Physics 141A
Problem Set 2

Due Monday, Oct. 15 (hand in in class). This set is long– apologies. Start early.

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General: Organization, Web page, Labs, Syllabus: See the P141A Information Sheet, Syllabus, and Textbooks posted in 'Files' on Canvas

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Time Management and Study Groups You need to work with your study group. The problem sets will go faster if you discuss the problems, with friends/colleagues, and you will have a deeper understanding. However, the work you hand in **has to be your own**.[1]

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Problems with answers, and recycled problems There are a limited number of easily-solved mechanics problems, and so one can find answers to most by searching on the web. We trust you to instead work them yourself; ask the TA's, me, or fellow students for help if you need it. Using books is not only fair but recommended; however, you should write out the solution with the book closed to make sure you *know* how.

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Formulae: $\beta = v/c$; $\quad \gamma^2 = 1/(1-\beta^2)$; $\quad \beta^2 = (\gamma^2-1)/\gamma^2$

The invariant length of the 4-vector x_0, x_1, x_2, x_3: $|x^\mu| = \sqrt{x_0^2 - x_1^2 - x_2^2 - x_3^2}$

Lorentz Transformation for a 'Boost' of Frame F along the x direction relative to frame F':

$$t' = \gamma t + \beta\gamma x \tag{1}$$

$$x' = \beta\gamma t + \gamma x \tag{2}$$

$$y' = y \tag{3}$$

$$z' = z \tag{4}$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \tag{5}$$

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Problem 1: Using the Lorentz Transformation- Einstein's 3rd Gedanken Experiment Casals is on a train moving at speed $\beta = 0.866$, corresponding to a Lorentz factor of $\gamma = 2$, down a set of tracks past a platform on which Primrose is standing very (too) close to the tracks. Casals is seated by a window in the middle of the train, equidistant-distant from both ends. Just as Casals is opposite Primrose both of them see two flashes of light, produced by a fast Light-Emitting Diode (LED) at each end of the train, which came towards them from opposite directions.

Please ignore the (transverse) distance between the Casals and Primrose at the moment they both see the flashes, as the train length is much longer than the transverse distance between their heads at that moment.). I recommend working in units in which $c = 1$ so that in F the 4-vector for the lamp at the head of the train is $(L/2, L/2, 0, 0)$, for example.

[1] If you have any questions on where the line is I recommend getting and reading Charles Lipson's book, *Doing Honest Work in College*, UC Press. I also once required two straying students to read Egil Krogh's book *Integrity*.

Please draw a good (not too small, clear, complete) diagram of the train in frame F.

1. In Casal's frame, write down the 4-vectors of the light flashes at the head and rear of the train.
2. In Primrose's frame, the frame of the platform, use the Lorentz transformation to find the 4 vectors for the flashes at the head and rear of the train.
3. Which lamp flashed first in each frame, and by how much?

Problem 2: Using the Lorentz Transformation- The 'Barn and the Pole Paradox'

Consider the following folk fable:

A rapidly-moving trolley car enters a shed used for washing the cars, with the track running into it and out the other side. The trolley is 30-feet long as measured by the conductor riding in it. The shed is measured to be 60 feet long by its staff. The shed has a door on each end that opens just as the front of the trolley reaches it and shuts just after the back of the train has gone by.

This is an express trolley, moving at $\gamma = 100$, $\beta = 0.99995$. A 'paradox' is that the staff inside the shed see the trolley as Lorentz-contracted so that at least for a time it was inside the shed with both doors closed. However the conductor sees the shed as shorter than the trolley, and so there's no way both doors could be closed at the same time with the car inside. Show that there is no paradox by calculating the arrival of the front and back ends of the trolley at each of the doors in each frame.
Step-wise:

1. Draw a picture using the conventions we use in class (i.e. the primed frame is the frame of the shed) showing the two coordinate frames, with origins and labeled axes. Label the lengths of the shed and trolley [2] . I suggest taking the origins of both frames to be $t = t' = 0$ and $x = x' = 0$ when the front of the trolley arrives at the first door.
2. Write down the 4-vectors for the following four events in the trolley's frame:
   (a) Front end of trolley reaches shed front door;
   (b) Front end of trolley reaches shed back door;
   (c) Back end of trolley reaches shed front door;
   (d) Back end of trolley reaches shed back door;
3. Transform the 4-vectors for the same four events into the frame of the shed.
4. Using appropriate numerical values, order the four events in time in the trolley's frame.
5. Using appropriate numerical values, order the four events, in time in the shed's frame.
6. In each of the 2 frames make a short summary of the time sequence of the 4 events. There will be a prize for inventing a test to find which history is correct.

[2] Please note that lengths and times in the primed frame are 'primed', and in the unprimed frame have no prime. For example, a length in frame F' is denoted by L'; the corresponding length in F is denoted by L.

Problem 3: Indices, Unit Vectors and Projection Operators

1. In Cartesian 3-space, give the explicit representation of $\hat{x}$,$\hat{y}$,$\hat{z}$
2. Show that $\hat{x_i} \cdot \hat{x_j} = \delta_{ij}$, where the Kronecker δ_{ij} is equal to 1 if $i = j$ and 0 otherwise (the indices i and j run from 1 to 3).
3. Show that $\sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} = 3$ (this is the sum of the diagonal elements, called the trace, of the matrix δ_{ij})
4. Find the unit vector in the $\vec{A}$ direction for$\vec{A} = (2, -3, 5)$
5. Using the geometric relationship $\vec{A} \cdot \vec{B} = |A||B|cos(\theta)$, show that the vector operator $\hat{n}\cdot$ applied to a vector $\vec{A}$ returns the projection of $\vec{A}$ along the $\vec{n}$ direction.

Problem 4: Rotations and Matrix Multiplication

1. Consider the vector $\vec{A} = (1, -2, 5)$. Find $\vec{A}\prime$ in a frame rotated about the y axis by -45^o.
2. Consider the vector $\vec{A} = (3, 2, 1)$. Find $\vec{A}\prime$ in a frame rotated about first the z axis and then the x axis, in each case by 45^o.
3. Consider the 4-vector $A^\mu = (3, 1, 2, 1)$. Find $A^{\mu\prime}$ in a frame rotated around the z axis by 45^o and then boosted along the y axis by velocity β. Calculate the invariant length before and after the transformations.

Problem 5: Addition of Velocities in the Same Direction(Easy)

1. A rocket ship moving at $\beta = 0.995$ away from Earth launches a small ship in the same direction with $\beta = 0.995$ relative to the mother ship. What is the velocity measured from earth?
2. A rocket ship approaching Earth at $\beta = 0.9999$ turns on a headlight pointed right at earth. Use the law of addition of velocities to calculate the velocity of the light from the headlight as seen on Earth.
3. A bottle is thrown backward from a moving car. If the car is moving at 80 feet/second and the bottle is thrown at 60 feet/second, how fast is is the bottle moving with respect to the ground?

Problem 6: Addition of Velocities- different directions
Consider a frame F moving in the x direction at velocity β_1 relative to a frame F', which is itself moving at velocity β_2 *in the y direction* relative to a frame F''. Find the Lorentz transformation from F to F''.

Problem 7: Conservation of Energy and Momentum

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Please don't freak when reading this- it's much easier than poles and sheds and flashing lights on trains. The key is to know that (E, p_x, p_y, p_z) is a 4-vector, with all quantities measured in electron volts ($c = 1$), and that each component is independently conserved.

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A Higgs boson is created at rest in a proton-antiproton collision at Fermilab. It decays into . The mass of a Higgs boson is 125 GeV (Joe Incandela, a former UC undergrad Physics major made the discovery announcement at CERN as Spokesperson for the CMS experiment); the mass of a b-quark (which is equal to the mass of an anti-b-quark) is $m_b = 5$ GeV [3]. The energies and momenta of the b-quark and the anti-bquark are equal (they have to be by momentum conservation). Find the momentum of the b-quark after the decay by making the following steps:

1. Write down the 4-vector for the Higgs boson in its rest frame before the decay (use M for the mass of the Higgs);
2. Write the 4-vector momenta for the b-quark and anti-quark in the Higgs rest frame after the decay (Use the symbols E_b and p_b, for example, for the energy and (magnitude of) momentum along the x axis for both the b and anti-b quarks.
3. Use conservation of energy to find the energy of the b-quark in the Higgs rest frame.
4. Use the invariance of the 'length' of the 4-vector of the b-quark to find the magnitude of the momentum $|\vec{p}|$ of the b-quark in the Higgs rest frame.
5. Plug in the numbers to get the momentum in GeV.

Extra Credit: Problem 8– the Twin 'Paradox' (not so easy)

Two identical twins flip a coin as to who gets to go on a trip to a distant inter-galactic resort on Virgin Spaceways. The ship accelerates rapidly to $v = 0.99995c$ ($\gamma = 100$), goes for 1 year, and rapidly decelerates to land at a distant rest-stop. After a brief visit, the process is reversed. Ignoring the time spent accelerating and visiting, the traveling twin is now 2 years older. How much time has elapsed on earth when she returns?

The following may (or may not) be helpful:

Suppose a light flashes every New Year's (midnight Dec 31, local time) on Earth and another flashes also on New Year's (local time) on the spaceship. Draw a space-time diagram (t vs x) of the motion of the ship and the propagation of the light flashes. List the times when the twin on the ship sees flashes from Earth and when the twin on Earth sees flashes from the ship. (you are welcome to code this up if you program- but it's instructive to draw it by hand first.)

[3] c=1, so mass, momentum, and energy all have the same units, GeV.