T2);

T1.AddChildFrame(Bjet1);
T1.AddChildFrame(W1);
T2.AddChildFrame(Bjet2);
T2.AddChildFrame(W2);

W1.AddChildFrame(Lep1);
W1.AddChildFrame(Neu1);
W2.AddChildFrame(Lep2);
W2.AddChildFrame(Neu2);
```

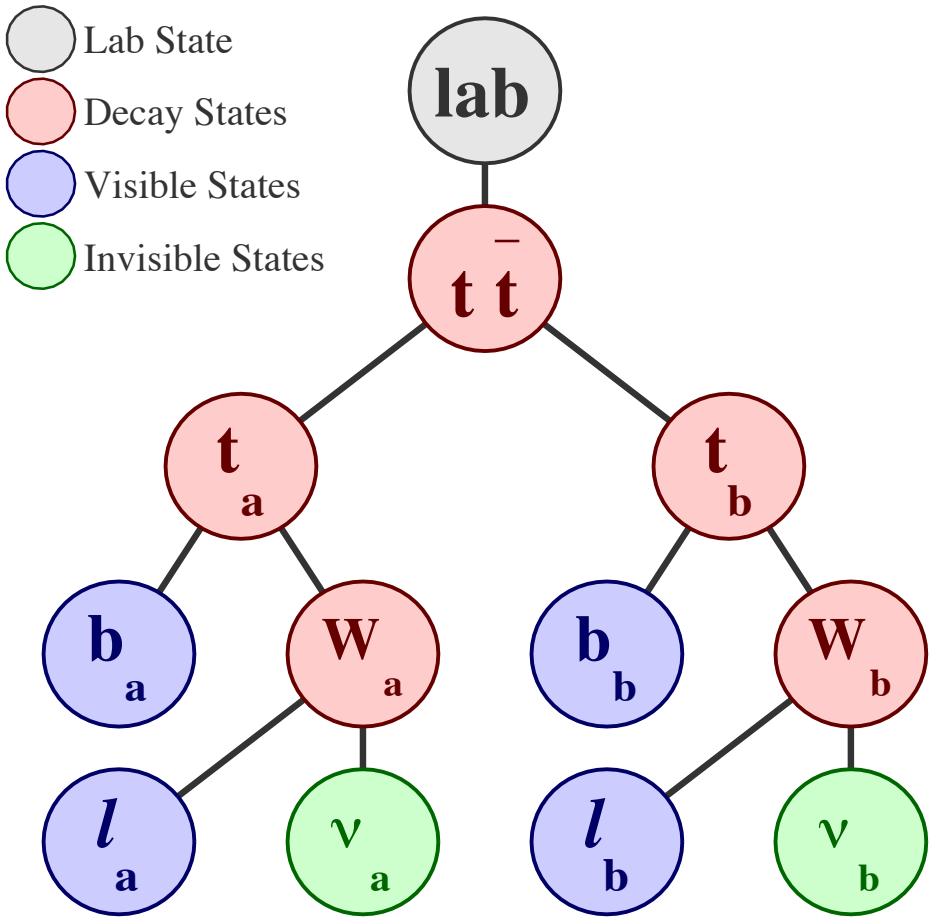
connect them according to the decay tree you want to impose on the event

RestFrames software library

```
// draw your decay tree
FramePlot* DecayTree_plot = new FramePlot("tree","Decay Tree");
DecayTree_plot->AddFrameTree(LAB);
DecayTree_plot->DrawFramePlot();
```

draw your decay tree…

…and you'll see this



RestFrames example: di-leptonic top

```
Lep1.SetLabFrameFourVector(lepA);
Lep2.SetLabFrameFourVector(lepB);
BTAGS.AddLabFrameFourVector(b1);
BTAGS.AddLabFrameFourVector(b2);
INV.SetLabFrameFourVector(MET);
```

Set the 4-vector input and MET for an event

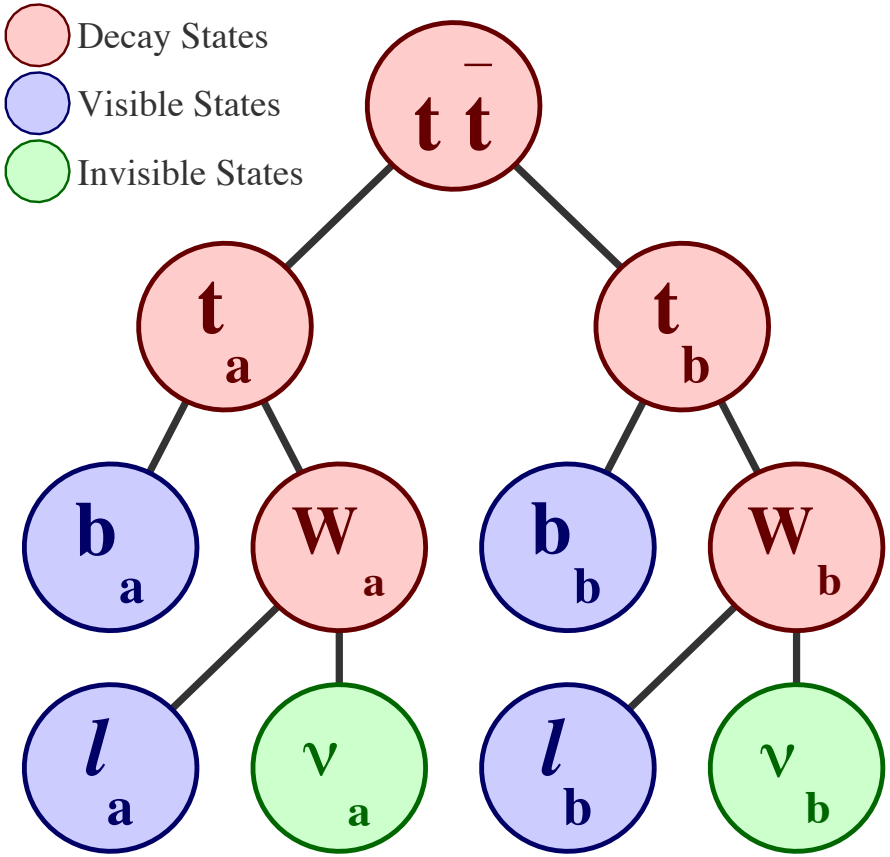
```
cout << "Analyzed event ok? : " << LAB.AnalyzeEvent() << endl;
```

Get observables directly from decay tree

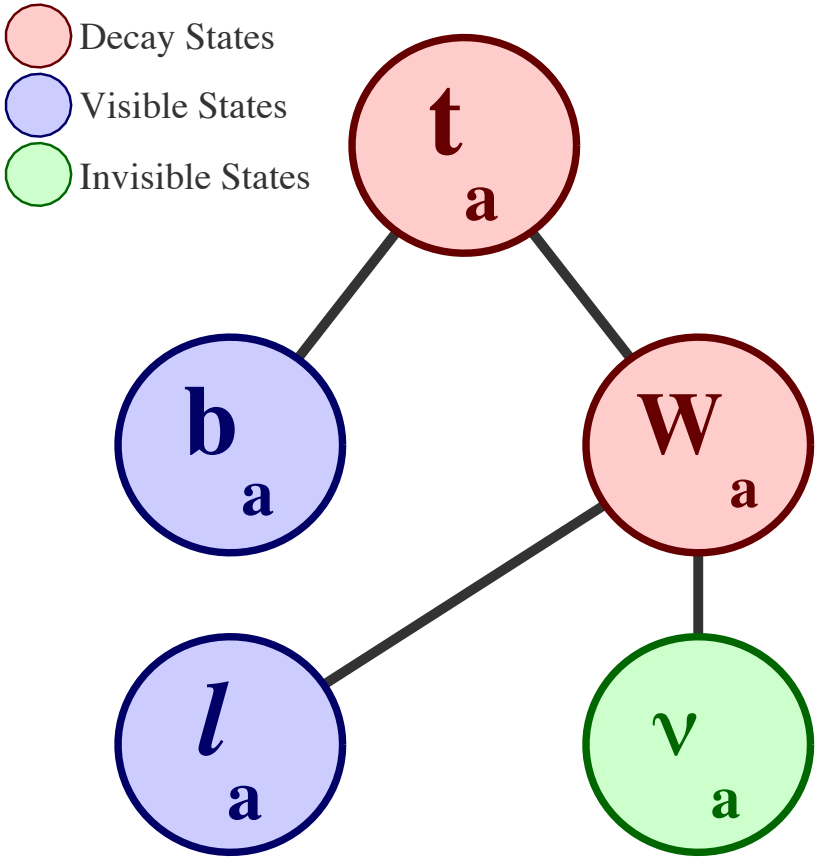
```
// example analysis methods
double TTmass      = TT.GetMass();                    // mass
double costhetaT1 = T1.GetCosDecayAngle();            // decay angle
double P_T2_inTT   = T2.GetMomentum(TT);              // momentum in some frame
TLorentzVector V_W1_in_W2 = W1.GetFourVector(W2);  // 4-vector in some frame
double dphi_T1_W1 = T1.GetDeltaPhiDecayPlanes(W1); // angle between decay planes
```

RestFrames example: di-leptonic top

```
DecayTree_plot->AddFrameTree(TT);
DecayTree_plot->DrawFramePlot();
```




```
DecayTree_plot->AddFrameTree(T1);
DecayTree_plot->DrawFramePlot();
```



RestFrames example: di-leptonic top

```
// Invisible Frame Group
InvisibleGroup INV("INV","Invisible Frame Jigsaws");
INV.AddFrame(Neu1);
INV.AddFrame(Neu2);
```

RestFrames example: di-leptonic top

```
CombinatoricGroup BTAGS("BTAGS","B-tagged jet Jigsaws");
BTAGS.AddFrame(Bjet1);
BTAGS.AddFrame(Bjet2);
```

```
BTAGS.SetNElementsForFrame(Bjet1,1,true);
BTAGS.SetNElementsForFrame(Bjet2,1,true);
```

number of elements that _have_ to go to this frame from the group each event

exclusive (true) or inclusive (false) counting

RestFrames example: di-leptonic top

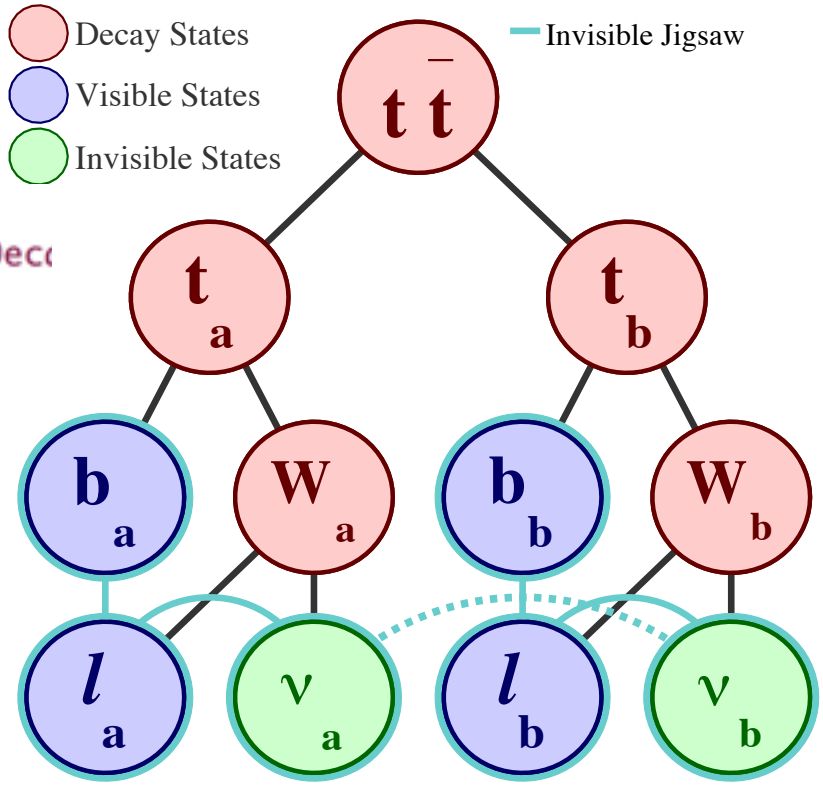
```
InvisibleMassJigsaw MinMassJigsaw("MINMASS_JIGSAW",
                                  "Invisible system mass Jigsaw");
INV.AddJigsaw(MinMassJigsaw);
```

```
InvisibleRapidityJigsaw RapidityJigsaw("RAPIDITY_JIGSAW",
                                       "Invisible system rapidity Jigsaw");
INV.AddJigsaw(RapidityJigsaw);
RapidityJigsaw.AddVisibleFrame((LAB.GetListVisibleFrames()));
// Same as this
RapidityJigsaw.AddVisibleFrame(T1.GetListVisibleFrames());
RapidityJigsaw.AddVisibleFrame(T2.GetListVisibleFrames());
// Same as this
RapidityJigsaw.AddVisibleFrame(Bjet1);
RapidityJigsaw.AddVisibleFrame(Bjet2);
RapidityJigsaw.AddVisibleFrame(Lep1);
RapidityJigsaw.AddVisibleFrame(Lep2);
```

RestFrames example: di-leptonic top

```
ContraBoostInvariantJigsaw TTJigsaw("TT_JIGSAW","Contraboost invariant Jigsaw");
INV.AddJigsaw(TTJigsaw);
TTJigsaw.AddVisibleFrame((T1.GetListVisibleFrames()), 0);
TTJigsaw.AddInvisibleFrame((T1.GetListInvisibleFrames()), 0);

TTJigsaw.AddInvisibleFrame((T2.GetListInvisibleFrames()), 1);
TTJigsaw.AddVisibleFrame((T2.GetListVisibleFrames()), 1);
```

```
// draw your decay tree with jigsaws
FramePlot* JigsawTree_plot = new FramePlot("tree","Deca
JigsawTree_plot->AddFrameTree(TT);
JigsawTree_plot->AddJigsaw(TTJigsaw);
JigsawTree_plot->DrawFramePlot();
```



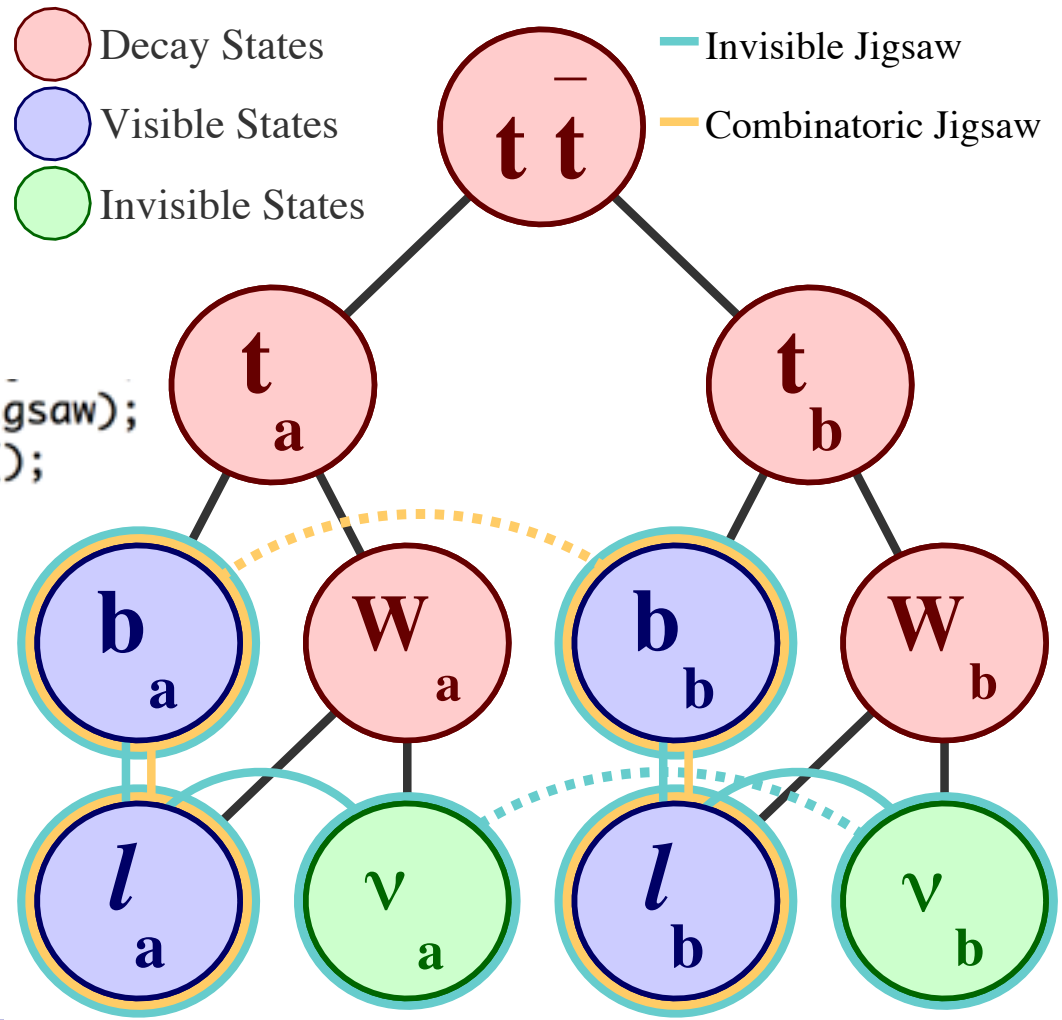
RestFrames example: di-leptonic top

```
MinimizeMassesCombinatoricJigsaw BLJigsaw("BL_JIGSAW",
                                          "Minimize m_{b#it{l}}'s Jigsaw");

BTAGS.AddJigsaw(BLJigsaw);
BLJigsaw.AddFrame(Bjet1,0);
BLJigsaw.AddFrame(Lep1,0);

BLJigsaw.AddFrame(Bjet2,1);
BLJigsaw.AddFrame(Lep2,1);
```

event-by-event will choose b-tag jet combinatoric assignment that minimizes the masses of these two collections

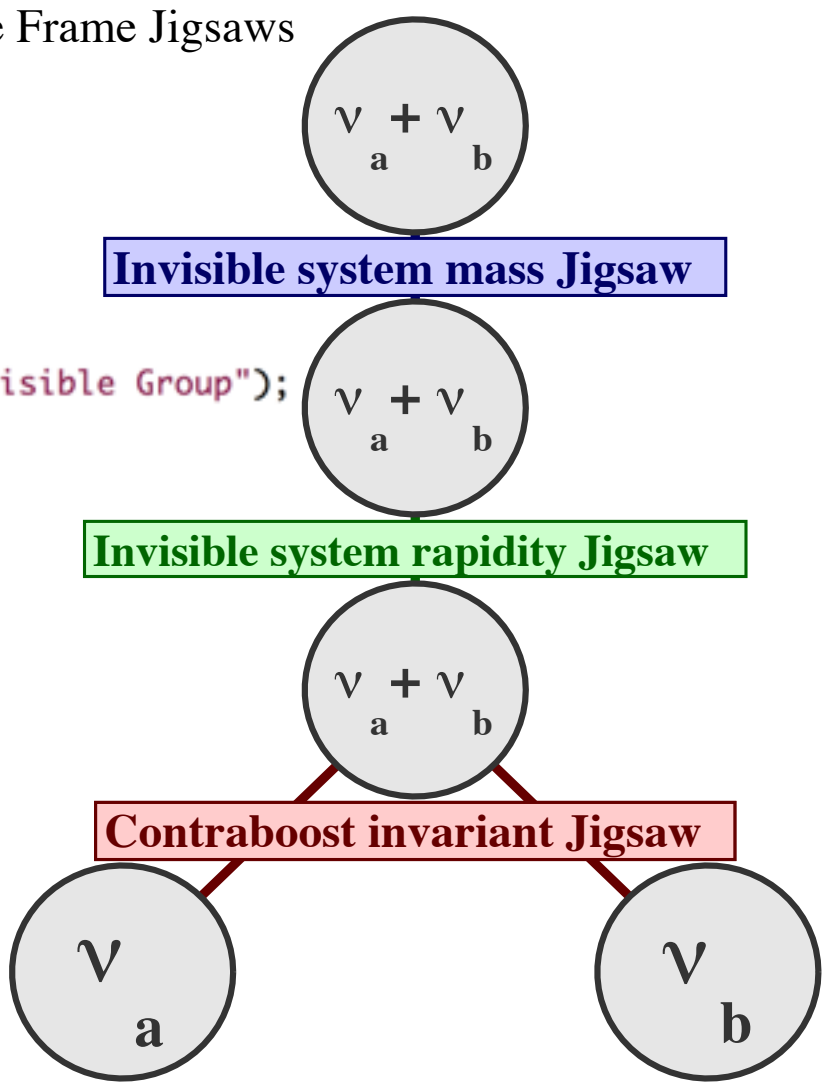
RestFrames example: di-leptonic top

```
MinimizeMassesCombinatoricJigsaw BLJigsaw("BL_JIGSAW",
                                          "Minimize m_{b#it{l}}'s Jigsaw");

BTAGS.AddJigsaw(BLJigsaw);
BLJigsaw.AddFrame(Bjet1,0);
BLJigsaw.AddFrame(Lep1,0);

BLJigsaw.AddFrame(Bjet2,1);
BLJigsaw.AddFrame(Lep2,1);

JigsawTree_plot->AddJigsaw(BLJigsaw);
JigsawTree_plot->DrawFramePlot();
```



RestFrames example: di-leptonic top

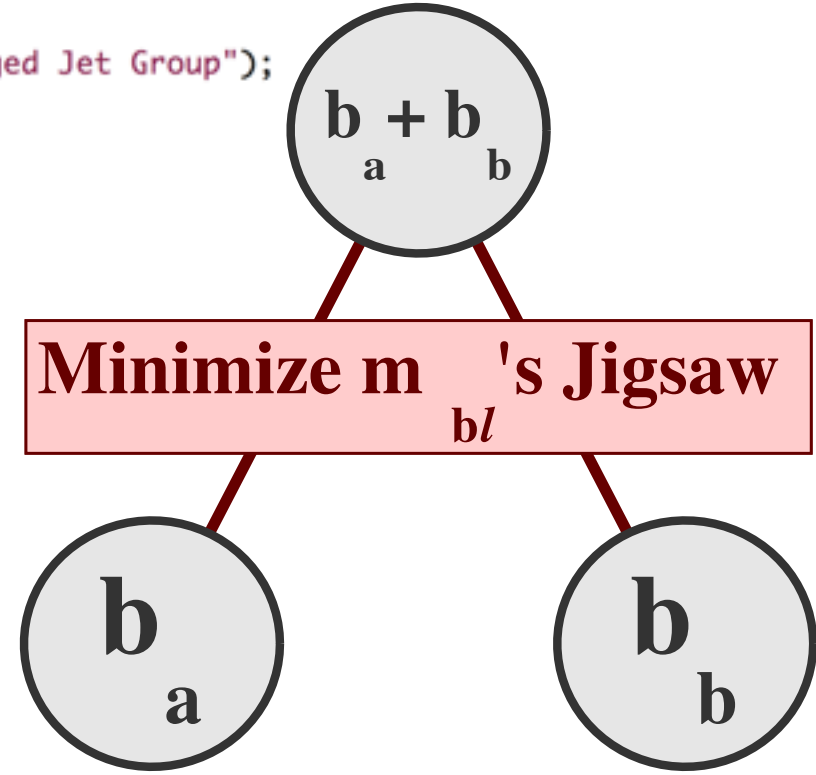
```
//draw invisible group tree
FramePlot* INVgroup_plot = new FramePlot("INVtree","Invisible Group");
INVgroup_plot->AddGroupTree(INV);
INVgroup_plot->DrawFramePlot();
```



RestFrames example: di-leptonic top

B-tagged jet Jigsaws

```
//draw b-tag group tree
FramePlot* BTAGgroup_plot = new FramePlot("BTAGtree","B-tagged Jet Group");
BTAGgroup_plot->AddGroupTree(BTAGS);
BTAGgroup_plot->DrawFramePlot();
```



RestFrames example: di-leptonic top

```
//check that tree topology is consistent
cout << "Is consistent decay topology? : " << LAB.InitializeTree() << endl;
```

```
cout << "Is the tree ok for analysis? : " << LAB.InitializeAnalysis() << endl;
```

RestFrames example: di-leptonic top

```
TLorentzVector lepA, lepB, b1, b2, MET;
lepA.SetPxPyPzE(1.,0.,0.,1.);
lepB.SetPxPyPzE(0.,1.,0.,1.);
b1.SetPxPyPzE(1.,0.,0.,1.);
b2.SetPxPyPzE(0.,1.,0.,1.);
MET.SetPxPyPzE(-.5,-.5, 0., 1.);
```

some toy input

```
LAB.ClearEvent();

Lep1.SetLabFrameFourVector(lepA);
Lep2.SetLabFrameFourVector(lepB);
BTAGS.AddLabFrameFourVector(b1);
BTAGS.AddLabFrameFourVector(b2);
INV.SetLabFrameFourVector(MET);
// same as
INV.SetLabFrameThreeVector(MET.Vect());

cout << "Analyzed event ok? : " << LAB.AnalyzeEvent() << endl;
```

analyze the event

RestFrames example: di-leptonic top

```
// example analysis methods
double TTmass     = TT.GetMass();                    // mass
double costhetaT1 = T1.GetCosDecayAngle();           // decay angle
double P_T2_inTT  = T2.GetMomentum(TT);              // momentum in some frame
TLorentzVector V_W1_in_W2 = W1.GetFourVector(W2);  // 4-vector in some frame
double dphi_T1_W1 = T1.GetDeltaPhiDecayPlanes(W1); // angle between decay planes
```

tracking of combinatoric elements

```
GroupElementID b1ID = BTAGS.AddLabFrameFourVector(b1);
GroupElementID b2ID = BTAGS.AddLabFrameFourVector(b2);

// object tracking in combinatoric groups
cout << "Did b1 end up as Bjet1? : ";
cout << BTAGS.GetFrame(b1ID)->IsSame(Bjet1) << endl;
```

RestFrames setup

- To make shared library ‘lib/libRestFrames.so’

```
<$ make
```

- To set environmental variables
(ex. for running ROOT macros):

```
<$ source setup.sh
```

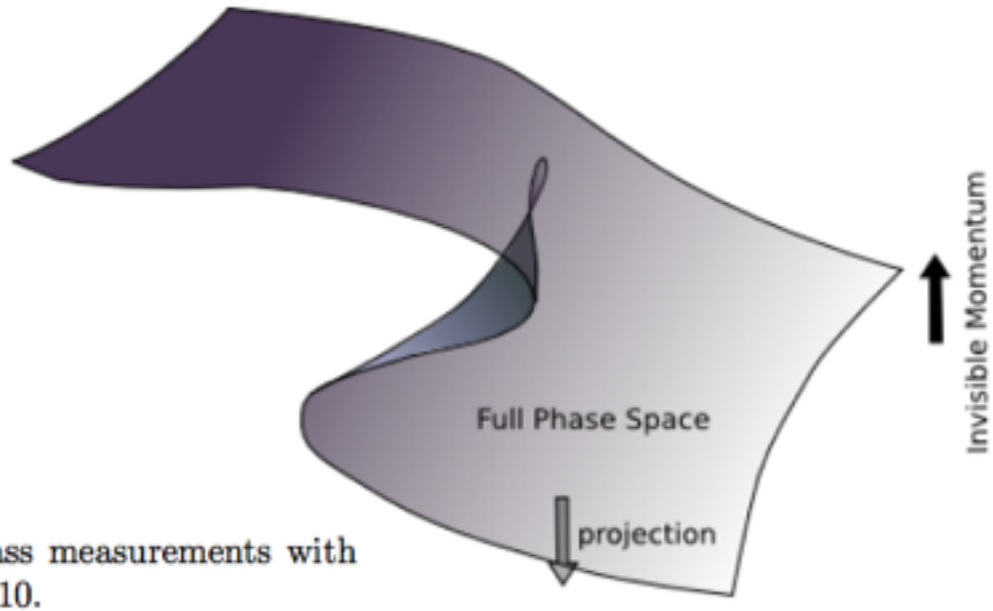
RestFrames example: di-leptonic top

- To run ‘macros/TestDiLeptonicTop.C’ macro:

```
<$ root
root [0] .x macros/TestDiLeptonicTop.C
```

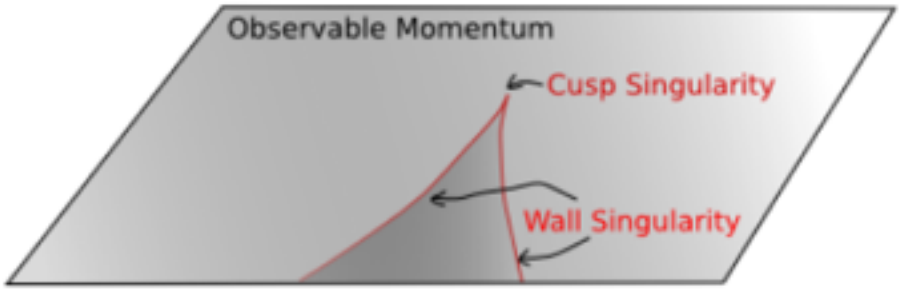
Singularity variables

Kinematic Singularities. A singularity is a point where the local tangent space cannot be defined as a plane, or has a different dimension than the tangent spaces at non-singular points.



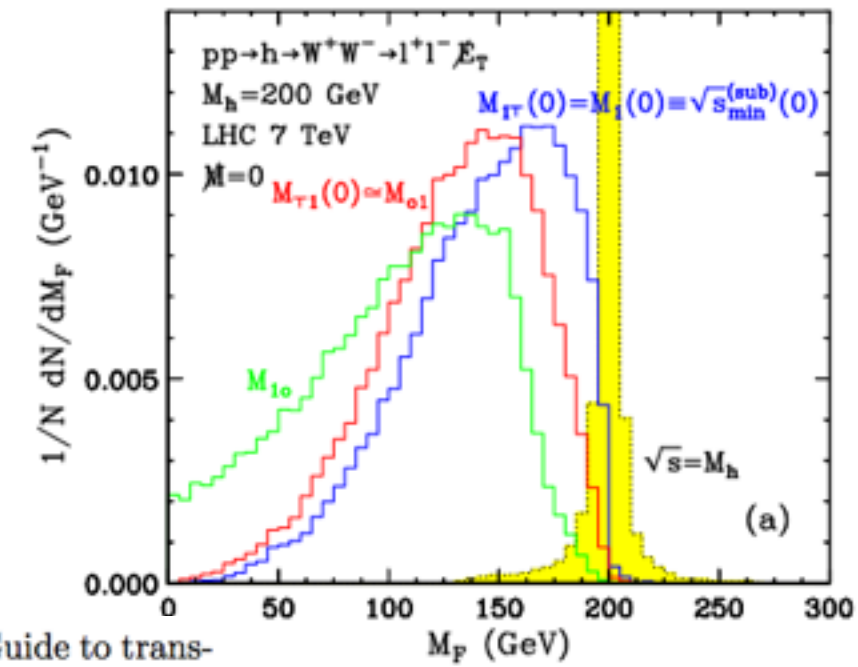
From:

Ian-Woo Kim. Algebraic singularity method for mass measurements with missing energy. *Phys. Rev. Lett.*, 104:081601, Feb 2010.



Singularity variables

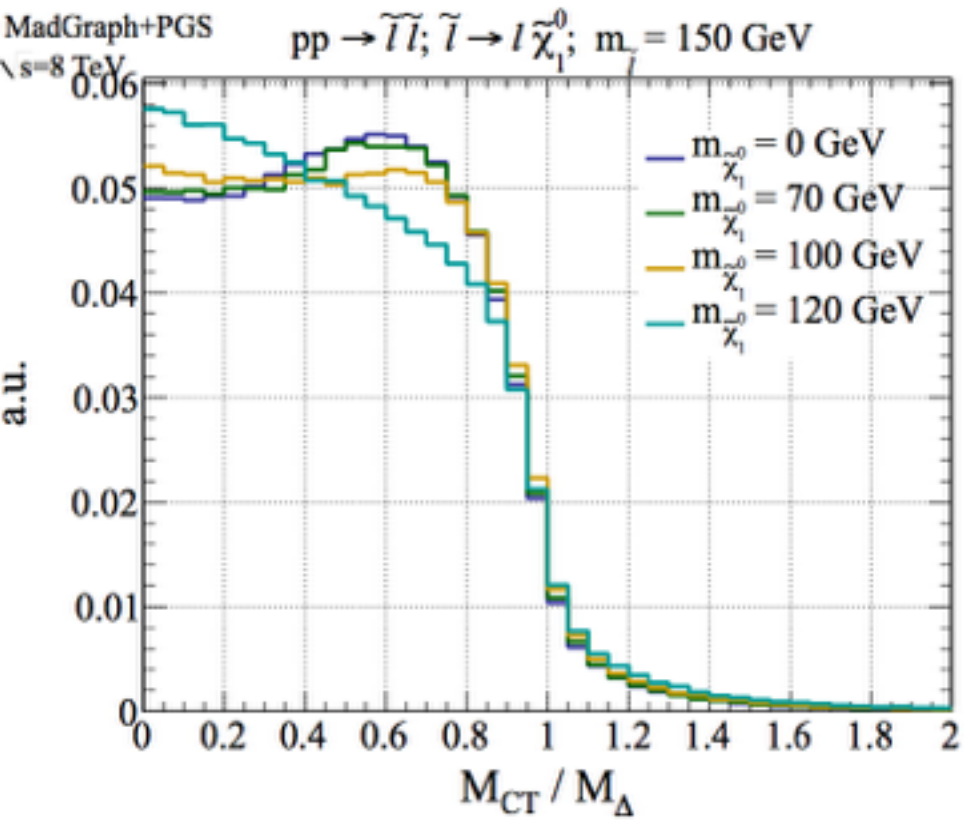
The guiding principle we employ for creating useful hadron-collider event variables, is that: *we should place the best possible bounds on any Lorentz invariants of interest, such as parent masses or the center-of-mass energy $\hat{s}^{1/2}$, in any cases where it is not possible to determine the actual values of those Lorentz invariants due to incomplete event information.*



From:

A.J. Barr, T.J. Khoo, P. Konar, K. Kong, C.G. Lester, et al. Guide to transverse projections and mass-constraining variables. *Phys.Rev.*, D84:095031, 2011.

Example: M_{CT}



assuming ~mass-less leptons

$$M_{CT}^2 = 2\left(p_T^{\ell_1} p_T^{\ell_2} + \vec{p}_T^{\,\ell_1} \cdot \vec{p}_T^{\,\ell_2}\right)$$

Constructed to have a kinematic endpoint at:

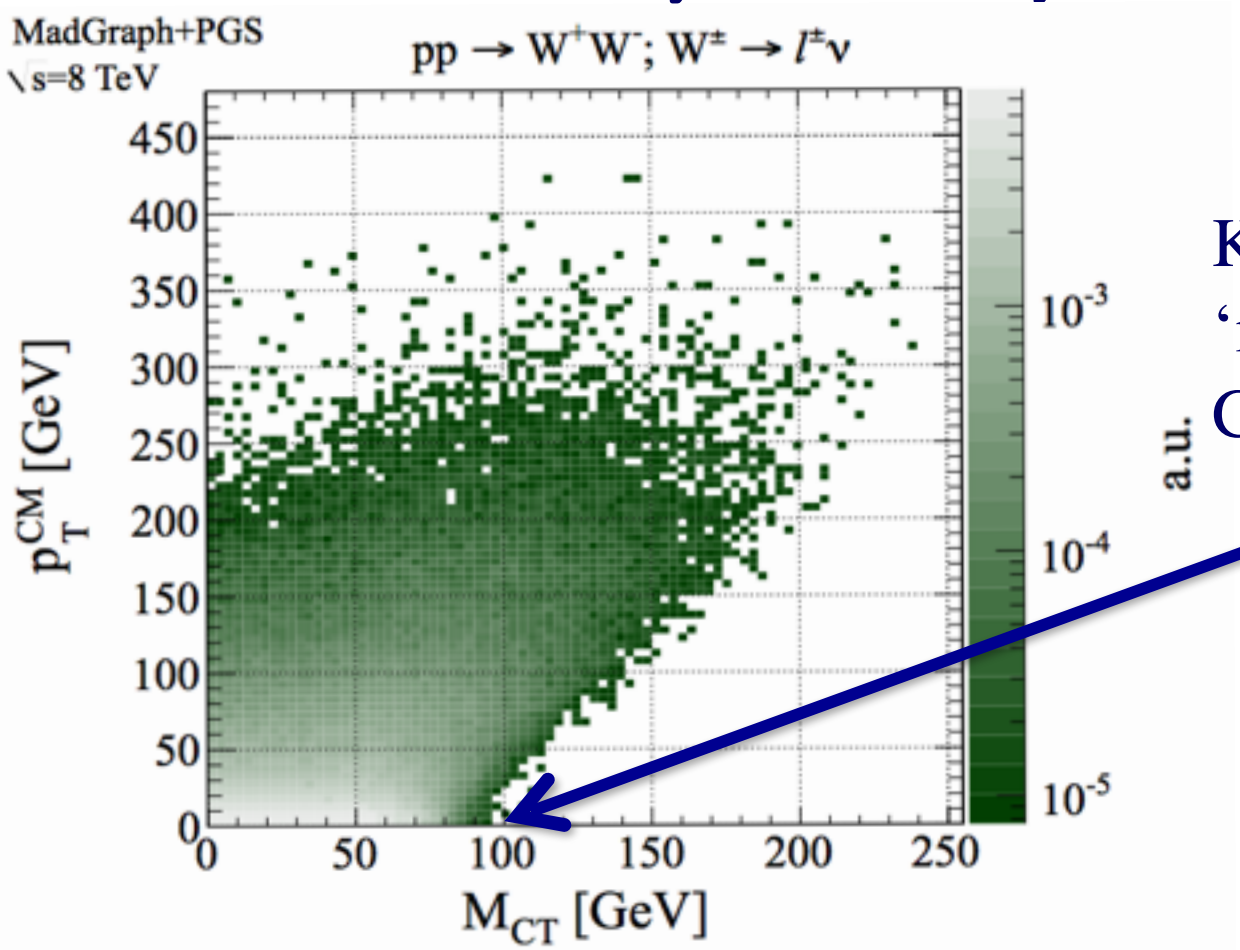
$$M_{CT}^{\max} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2}{m_{\tilde{\chi}_1^0}} = M_\Delta$$

From:

Daniel R. Tovey. On measuring the masses of pair-produced semi-invisibly decaying particles at hadron colliders. *JHEP*, 0804:034, 2008.

M_{CT} in practice

Singularity variables (like M_{CT}) can be sensitive to quantities that can vary dramatically event-by-event



Kinematic endpoint 'moves' with nonzero CM system p_T

The mass challenge

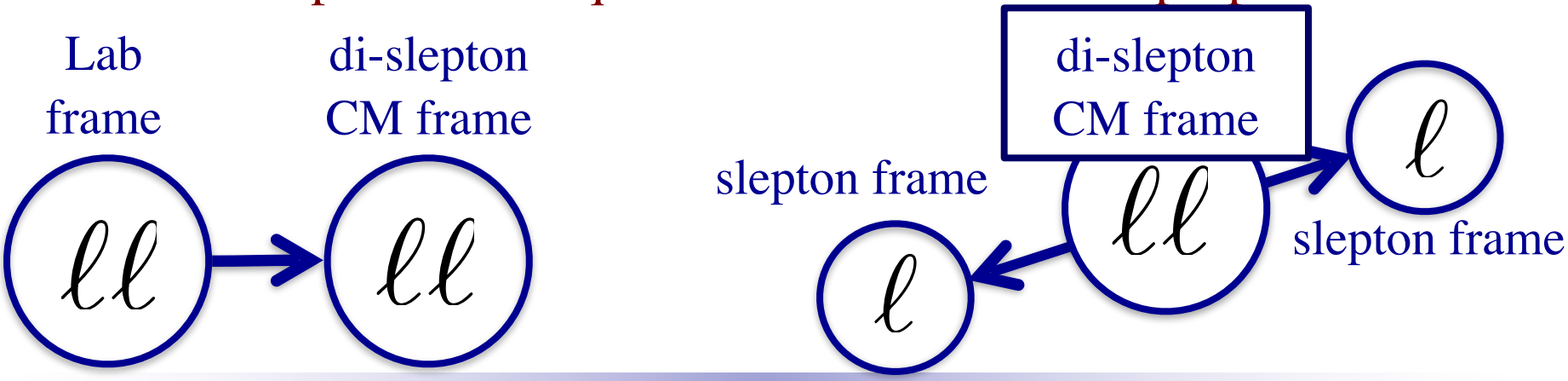
The invariant mass is invariant under coherent Lorentz transformations of two particles

$$m^2_{inv}(p_1, p_2) = m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2)$$

The Euclidean mass (or contra-variant mass) is invariant under anti-symmetric Lorentz transformations of two particles

$$m^2_{eucl}(p_1, p_2) = m_1^2 + m_2^2 + 2(E_1 E_2 + \vec{p}_1 \cdot \vec{p}_2)$$

Even the simplest case requires variables with both properties!



Correcting for CM p_T

- Want to boost from lab-frame to CM-frame
- We know the transverse momentum of the CM-frame:

$$\vec{p}_T^{\,CM} = \vec{p}_T^{\,\ell_1} + \vec{p}_T^{\,\ell_2} + \vec{E}_T^{\,\mathrm{miss}}$$

- But we don't know the energy, or mass, of the CM-frame:

$$\vec{\beta}_{lab \to CM} = \frac{\vec{p}_T^{\,CM}}{\sqrt{|\vec{p}_T^{\,CM}|^2 + \hat{s}}}$$

p_T corrections for M_{CT}

Attempts have been made to mitigate this problem:

(i) 'Guess' the lab ➔ CM frame boost:

$$M_{CT(\mathrm{corr})} = \begin{cases} M_{CT} & \text{after boosting by } \beta = p_b/E_{\mathrm{cm}} \quad \text{if } A_{x(\mathrm{lab})} \geq 0 \text{ or } A'_{x(\mathrm{lo})} \geq 0 \\ M_{CT} & \text{after boosting by } \beta = p_b/\hat{E} \quad \text{if } A'_{x(\mathrm{hi})} < 0 \\ M_{Cy} & \quad \text{if } A'_{x(\mathrm{hi})} \geq 0 \end{cases}$$

x – parallel to boost

y – perp. to boost

with:

$$A_x = p_x[q_1]E_y[q_2] + p_x[q_2]E_y[q_1]$$

$$M_{Cy}^2 = (E_y[q_1] + E_y[q_2])^2 - (p_y[q_1] - p_y[q_2])^2$$

Giacomo Polesello and Daniel R. Tovey. Supersymmetric particle mass measurement with the boost-corrected contransverse mass. *JHEP*, 1003:030, 2010.

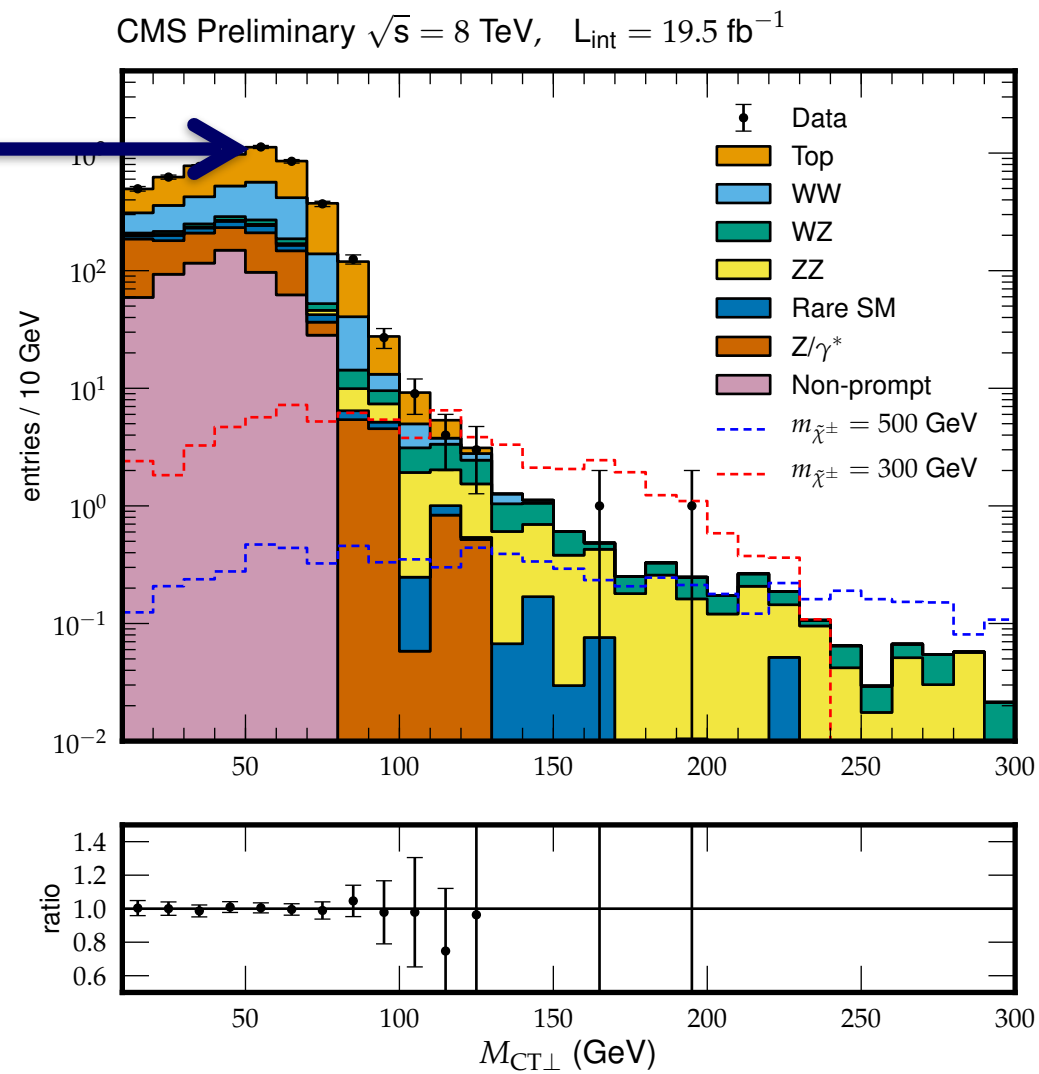
(ii) Only look at event along axis perpendicular to boost:

$$M_{CT\perp}$$

Konstantin T. Matchev and Myeonghun Park. A General method for determining the masses of semi-invisibly decaying particles at hadron colliders. *Phys.Rev.Lett.*, 107:061801, 2011.

M_{CTperp} in practice

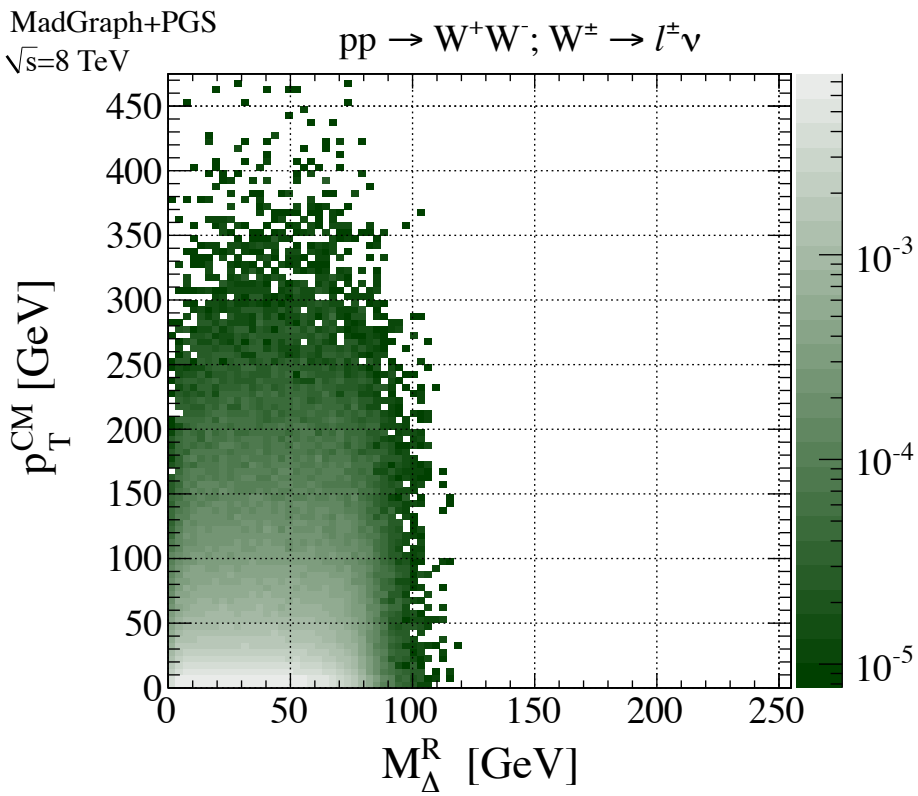
'peak position' of signal and backgrounds due to other cuts (p_T, MET) and only weakly sensitive to sparticle masses

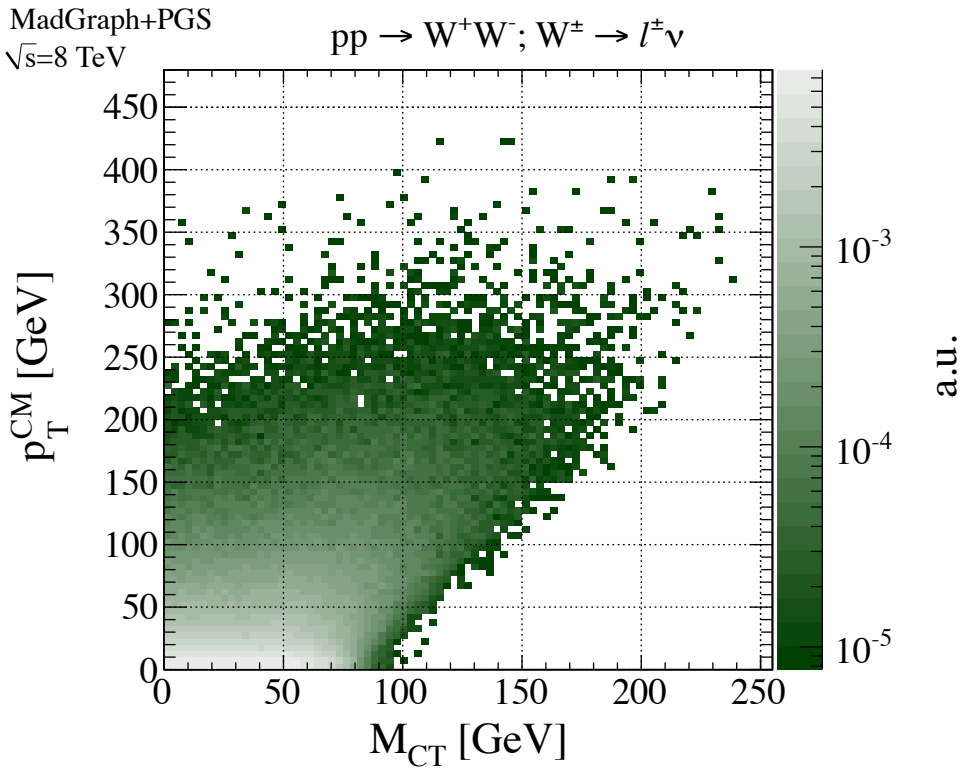


From:

CMS-SUS-PAS-13-006

Recursive rest-frame reconstruction

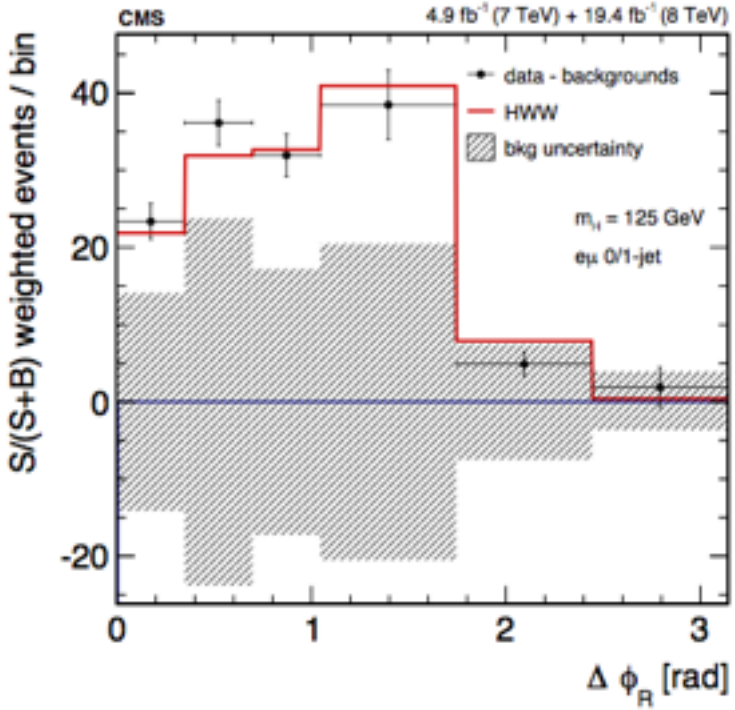




M_{Δ}^{R} is a singularity variable – in fact it is essentially identical to M_{CT} but evaluated ***in a different reference frame***. Boost procedure ensures that new variable is invariant under the previous transformations

Resonant Higgs production

$H \to WW^* \to 2\ell 2\nu$



The $\Delta\phi$ between the leptons is evaluated in the *R*-frame, removing dependence on the p_T of the Higgs and correlation with $\sqrt{\hat{s}_R}$

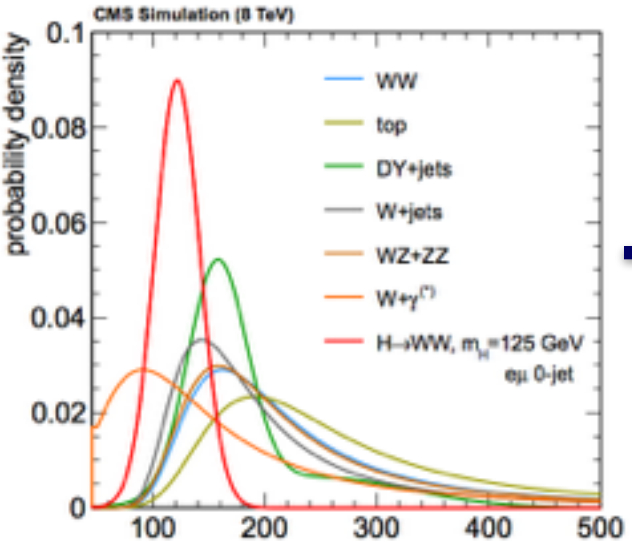
CMS uses 2D fit of variables to measure Higgs mass in this channel

From:

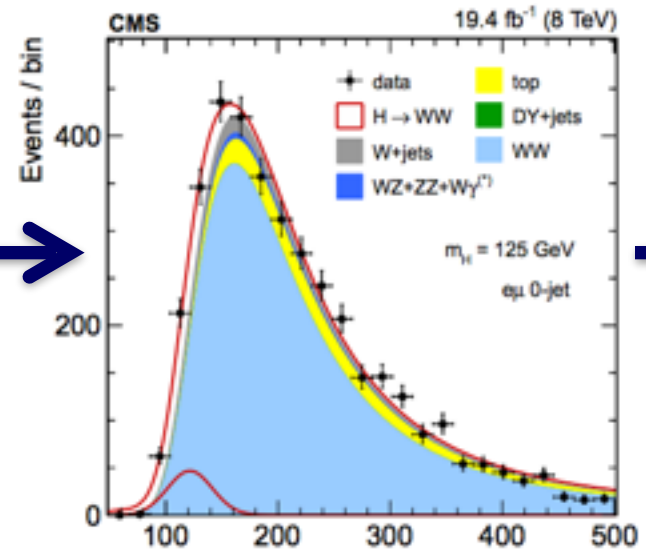
CMS Collaboration, *Measurement of Higgs boson production and properties in the WW decay channel with leptonic final states*, arXiv:1312.1129v1 [hep-ex]

Resonant Higgs production

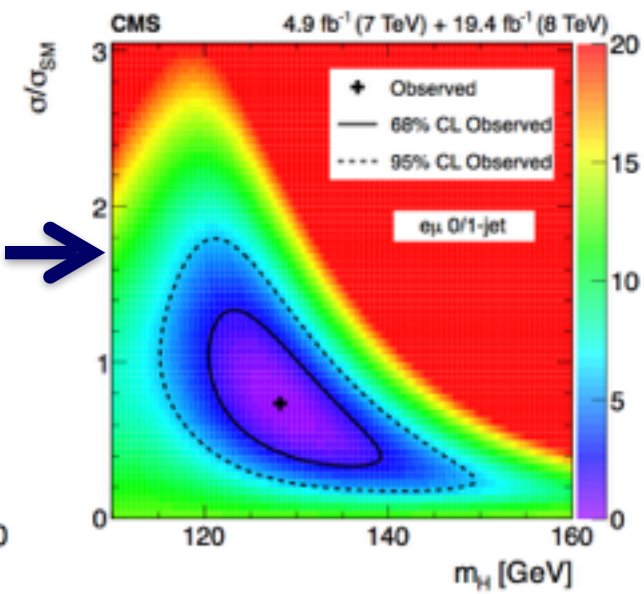
$H \to WW^* \to 2\ell 2\nu$



$\sqrt{\hat{s}_R}$ [GeV]



$\sqrt{\hat{s}_R}$ [GeV]



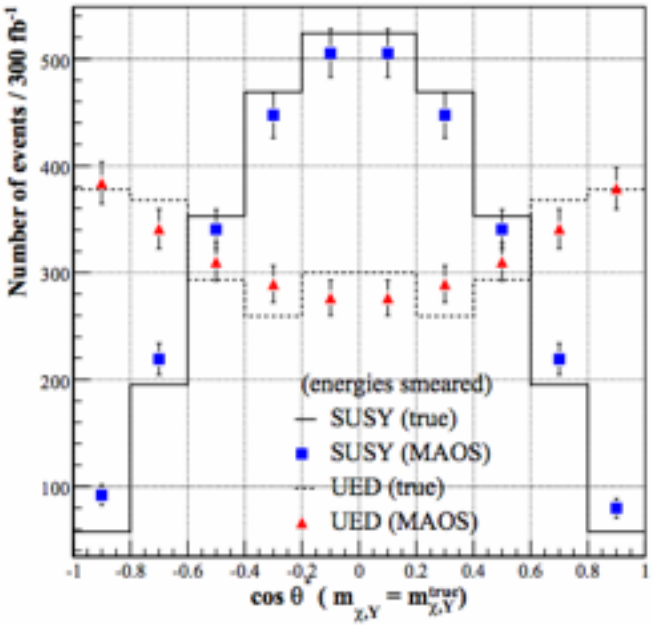
The shape of the $\sqrt{\hat{s}_R}$ distribution, for the Higgs signal and backgrounds, is used to extract both the Higgs mass and signal strength – even while information is lost with the two escaping neutrinos

From:

CMS Collaboration, *Measurement of Higgs boson production and properties in the WW decay channel with leptonic final states*, arXiv:1312.1129v1 [hep-ex]

What other info can we extract?

Ex. M_{T2} extremization assigns values to missing degrees of freedom – if one takes these assignments literally, can we calculate other useful variables?

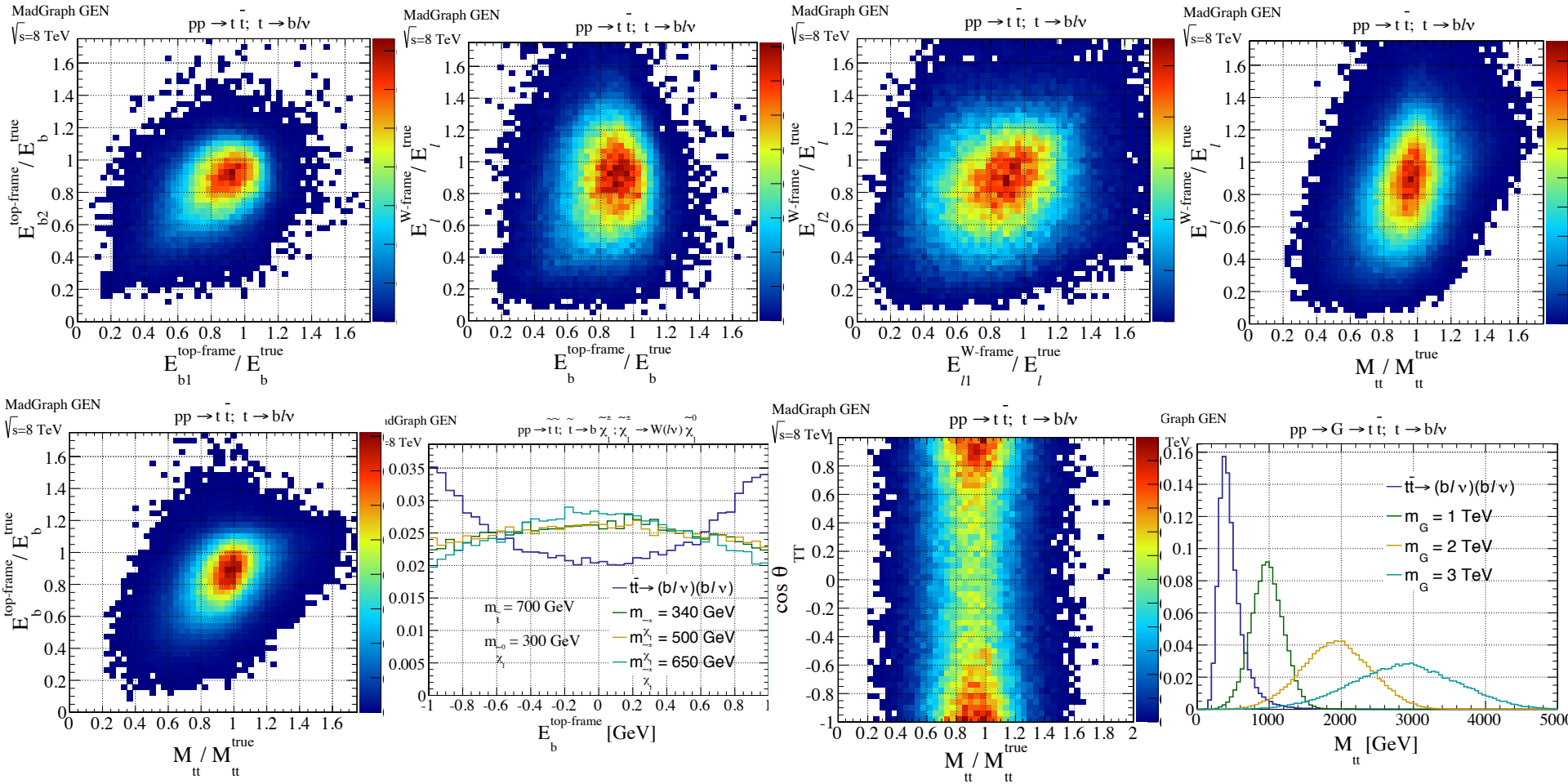


From:

Mass and Spin Measurement with M(T2) and MAOS Momentum - Cho, Won Sang et al. Nucl.Phys.Proc.Suppl. 200-202 (2010) 103-112 arXiv:0909.4853 [hep-ph]

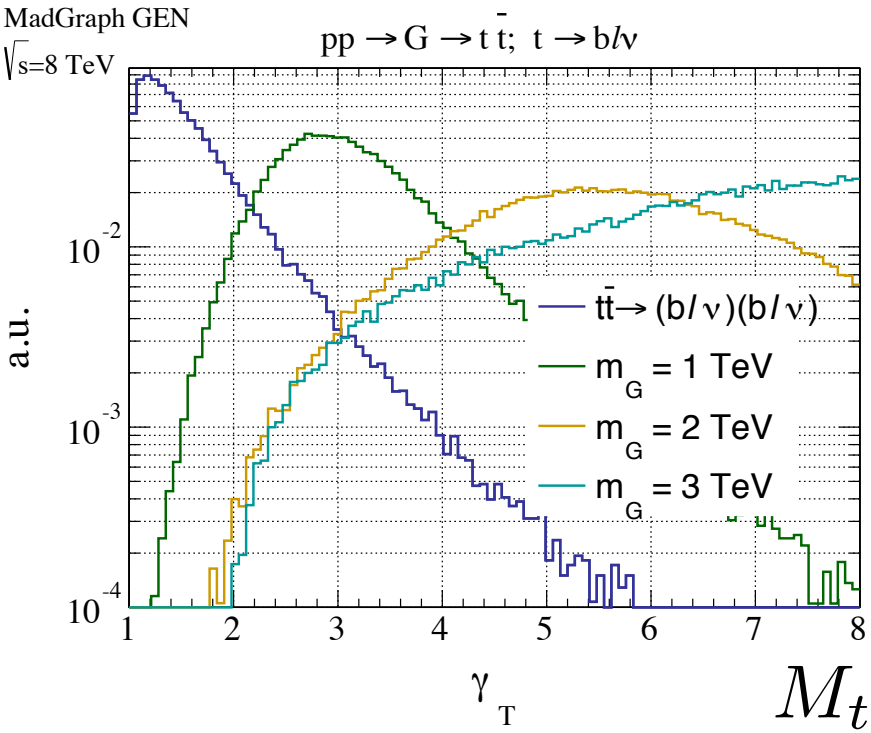
When we assign unconstrained d.o.f. by extremizing one quantity, what are the general properties of other variables we calculate? What are the correlations among them?

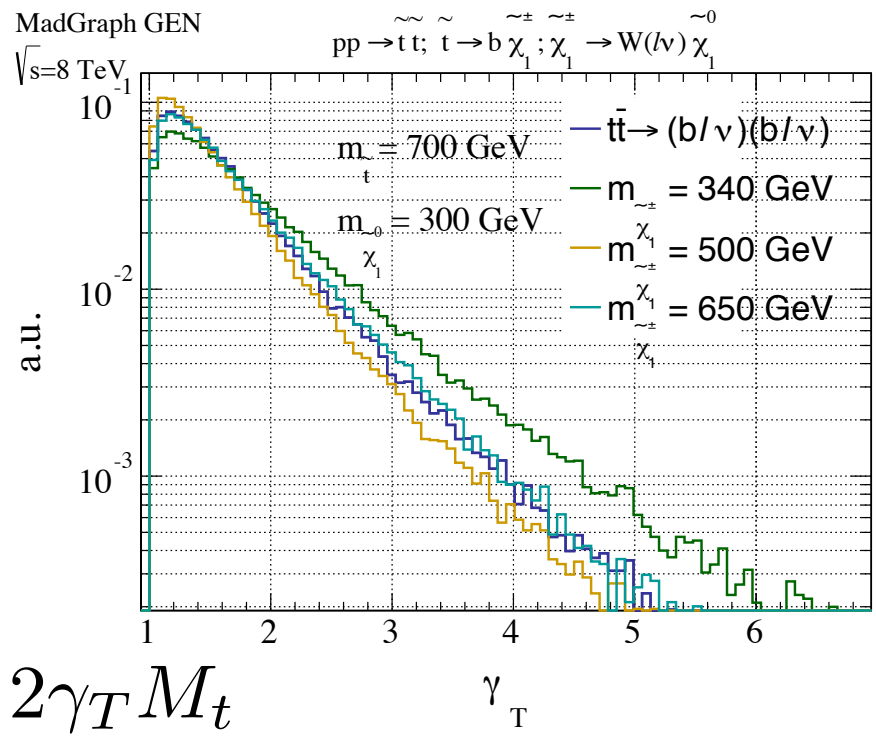
Example: the di-leptonic top basis



A rich basis of useful Recursive Jigsaw observables can be calculated, each with largely independent information

The di-leptonic top basis vs. stops

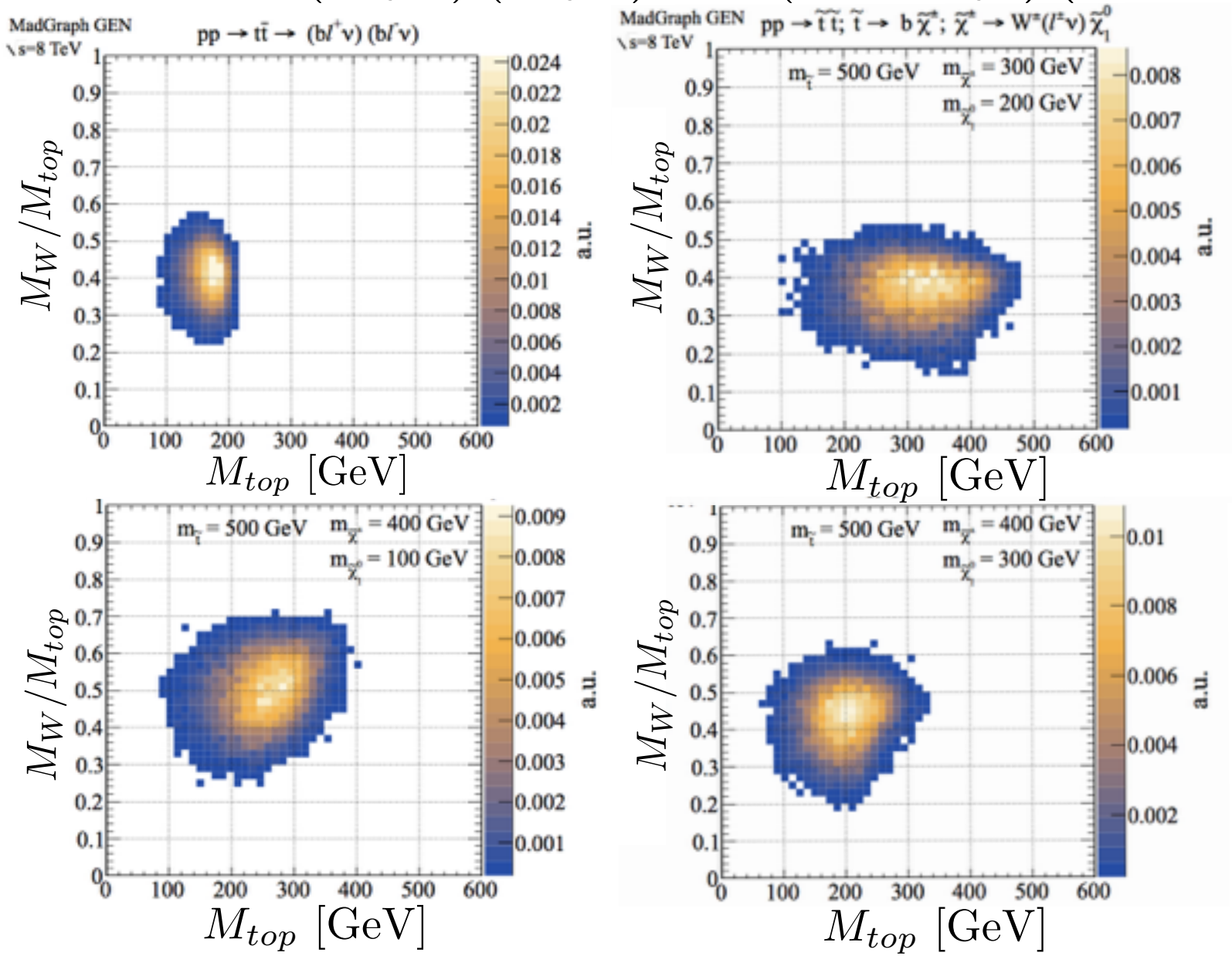




$$M_{tt} = 2\gamma_T M_t$$

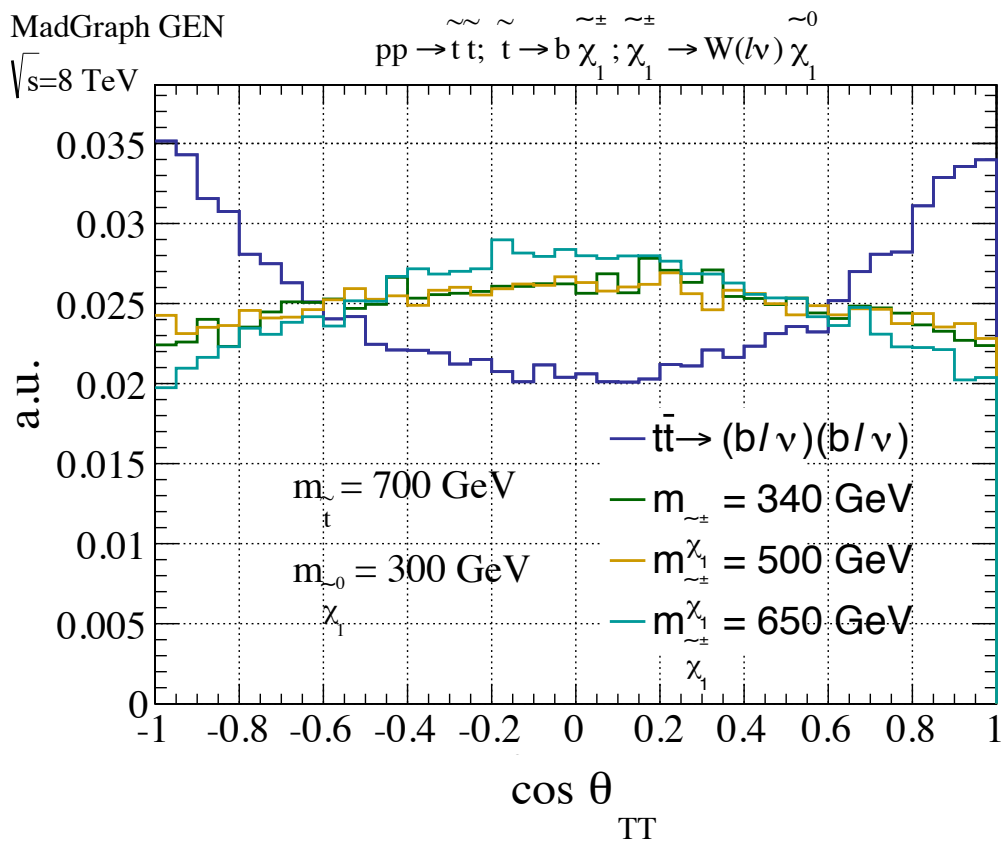
Observables sensitive to intermediate resonances cannot distinguish between non-resonant signals and background

$t\bar{t} \to (b\ell\nu)(b\ell\nu)$ vs. $\tilde{t}\tilde{t} \to (b\tilde{\chi}^{\pm})(b\tilde{\chi}^{\pm}) \to (b\ell^{\pm}\nu\tilde{\chi}^{0})(b\ell^{\pm}\nu\tilde{\chi}^{0})$



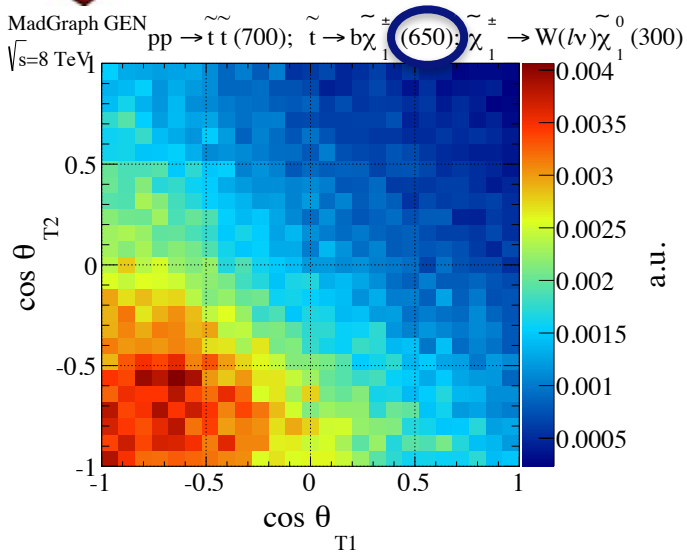
Multi-dimensional bump-hunting

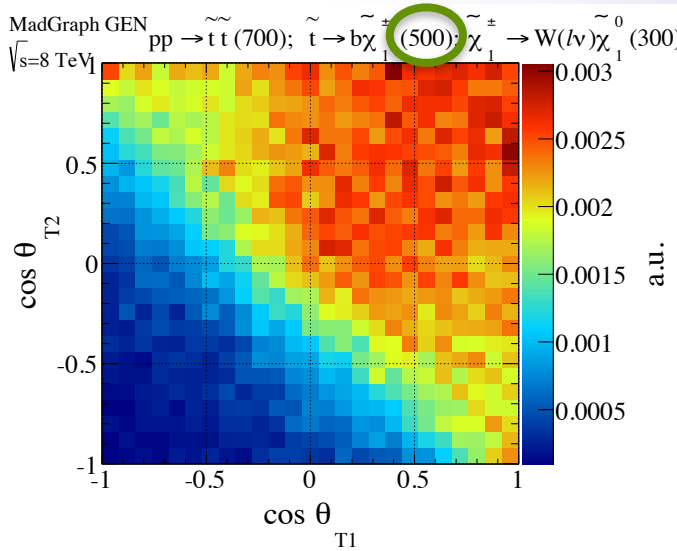
The di-leptonic top basis vs. stops



Decay angles are also sensitive to differences between stop signals and ttbar background

The di-leptonic top basis vs. stops

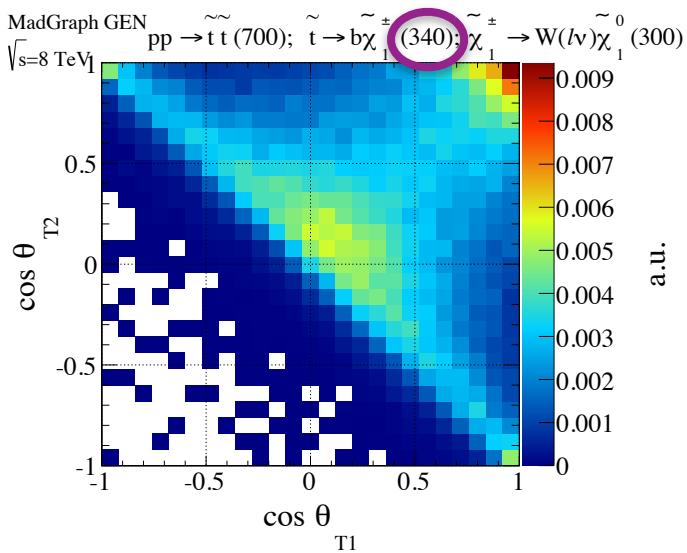


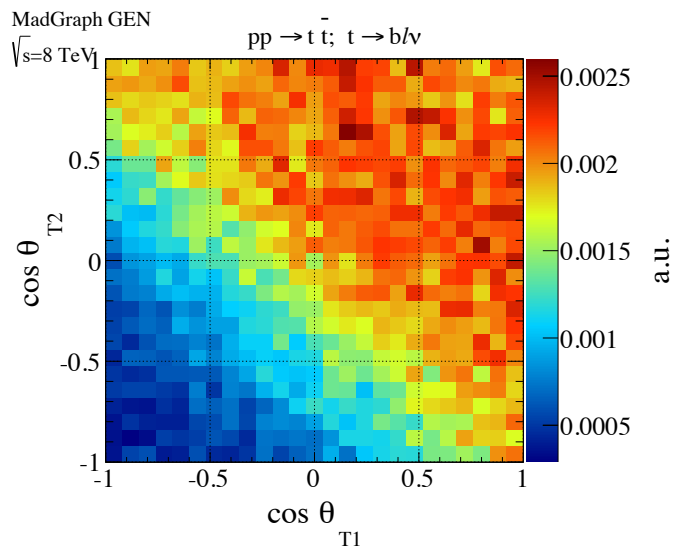


$m_{\tilde{t}} = 700$ GeV

$m_{\tilde{\chi}^{\pm}} =$ 650 / 500 / 340 GeV

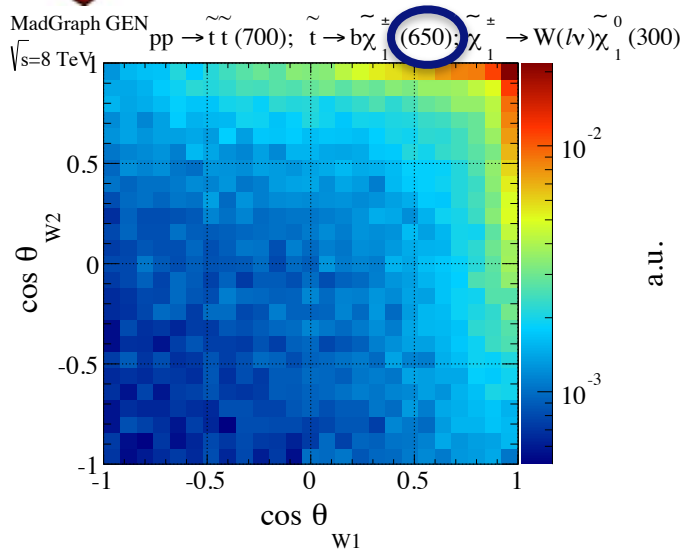
$m_{\tilde{\chi}^0} = 300$ GeV

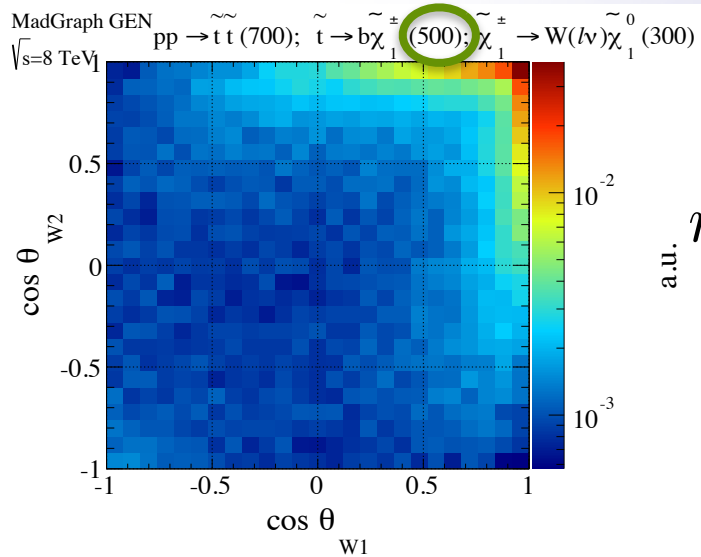




Decay angles are also sensitive to differences between stop signals and ttbar background

The di-leptonic top basis vs. stops





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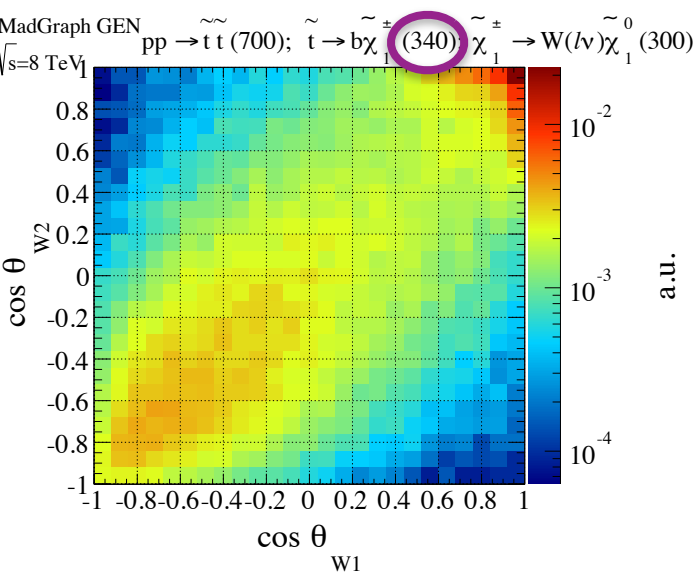
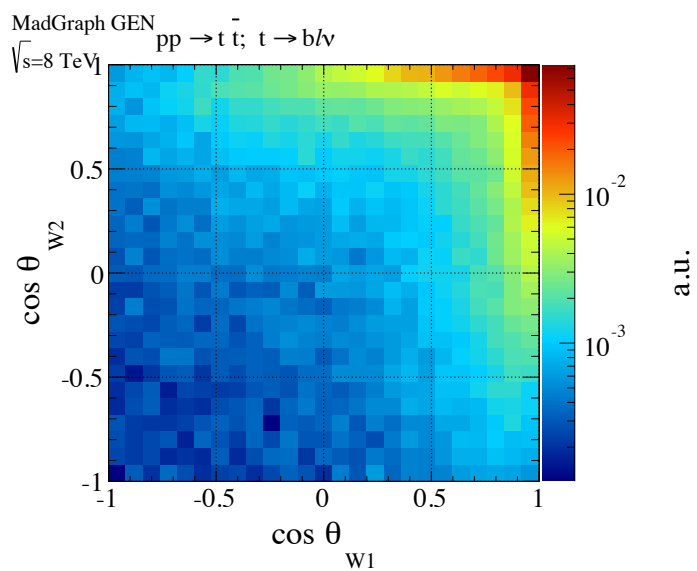
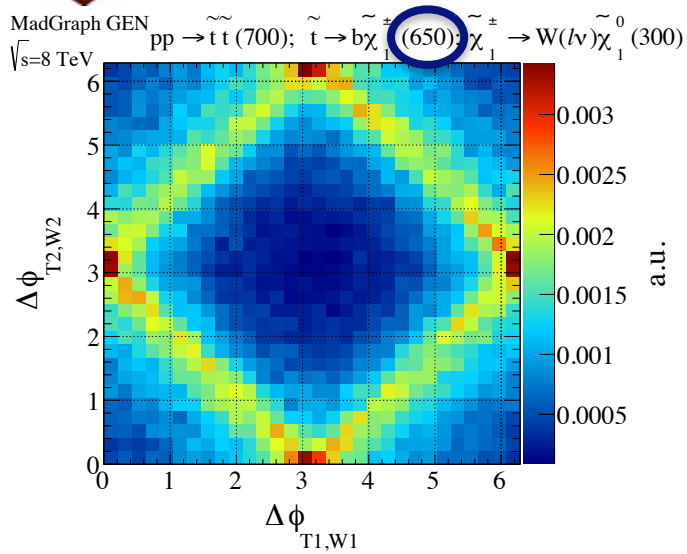
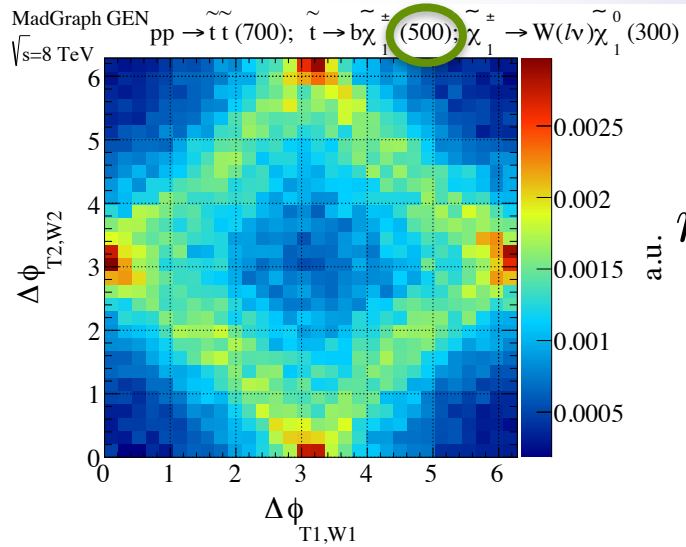




Decay angles are also sensitive to differences between stop signals and ttbar background

The di-leptonic top basis vs. stops





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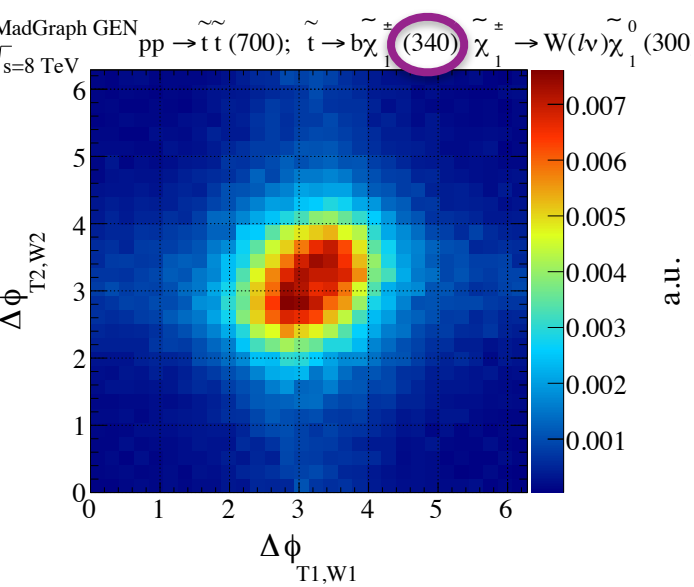
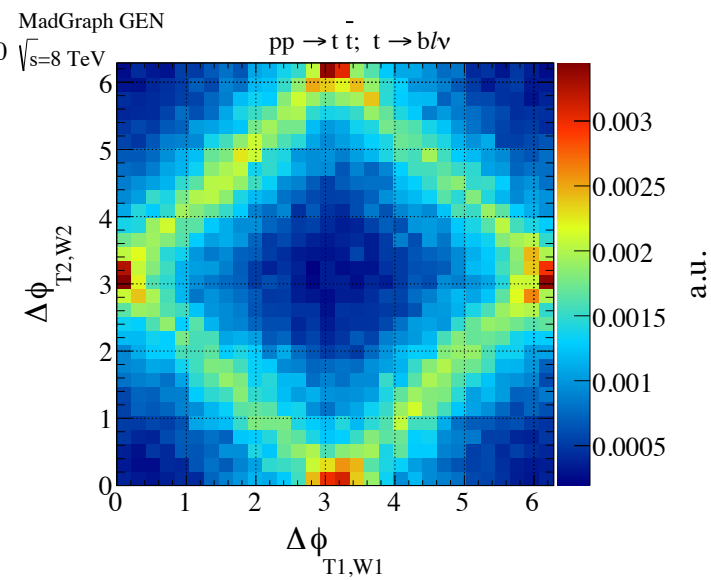
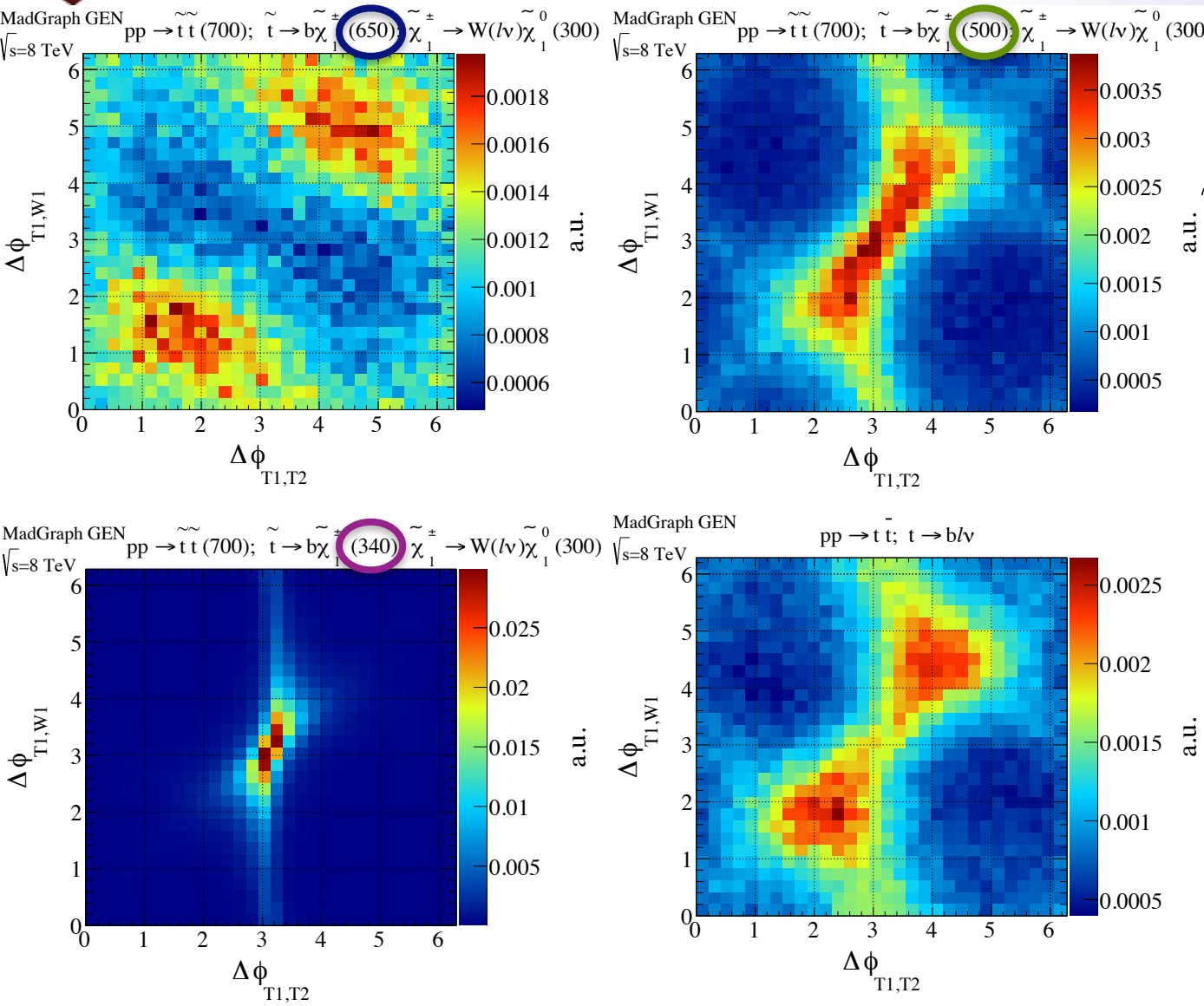




Here, the azimuthal angle between the the top and W decay planes $\Delta\phi_{T1,W1}$, for each of the two decay chains

The di-leptonic top basis vs. stops



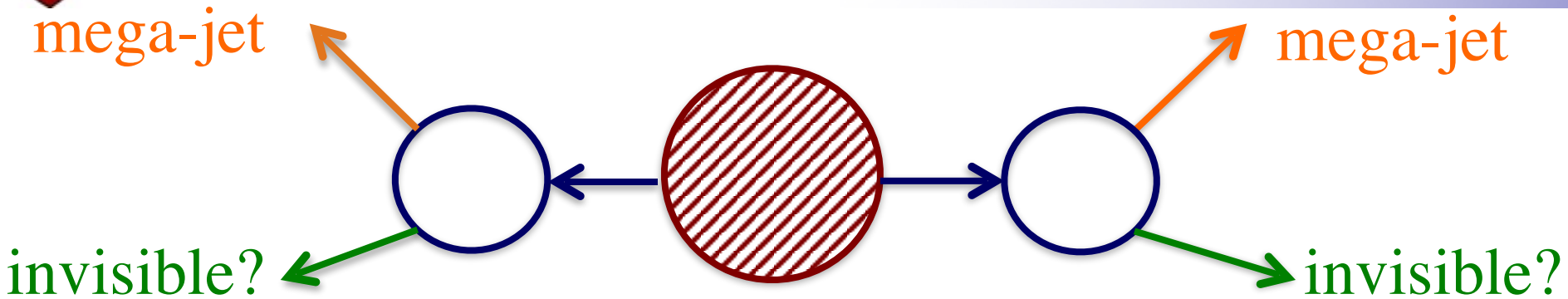
$m_{\tilde{t}} = 700$ GeV

$m_{\tilde{\chi}^{\pm}} =$ 650 / 500 / 340 GeV

$m_{\tilde{\chi}^{0}} = 300$ GeV

Here, the azimuthal angle between the the top and W decay planes $\Delta\phi_{T1,W1}$ and the angle between the two top decay planes $\Delta\phi_{T1,T2}$

Razor kinematic variables

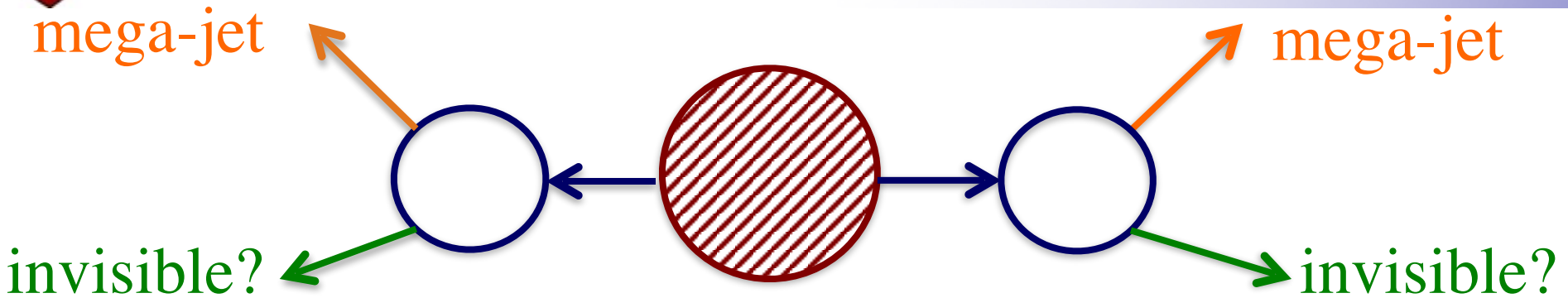


- Assign every reconstructed object to one of two mega-jets
- Analyze the event as a 'canonical' open final state:
  - two variables: M_R (mass scale) , R (scale-less event imbalance)
- An inclusive approach to searching for a large class of new physics possibilities with open final states

Razor variables — arXiv:1006.2727v1 [hep-ph]

CMS+ATLAS analyses — PRD 85, 012004 (2012); EPJC 73, 2362 (2013); PRL 111, 081802 (2013); CMS-PAS-SUS-13-004

Razor kinematic variables



- Assign every reconstructed object to one of two mega-jets
- Analyze the event as a 'canonical' open final state:
  - two variables: M_R (mass scale) , R (scale-less event imbalance)

$$M_R \sim \sqrt{\hat{s}} \qquad R = \frac{M_T^R}{M_R} \sim \frac{M_\Delta}{\sqrt{\hat{s}}}$$

Two distinct mass scales in event
Two pieces of complementary information

A Monte Carlo analysis to compare

- Baseline Selection From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])
  - Exactly two opposite sign leptons with $p_T > 20$ GeV/c and $|\eta| < 2.5$
  - If same flavor, $m(\ell\ell) > 15$ GeV/c2
  - ΔR between leptons and any jet (see below) > 0.4
  - veto event if b-tagged jet with $p_T > 25$ GeV/c and $|\eta| < 2.5$
- Kinematic Selection

'CMS selection'

$$|m(\ell\ell) - m_Z| > 15 \text{ GeV}$$

$$E_T^{miss} > 60 \text{ GeV}$$

CMS-PAS-SUS-12-022

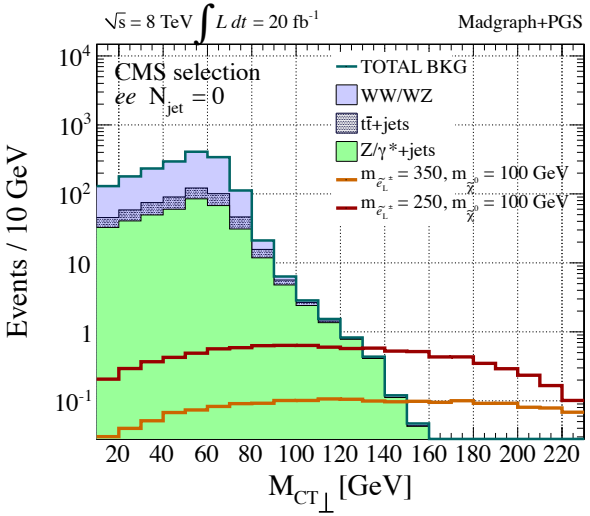
'ATLAS selection'

$$|m(\ell\ell) - m_Z| > 10 \text{ GeV}$$

$$E_{\mathrm{T}}^{\mathrm{miss,rel.}} = \begin{cases} E_{\mathrm{T}}^{\mathrm{miss}} & \text{if } \Delta\phi_{\ell,j} \geq \pi/2 \\ E_{\mathrm{T}}^{\mathrm{miss}} \times \sin\Delta\phi_{\ell,j} & \text{if } \Delta\phi_{\ell,j} < \pi/2 \end{cases} > 40 \text{ GeV}$$

ATLAS-CONF-2013-049

1D Shape Analysis



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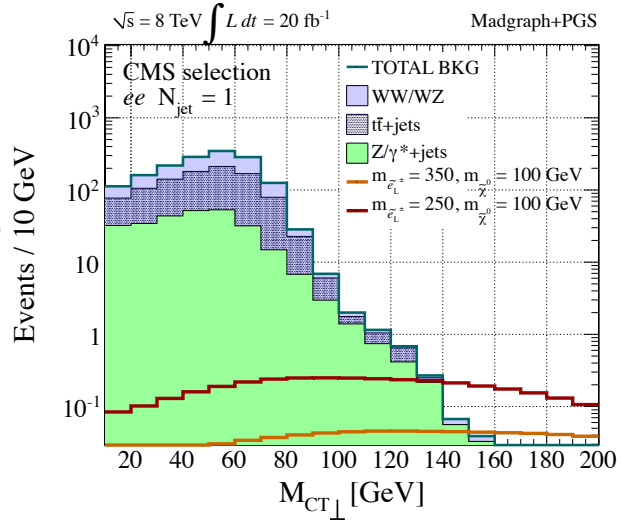


+

From PRD 89, 055020

Other jet multiplicity and lepton flavor categories

- Analysis Categories
  - Consider final 9 different final states according to lepton flavor and jet multiplicity – simultaneous binned fit includes both high S/B and low S/B categories

$$(ee, \mu\mu, e\mu) \times (0, 1, \geq 2 \text{ jets}) \quad \text{with } p_T^{jet} > 30 \text{ GeV}/c, \ |\eta^{jet}| < 3$$

Fit to kinematic distributions (in this case, $M_\Delta{}^R$, M_{T2} or M_{CTperp} in 10 GeV bins), over all categories for WW, $t\bar{t}$ and $Z/\gamma^* + \text{jets}$ yields

Systematic uncertainties

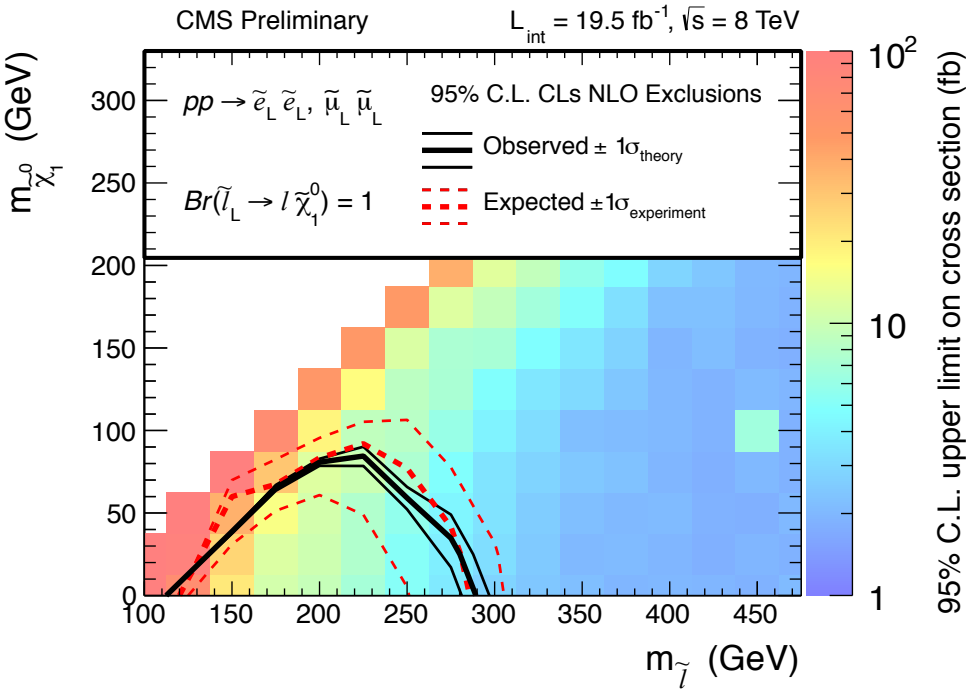
From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])

- 2% lepton ID (correlated btw bkgs, uncorrelated between lepton categories)
- 10% jet counting (per jet) (uncorrelated between all processes)
- 10% x-section uncertainty for backgrounds (uncorrelated) + theoretical x-section uncertainty for signal (small)
- 'shape' uncertainty derived by propagating effect of 10% jet energy scale shift up/down to MET and recalculating shapes templates of kinematic variables
- Uncertainties are introduced into toy pseudo-experiments through marginalization (pdfs fixed in likelihood evaluation but systematically varied in shape and normalization in toy pseudo-experiment generation)

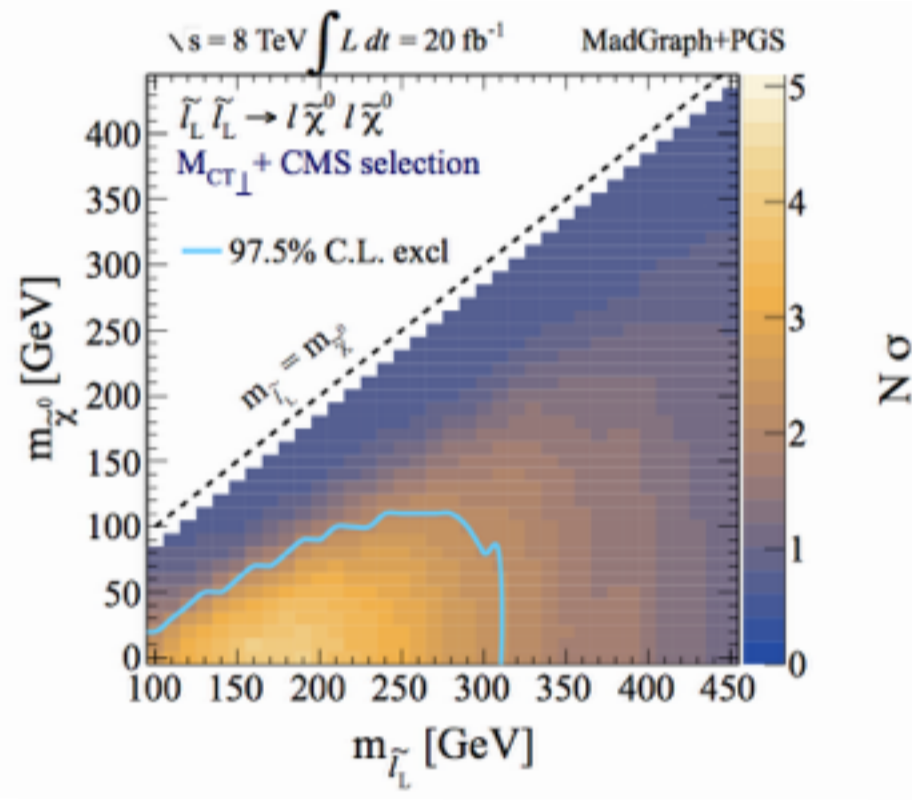
Compared to Reality

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])

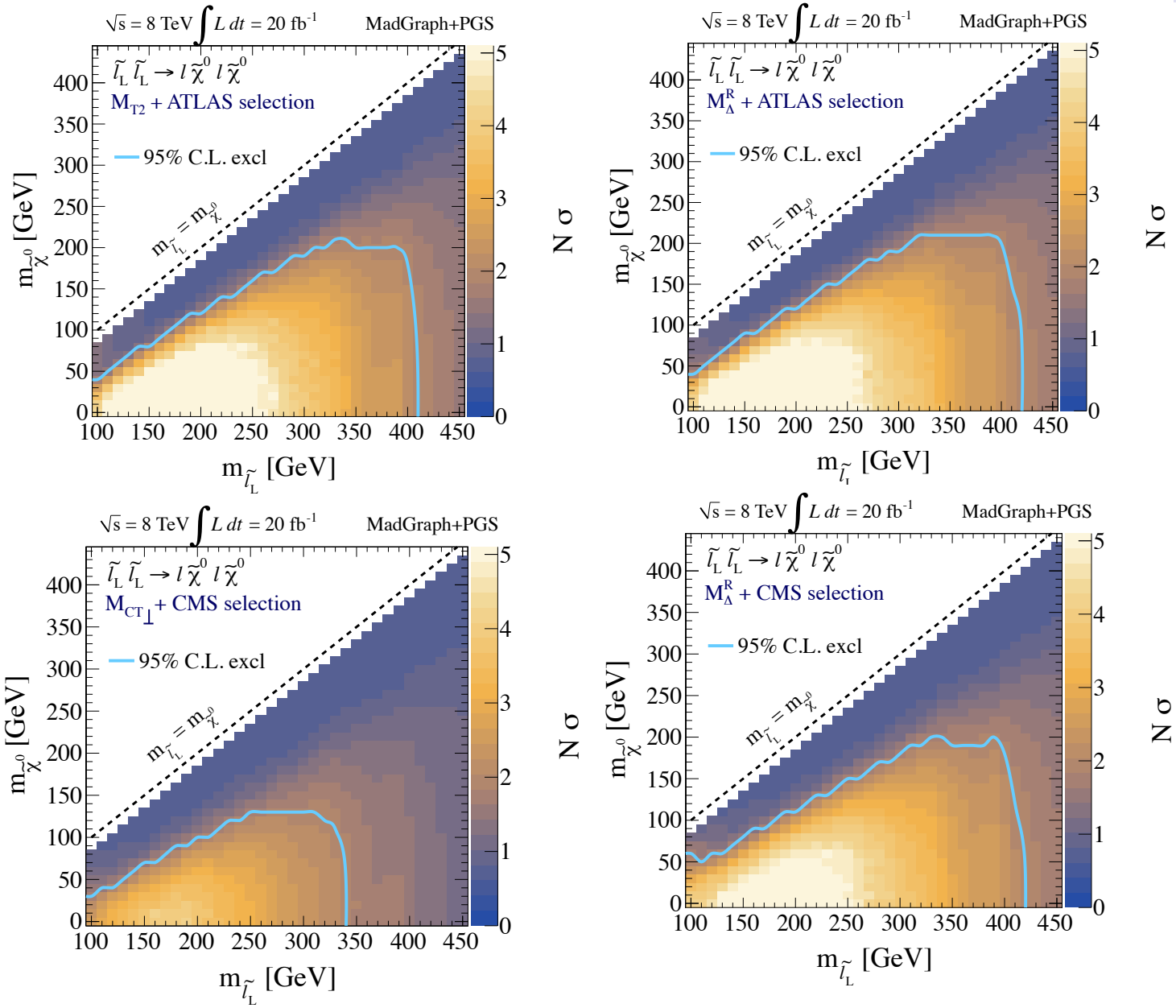
$$pp \to \tilde{\ell}_L \tilde{\ell}_L; \quad \tilde{\ell}_L \to \tilde{\chi}_1^0 \ell$$



CMS-PAS-SUS-12-022



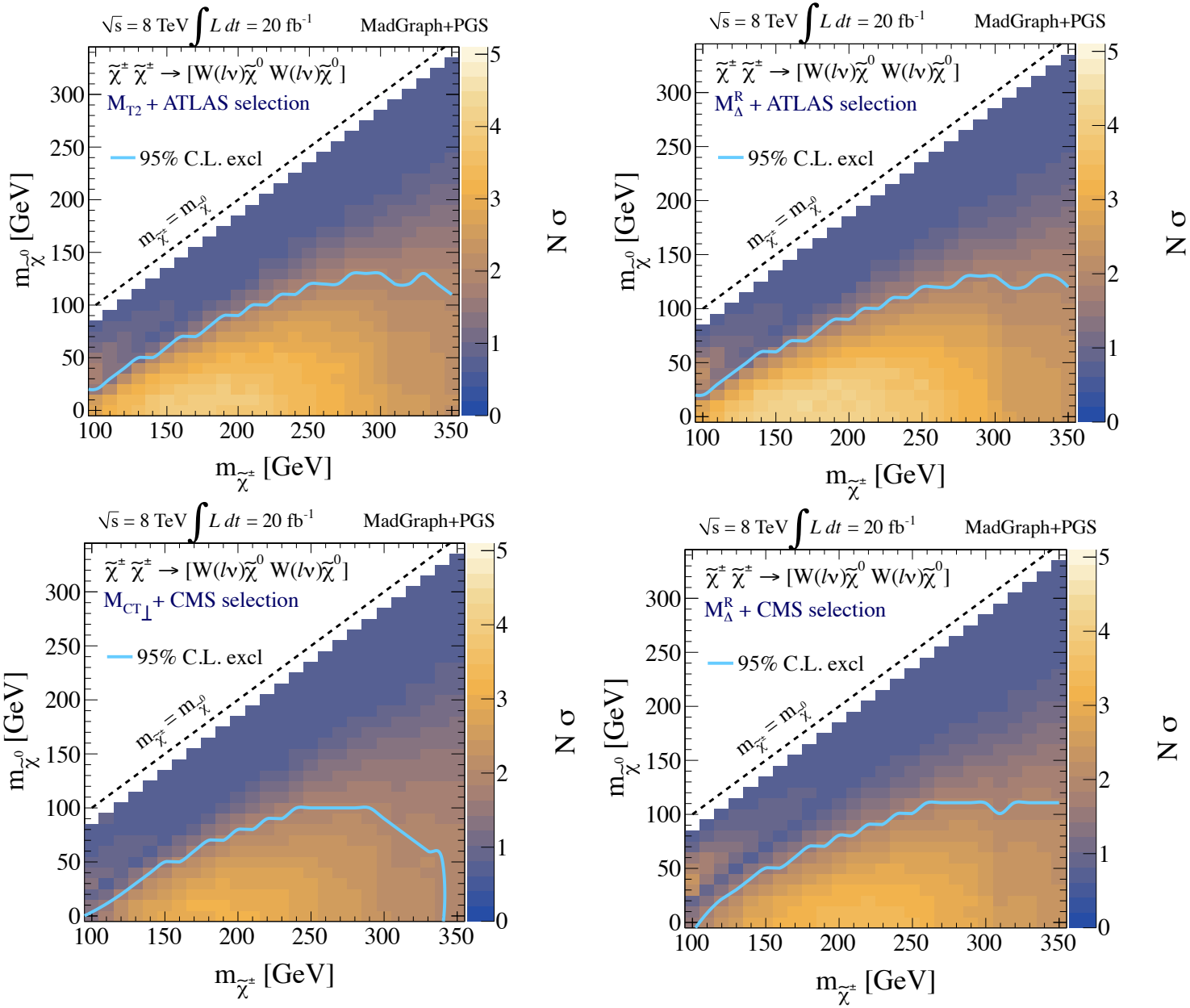
Expected Limit Comparison



$pp \to \tilde{\ell}_L \tilde{\ell}_L; \ \tilde{\ell}_L \to \tilde{\chi}_1^0 \ell$

From PRD 89, 055020 (arXiv:1310.4827)

Charginos

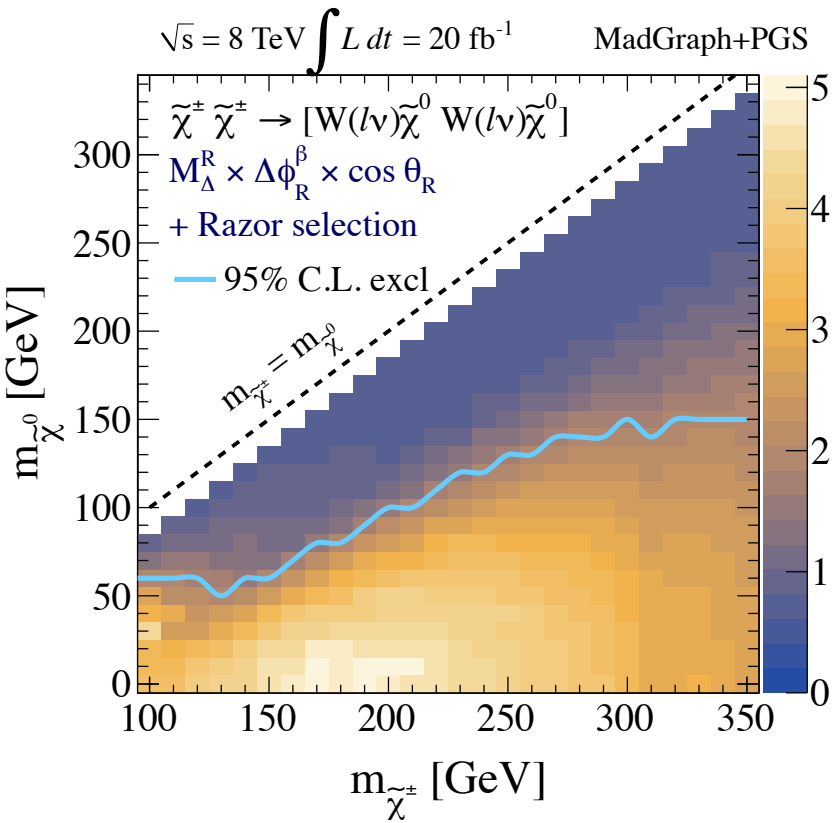


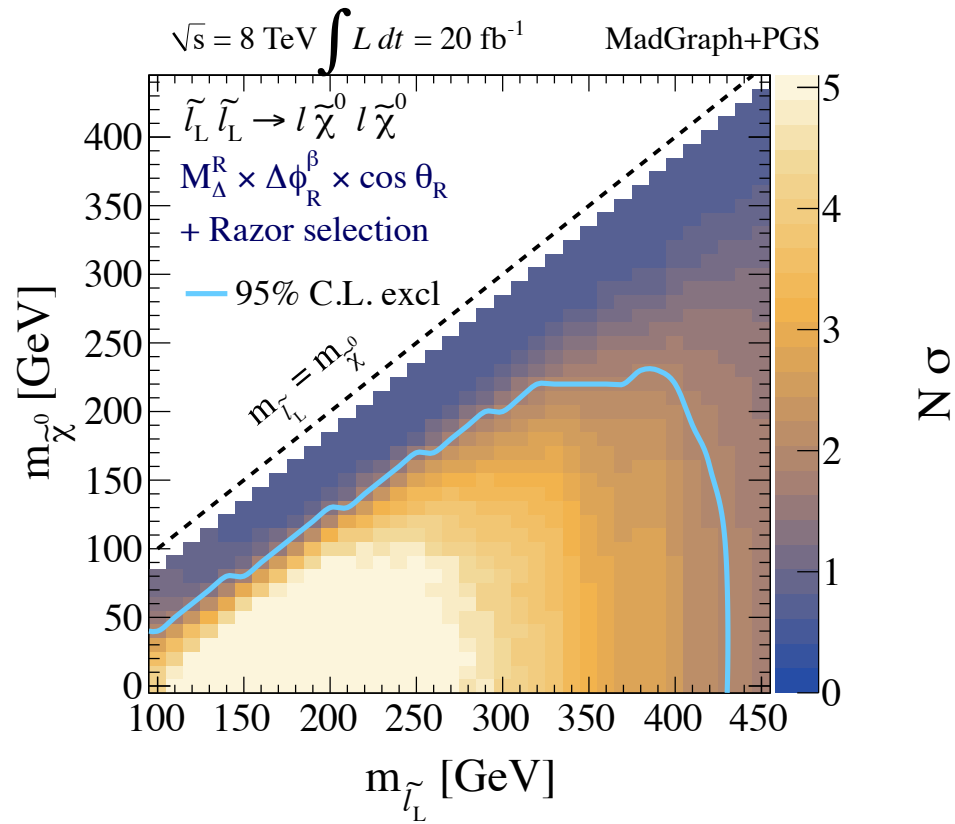
$pp \to \tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\pm}$; $\tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 W^{\pm}(\ell^{\pm}\nu)$

From PRD 89, 055020 (arXiv:1310.4827)

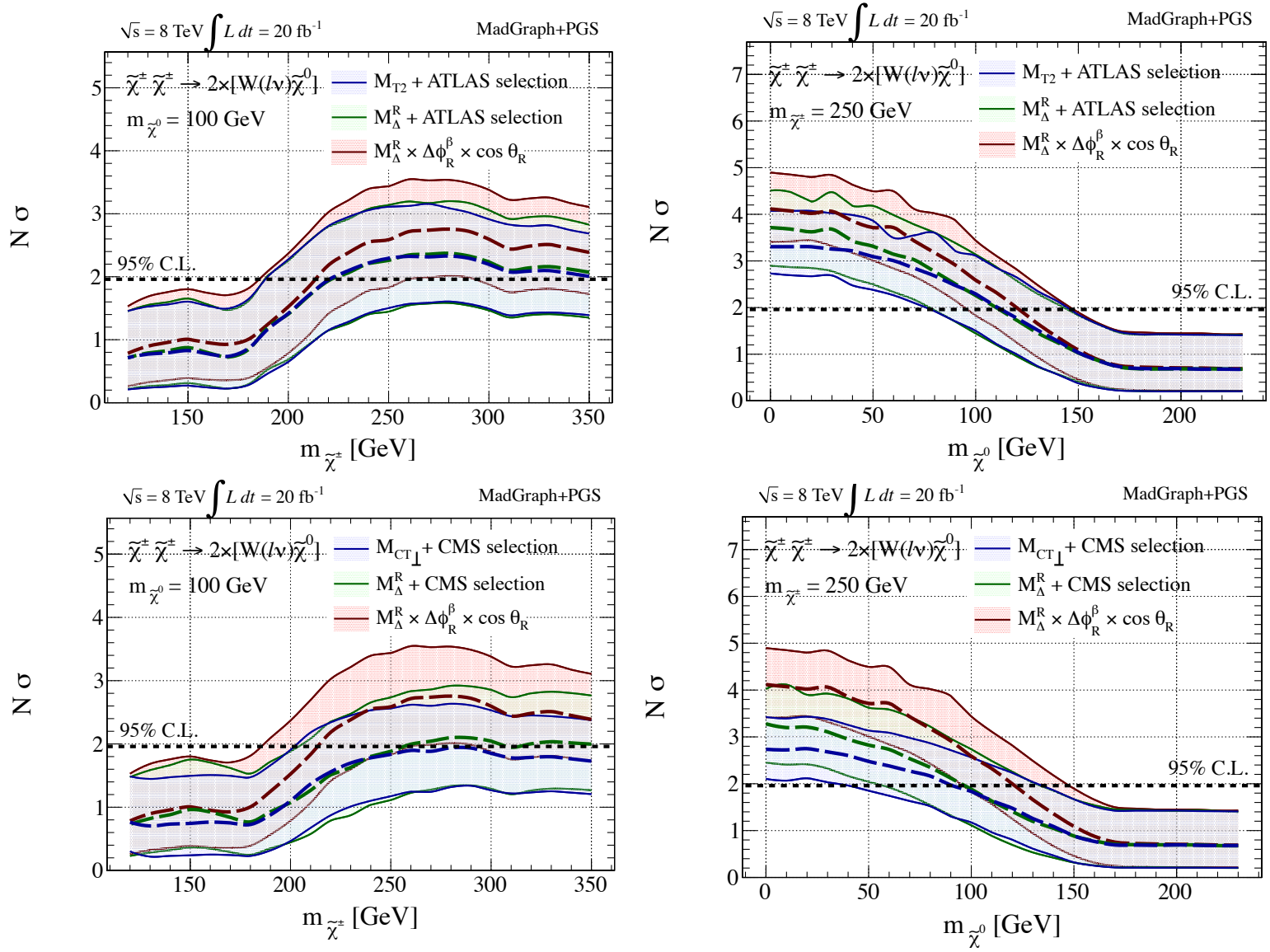
Super-Razor Basis Selection

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])





Comparisons



From PRD 89, 055020 (arXiv:1310.4827)