재미한인물리학자협회
Association of Korean Physicists in America
Institute for Basic Science

Holographic Principle
–towards the new paradigm–

Bum-Hoon Lee
Center for Quantum Spacetime(CQUeST)/Dept. of Physics
Sogang University
Seoul, Korea

서강대학교 Sogang University

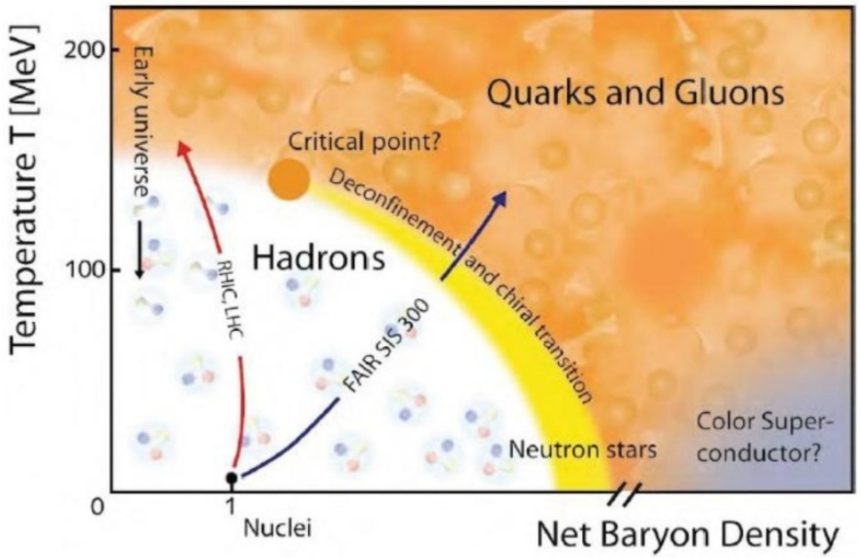
AKPA-IBS Symposium
@Chicago, May 9, 2014

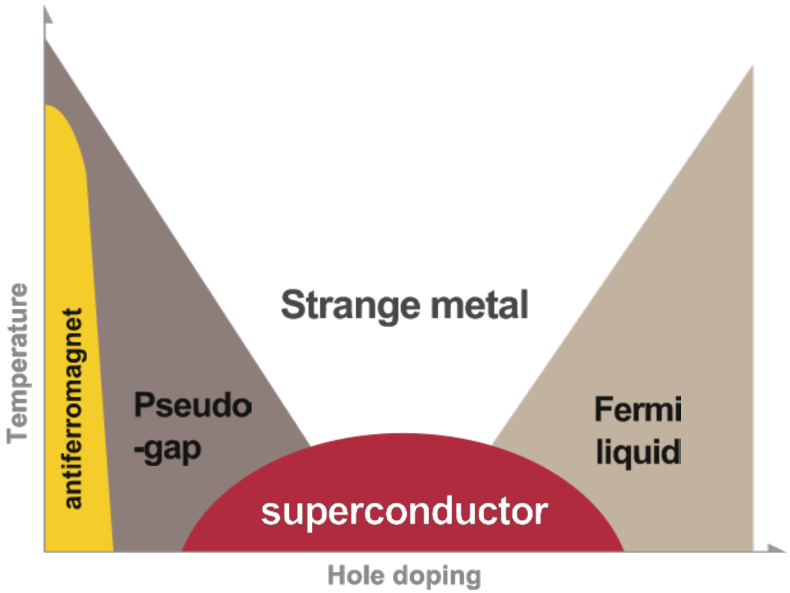
I. Introduction : Motivation & Basics

Q : How to understand the nonperturbative physics of the strongly interacting systems ?

Ex) In QCD, how to explain confinement, chiral symmetry breaking, phases (with or w/o chemical potential), meson spectra etc. ?

Ex) How to understand the phenomena in the Strongly correlated condensed matter systems?





AdS−CFT Holography : 3+1 dim. QFT ⇔ 4+1 Classical Gravity Theory

- Useful tool for strongly interacting systems such as QCD, Composite (Higgs) particles?, Condensed Matter, etc.

Contents

II. Holography Principle (AdS/CFT Correspondence)

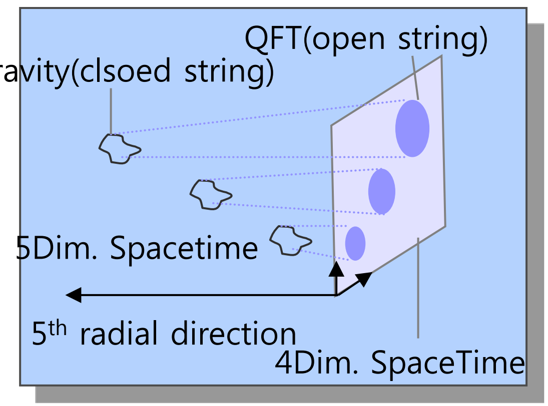
"2nd revolution of the string theory (1994)

「Quantum Field Theory in a given space time dimension (Ex):3+1=4 dim) can be equivalently described by the classical gravity theory in one higher dimensional spacetime (Ex): 4+1=5dim).

(Ex: p = 3)

(Classical) Gravity Theory
Anti de Sitter (AdS) Space
in (p+1)+1 dimension

time radial



(Quantum) Field Theory
Conformal Field Theory (CFT)
in p+1 dimension

space time

- Quantum Field Theory is the natural language to describe the many-body system

Main idea on holography through the Dp branes

Dp−brane carry tension(energy) and charges (for p+ 2 form)

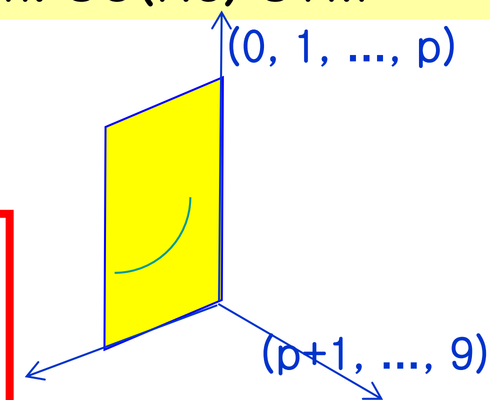
⇒ SUGRA on AdS (p+2) space

Dp−branes

Dp brane's low E dynamics by fluctuating Open Strings

= (p+1)dim. SU(Nc) SYM

#Dp−branes = Nc



Question : 4 = 5 ?

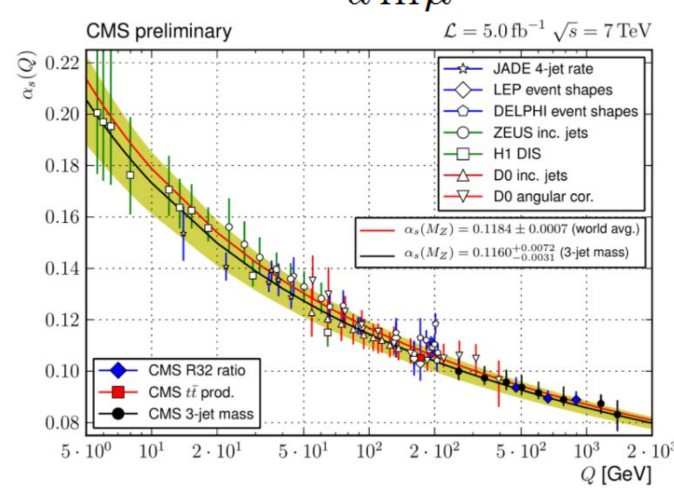
3+1 dim. QFT (large Nc)
⇔
4+1 dim. Effective Gravity description

Naïve Answer : Coupling constants are running in QFT !

$$\beta(g^2) = \frac{dg^2(\mu)}{d\ln\mu}$$

Energy scale in QFT corresponds to
the parameter in extra "dimension"
or radial direction in AdS5 space

$$g_s = e^{\phi(r)} = g^2_{YM}(\mu)$$



New Paradigm for the Strongly Interacting Quantum System

‘Size’ L of the 5dim is proportional to the coupling constant λ of the 4 dim.

4Dim QFT	Perturbative : Easy	Nonperturbative : Hard
Coupling constant λ	$\lambda \ll 1$	$\lambda \gg 1$
Size of the paramenter L	$L \ll 1$	$L \gg 1$
5Dim parameter	Quantum Gravity : Hard	Classical Gravity : “Easy”

- Strongly Interacting Quantum System ($\lambda >> 1$) $\leftrightarrow$ Classical Gravity ($L >> 1$)

New Methodolgoy : 4can use the 5 dim. classical gravity description for the 4dim. strongly interacting system.

4d QFT(on"boundary") ⇔ 5d Gravity (in"bulk")

Parameter $(g_{YM}^2\ ,\ N)$ $\quad 4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N \quad$ $(\ g_s\ ,\ R\ )$

Ex) Nc of D3 branes

N=4 SU(Nc) SYM ⟷ SUGRA on AdS5 x S5

Comments

- With $\beta(g^2) = 0$ (conf. inv.) In Anti-deSitter Space

N=4 SU(Nc) SYM ⟷ SUGRA on AdS5 x S5

Nc of D3 branes

$(g_{YM}^2\ ,\ N)$ $\quad 4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N \quad$ $(\ g_s\ ,\ R\ )$

- With $\beta(g^2) \to 0$ (conf. inv.) In asymptotic AdS Space

(ex) QCD ?? (in asymptotic AdS Space)

Intersecting D-Branes – Flavors, mesons, etc.

Ex) D3−D7 **Low energy dynamics** → **N=2 SYM with #M hyper**

CPX

0,1,2,3

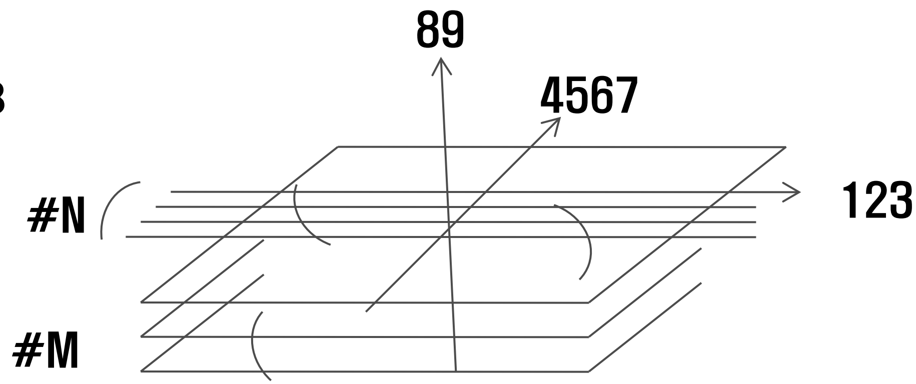
Strings

3−3 : A_μ, Φ, λ, χ

(N=2 Vector multiplet in adjoint

3−7 : Q, $\tilde{Q}$, ψ, $\tilde{\psi}$

(N=2 Hypermultiplet matter in fundamental)



7−7 open strings : Low energy dynamics for D7 branes (DBI action)

$$S_{D7} = -\mu_7 \int d^8\xi \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C^{(4)}] \wedge F \wedge F$$

$$\mu_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$$

Still far from QCD !

Extension of the AdS/CFT

- the gravity theory on the asymptotically AdS space
  -> modified boundary quantum field theory (nonconf, less SUSY, etc.)

- Gravity w/ black hole background corresponds to the finite T field theory

AdS/CFT Dictionary

Witten 98;
Gubser,Klebanov,Polyakov 98

Parameters (g_s , R) $4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N$ (g_{YM}^2 , N)

Partition function of bulk gravity theory (semi-classial)

$$Z_{str}[\phi_0(x)] = \int_{\phi_0} \mathcal{D}\phi \exp(-S[\phi, g_{\mu\nu}])$$

$$= e^{-S(\phi_{cl}[\phi_0])}$$

$\phi(t, \mathbf{x};\, u = \infty) = u^{\Delta-4}\phi_0(t, \mathbf{x})$ ϕ

ϕ_0 bdry value of the bulk field

Generating functional of bdry QFT for operator $\mathcal{O}$

$$Z[\phi_0(x)] = \left\langle \exp \int_{boundary} d^d x \phi_0 \mathcal{O} \right\rangle$$

$$= \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}$$

ϕ_0: source of the bdry op. $\mathcal{O}$

- ϕ: scalar → $S = \int d^4x du \sqrt{-g}\,(g^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2)$ $\phi(u) \sim u^{4-\Delta}\phi_0 + u^{\Delta}\langle\mathcal{O}\rangle$
- Correlation functions by $\frac{\delta^n Z_{\text{string}}}{\delta\phi_0(t_1,\mathbf{x}_1)\cdots\delta\phi_0(t_n,\mathbf{x}_n)} = \left\langle T\,\mathcal{O}(t_1,\mathbf{x}_1)\cdots\mathcal{O}(t_n,\mathbf{x}_n)\right\rangle_{\text{field theory}}$
- 5D bulk field Φ(t, x, u) ←→ Operator $\mathcal{O}$ (t, x)
  w/ 5D mass $E(\lambda, J_1, J_2, \cdots)$ ←→ w/ Operator dimesion $\Delta(\lambda, J_1, J_2, \cdots)$
- 5D gauge symmetry ←→ Current (global symmetry)
- Radial coord. r in the bulk is proportional to the energy scale E of QFT

$\mathcal{O}$(Operator in QFT) <-> ϕ (p-form Field in 5D)

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

Δ : Conformal dimension

m_5^2 : mass (squared)

Ex)

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3
$\langle \mathrm{Tr} G^2 \rangle$ Gluon cond .	dilaton	0	4	0
$\bar{q}_L \gamma^\mu q_L$ $\bar{q}_R \gamma^\mu q_R$ baryon density	vector w/ U(1)	1	3	0

fields in gravity

- massless dilaton
- scalar field with $m^2 = -\frac{3}{R^2}$
- m=0 vector field A_μ in the $SU(N_f)$ gauge group

<-> dual

operators of QCD

- gluon condensation $\langle \mathrm{Tr} G^2 \rangle$
- chiral condensation $\bar{q}_R q_L$
- mesons in the $SU(N_f)$ flavor group

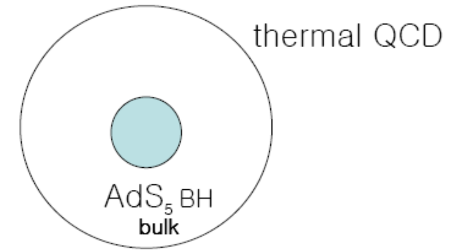
Temperatue

- Black hole gemometry
- $T = \frac{r_T}{\pi R^2}$

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\Big(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\Big),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



Flavor degrees of freedom

- Adding probe brane
- $y(\rho) = M_q + \frac{<\bar{\psi}\psi>}{\rho^2} + \cdots \qquad (\rho >> 1)$

Chemical potential or Density

- Turning on $U(1)$ gauge field on probe brane
- $A_\mu \leftrightarrow < J^\mu > = \bar{\psi}\gamma^\mu\psi$
- $A_t = \mu + \frac{Q}{\rho^2} + \cdots \qquad (\rho >> 1)$

Source of gauge field

- End point of fundamental strings
- Physical object which carry $U(1)$ baryon charge
- Fundamental strings which connect probe brane and black hole $\rightarrow$ Quarks
- Fundamental strings which connect probe brane and baryon vertex $\rightarrow$ Baryons

III. Application

Needs the dual geometry !.

Approaches :

- Top-down Approach : rooted in string theory
  Find brane config. or SUGRA solution giving the gravity dual (May put the probe brane)

  Ex) Nc of D3(D4) + M of D7(D8),
  10Dim. SUGRA solution etc.

- Bottom-up Approach:phenomenological
  Introduce fields, etc. (as needed based on AdS/CFT)
  5D setup → 4D effective Lagrangian

1. AdS/QCD (Holographic QCD)

Witten '98

Goal : Using the 5 dim. dual gravity, study 4dim. strongly interacting QCD such as spectra & Phases, etc.

parameters (Nc, Nf, m_q, T and μ, χ−symm., gluon condensation, etc.)

Ex) finte temperature for the pure Yang-Mills theory without quark matters

Low T QCD Phase transition High T

	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	Thermal AdS	AdS Black Hole

Hawking−Page transition
=Transition of bulk geometry

Hawking−Page phase transition

[Herzog , Phys.Rev.Lett.98:091601,2007]

The geometry is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

$\Lambda = -\frac{6}{R^2}$: cosmological constant

The geometry with smaller action is the stable one for given T.

$$\Delta S = \lim_{\epsilon\to 0}(S_{AdSBH} - S_{tAdS}) = \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4} - \frac{1}{2z_h^4}\right)$$

> 0 for T < Tc
< 0 for T>Tc

Holographic QCD for finite chemical potential

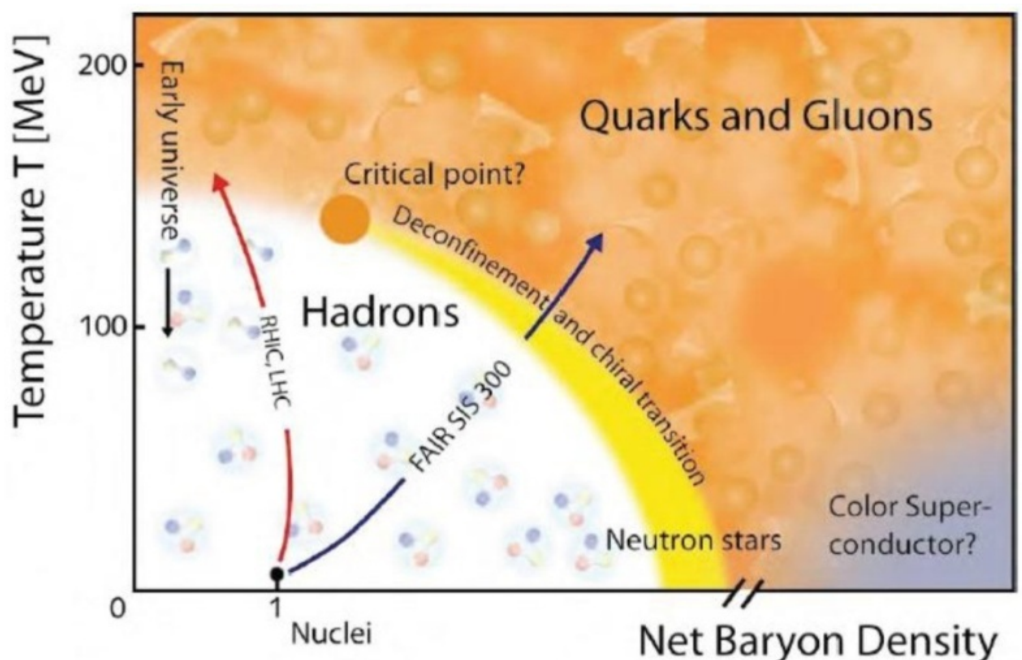
Low T QCD Phase transition High T

	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	thermal & charged AdS	RNAdS Black Hole

Hawking−Page transition (BHL, Park ,Sin JHEP 0907,(2009))

(q = 0)

Gravity (5Dim)	Thermal AdS	AdS Black Hole



For the fixed chemical potential

- dimensionless variables

$$\tilde{z}_c \equiv \frac{z_c}{z_{IR}}, \quad \tilde{\mu}_c \equiv \mu_c z_{IR}, \quad \tilde{T}_c \equiv T_c z_{IR},$$

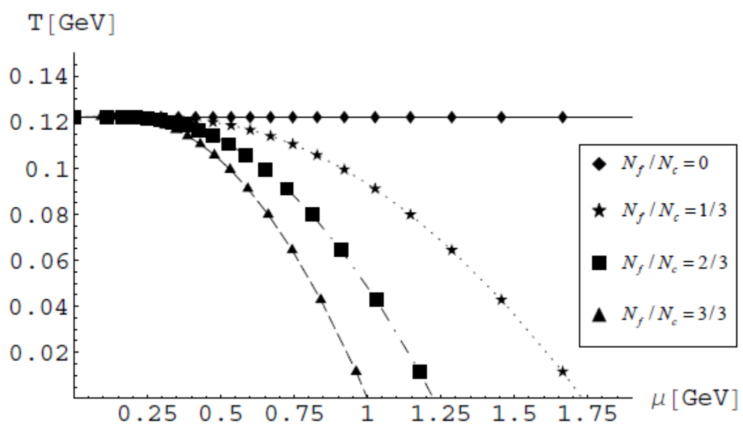
For the fixed number density

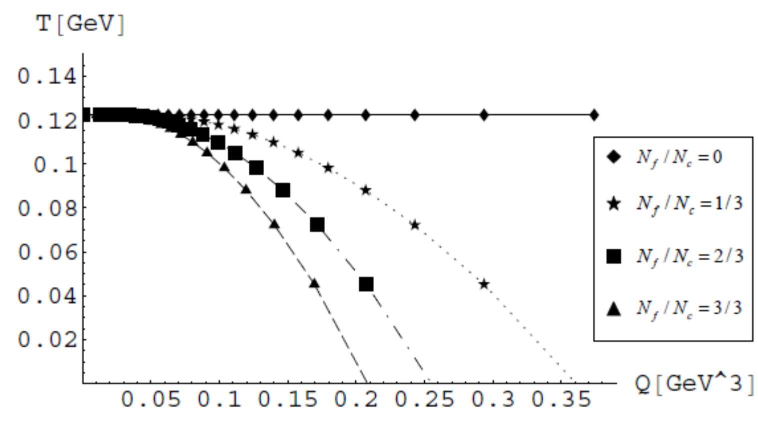
- Legendre transformation,

the Hawking-Page transition occurs at

$$\begin{cases} \tilde{\mu}_c = \sqrt{\frac{3N_c}{N_f}\frac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^2(9\tilde{z}_c^2-2)}}, \\ \tilde{T}_c = \frac{1}{\pi\tilde{z}_c}\left(1-\frac{1-2\tilde{z}_c^4}{9\tilde{z}_c^2-2}\right). \end{cases}$$

$$\begin{cases} \tilde{Q}_c = \sqrt{\frac{3N_c}{2N_f}\frac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^4(5\tilde{z}_c^2-2)}}, \\ \tilde{T}_c = \frac{1}{\pi\tilde{z}_c}\left[1-\frac{\tilde{z}_c^2}{2}\frac{(1-2\tilde{z}_c^4)}{(5\tilde{z}_c^2-2)}\right] \end{cases}$$





Light meson spectra in the hadronic phase

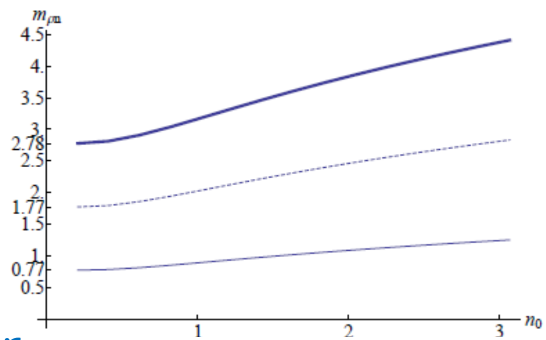
Jo, BHL, Park,Sin
JHEP 2010, arXiv:0909.3914

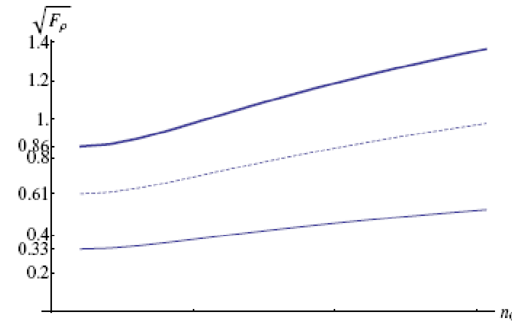
Turn on the fluctuation in bulk corresponding the meson spectra in QCD

$$\Delta S = \int d^5x\sqrt{G}\ \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right] \quad M_X^2 = -3/l^2$$

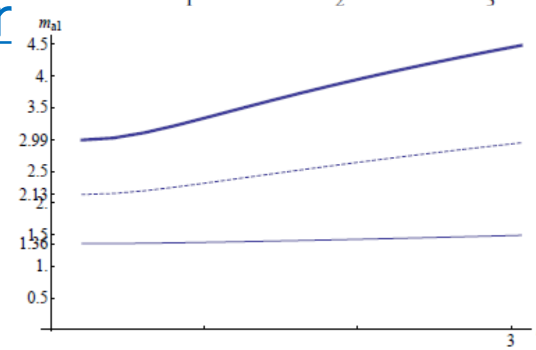
X is the dual to the quark bilinear operator <qbar q >.

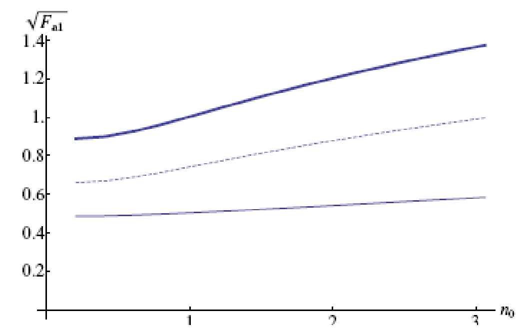
1. Vector meson



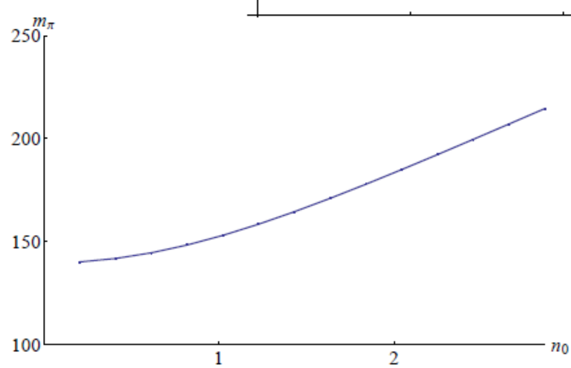


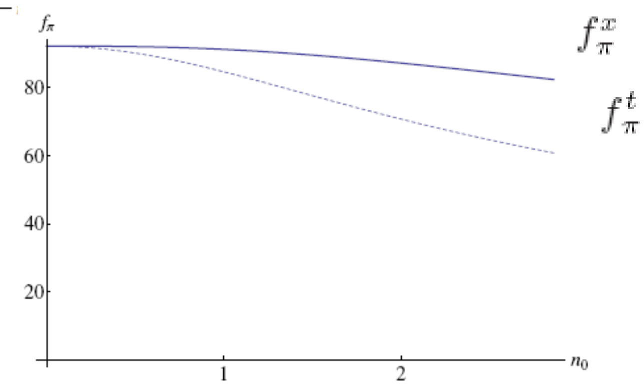
2. Axial vector meson





3. pion





Chiral Condensate Effect (w/ & w/o Density Effect)

C. Park, B-HL, S. Shin, arXiv:1112.2177.

The gravity action in the bulk is

$$\mathcal{S} = \int d^5x\sqrt{-G}\left\{\frac{1}{2\kappa^2}(\mathcal{R}-2\Lambda) - \mathrm{Tr}\left[|D\Phi|^2 + m^2|\Phi|^2 + \frac{1}{4g^2}\left(F^{(L)}_{MN}F^{(L)MN} + F^{(R)}_{MN}F^{(R)MN}\right)\right]\right\}$$

$$g^2 = \frac{12\pi^2 R}{N_c} \qquad \kappa^2 = \frac{\pi^2 R^3}{4N_c^2} \qquad N_f = 2 \quad N_c = 3 \qquad m^2 = -\frac{3}{R^2} \qquad \Lambda = -\frac{6}{R^2}$$

We set

$$\Phi(z) = \frac{1}{2\sqrt{N_f}}\phi(z)1_f e^{i2\pi^a(z)T^a}$$

The background geometries of our interest are obtained from a gravity action with the massive scalar field and U(1) gauge interaction.

$$\mathcal{S} = \int d^5x\sqrt{-G}\left\{\frac{1}{2\kappa^2}(\mathcal{R}-2\Lambda) - \frac{1}{4}\left[(\partial_M\phi)^2 + m^2\phi^2\right] - \frac{1}{4g^2}F_{MN}F^{MN}\right\}$$

an ansatz for the asymptotic AdS metric in the Fefferman-Graham coordinate as

$$ds^2 = \frac{R^2}{z^2}\left[-F(z)dt^2 + G(z)d\vec{x}^2 + dz^2\right]$$

We take R = 1.

$A_0 = A_0(z) \qquad A_M = 0$ M = 1, 2, 3, z

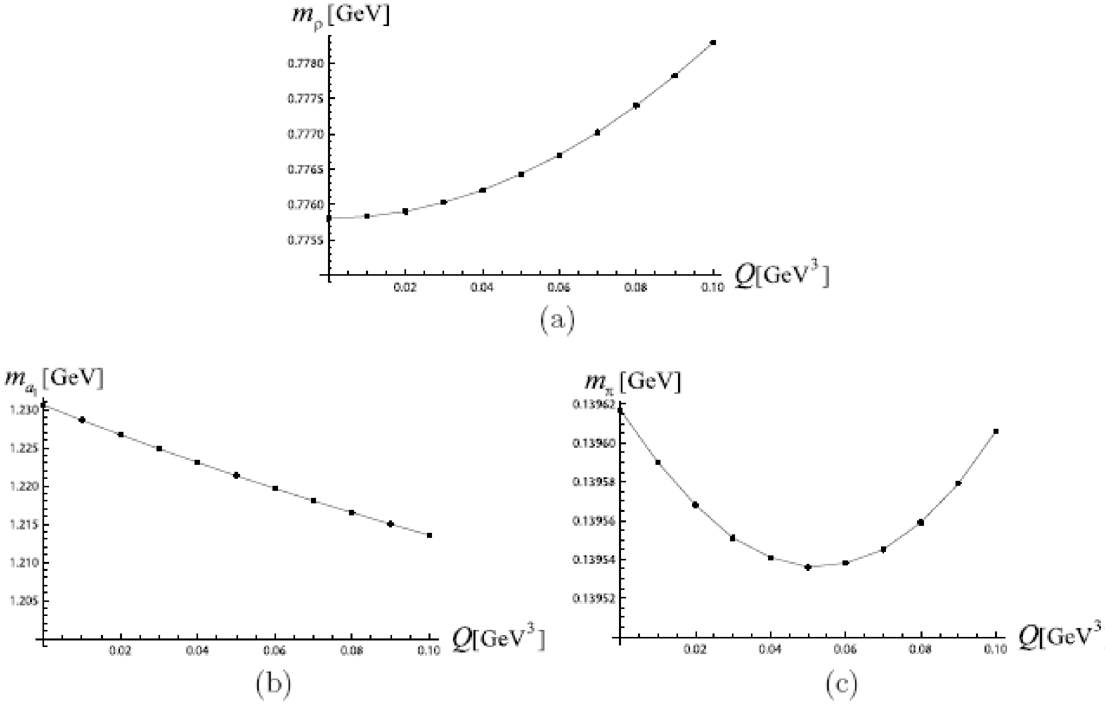


Figure 3: Meson masses with $m_q = 0.002383$GeV. (a)ρ-meson (b)a_1-meson (c)pion

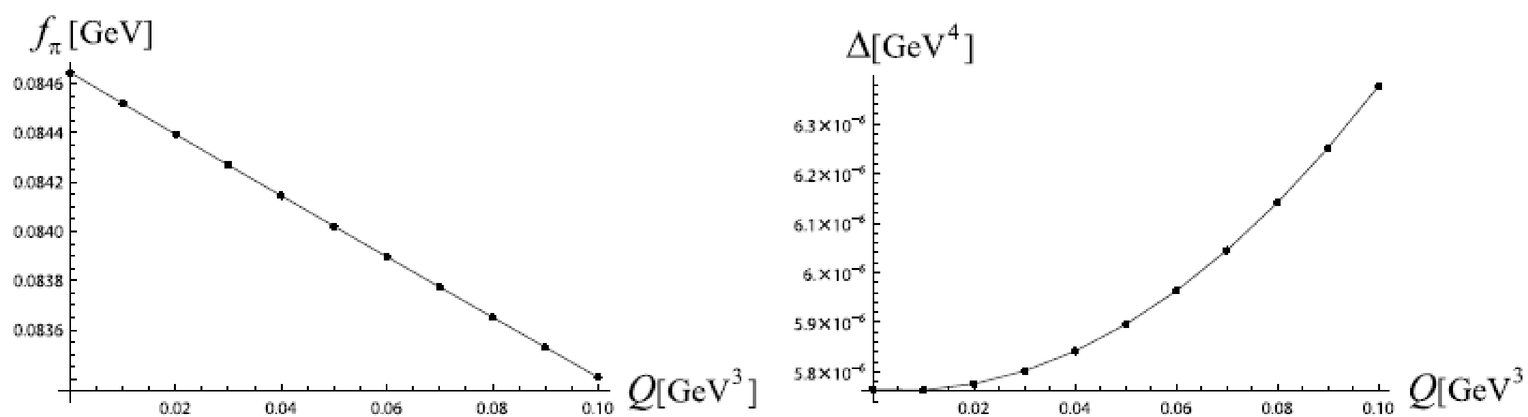


Figure 4: (a)pion decay constant (b)deviation from GOR relation. $m_q = 0.002383$GeV.

2. AdS/CMT

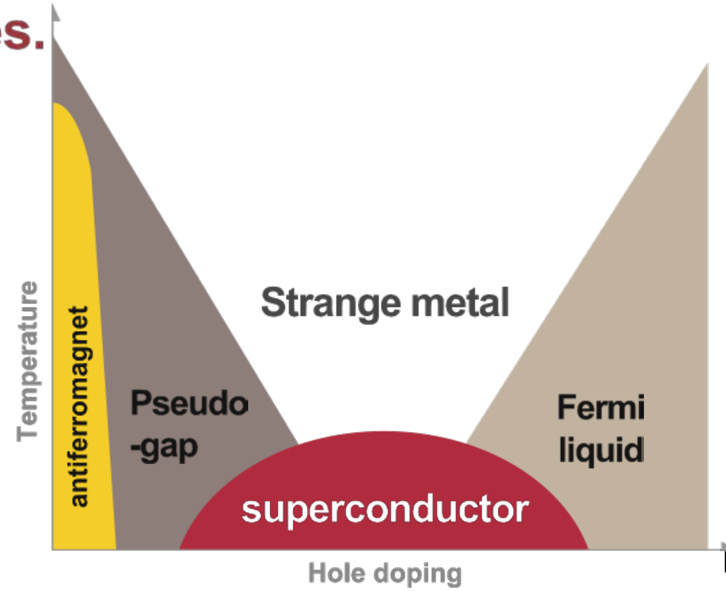
- Holographic superconductivity

Temperature	$T < T_c \quad (\psi \neq 0)$	$T > T_c \quad (\psi = 0)$
d dim. CMT	SC phase	Normal Phase
d+1 dim. 중력	AdS Black Hole with hair	AdS Black Hole no hair

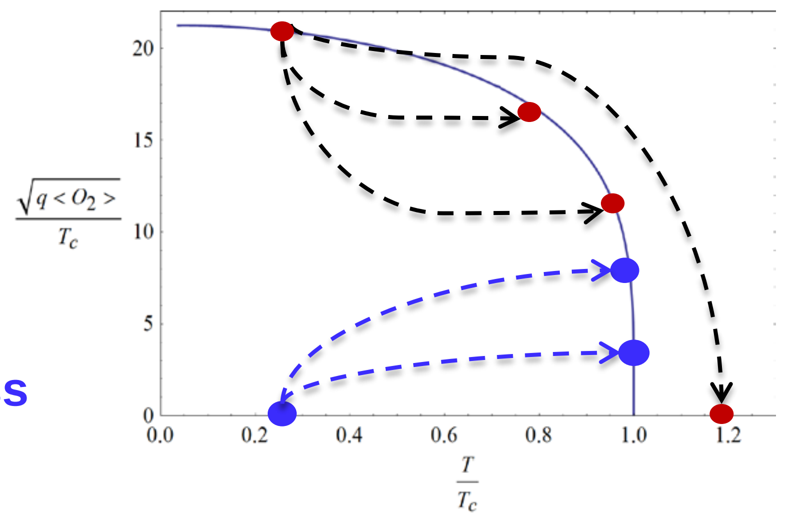
「**Evaluate the correlation functions, etc. to get the various physical quantities.**

- **2point correlation function**:
  Heat capacity, Conductivity,
  Magnetic susceptibility, Compressibility

▪ **Goal** :
Holographic Explanation for phenomena in the strongly correlated condensed matter systems, such as superconductivity, non Fermi liquid, Strange Metal, etc.



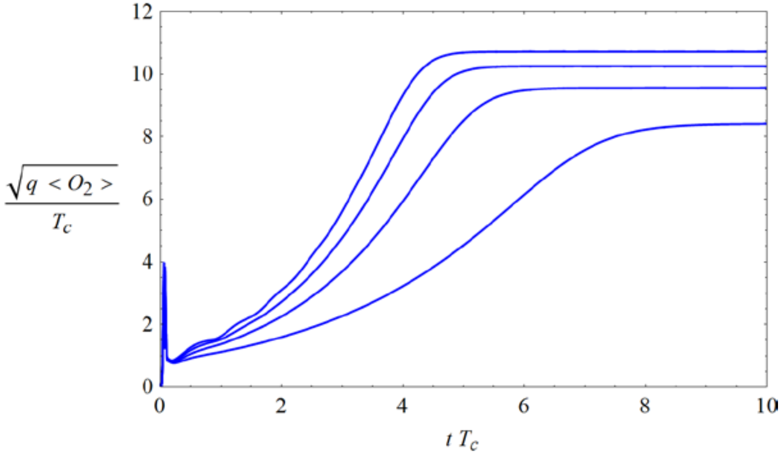
3.Holographic Approach to the nonequilibrium physics



「Two scenarios of Far-from-equilibrium dynamics

Blue routes : Condensation Process

· Non-equilibrium evolution of an unstable configuration

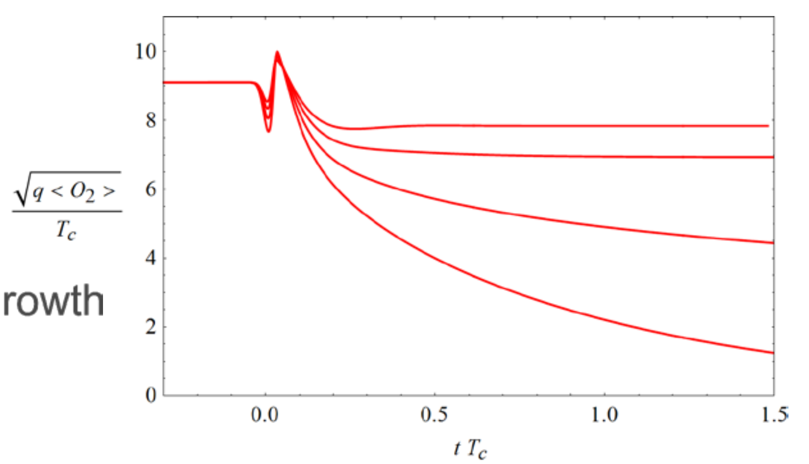


· Condensate undergoes an exponential growth

- Phase transition in "real" time !

Red routes: Quantum Quenching

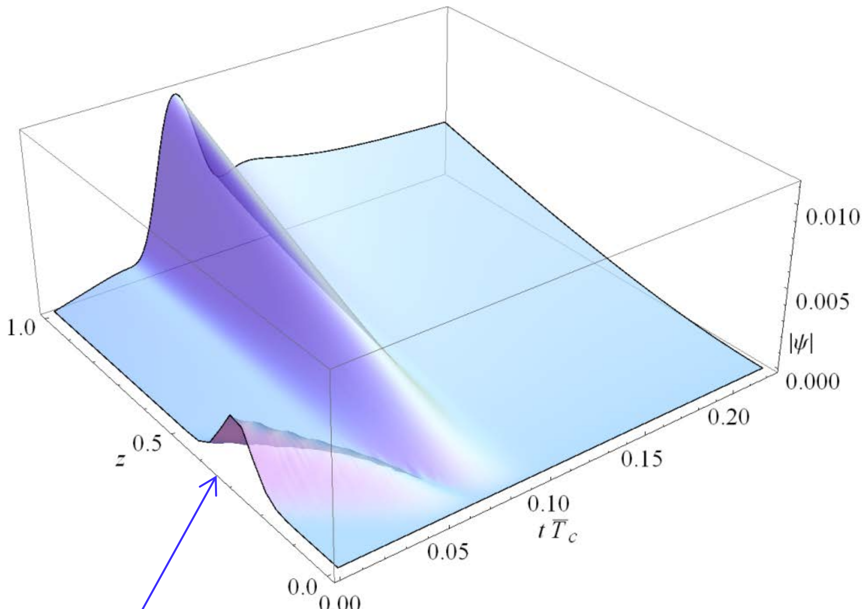
· Dynamical response to a sudden injection of energy



Condensation Process on Anisotropic Background
- evolution of scalar field

X. Bai, B-HL, M.Park, and K. Sunly arXiv:1405.1806

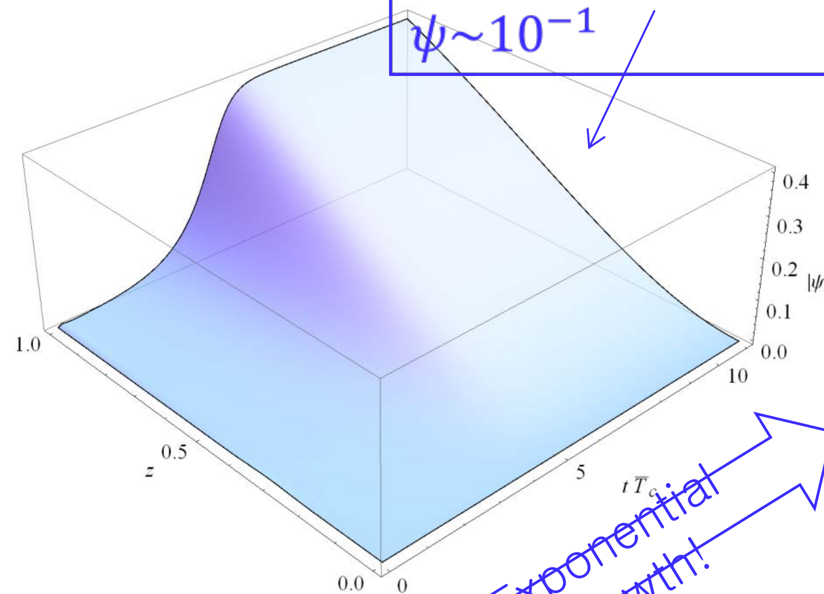
· Early time evolution : $t = 0 \sim 0.2$



initial perturbation
- scalar field $\psi \sim 10^{-3}$

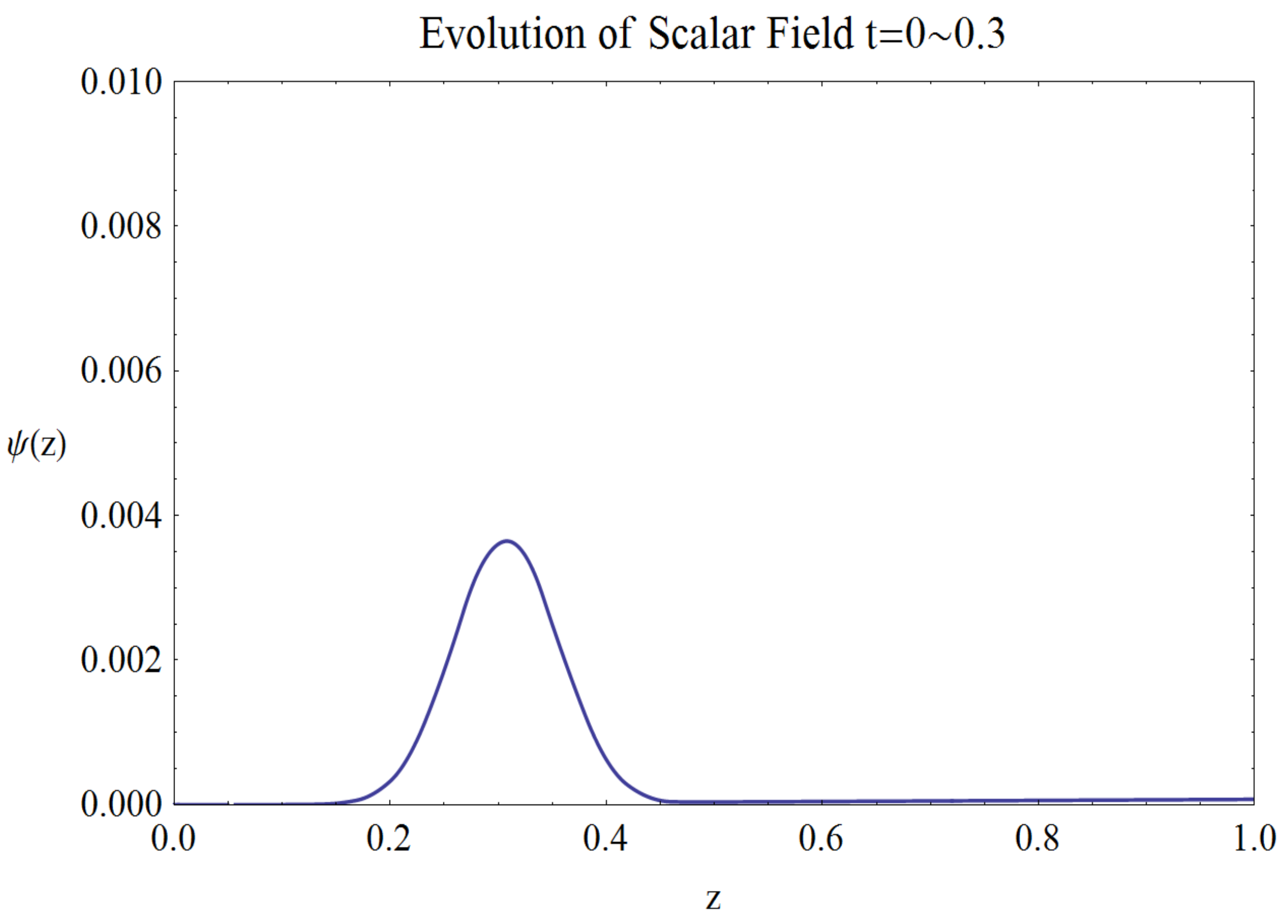
· Late time evolution : $t = 0.2 \sim 10$

final configuration
- scalar field $\psi \sim 10^{-1}$



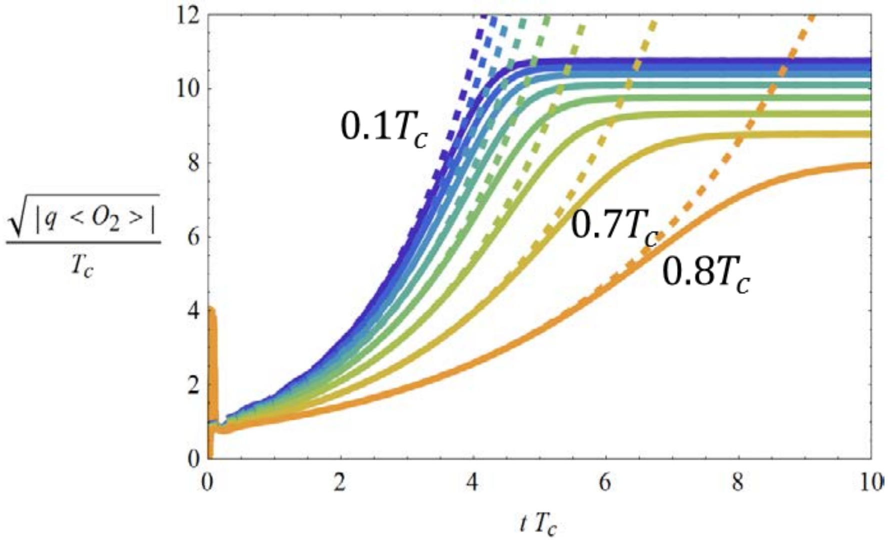
Exponential growth!

Temperature	$T < T_c \quad (\psi \neq 0)$	$T > T_c \quad (\psi = 0)$
d dim. CMT	SC phase	Normal Phase
d+1 dim. Gravity	Black Hole with hair	Black Hole no hair



Quasinormal modes (QNMs) - linearized fluctuation

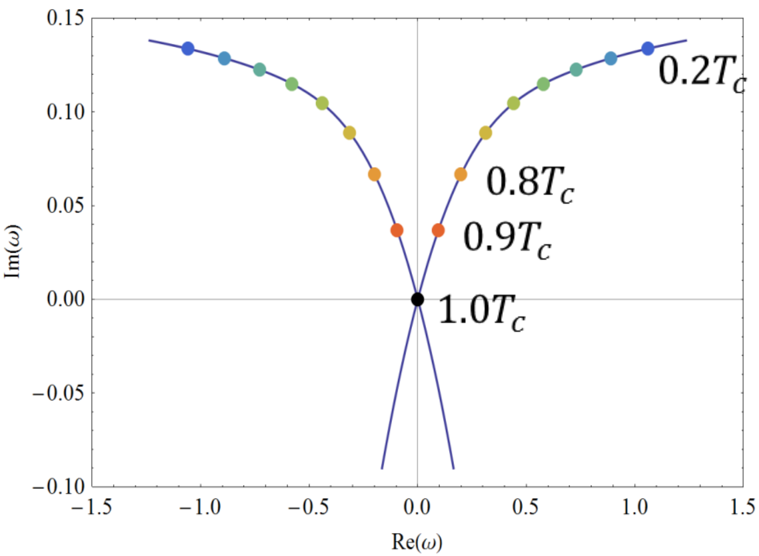
· Matching between QNMs (dashed) and non-equilibrium evolution (solid)



· Initial temperature : $0.1T_c \sim 0.8T_c$

· Leading three orders of QNMs

$T = 0.1T_c \sim 1.2T_c$



· QNMs move upward as T is lowered.

· Points indicate critical temperature T_c.

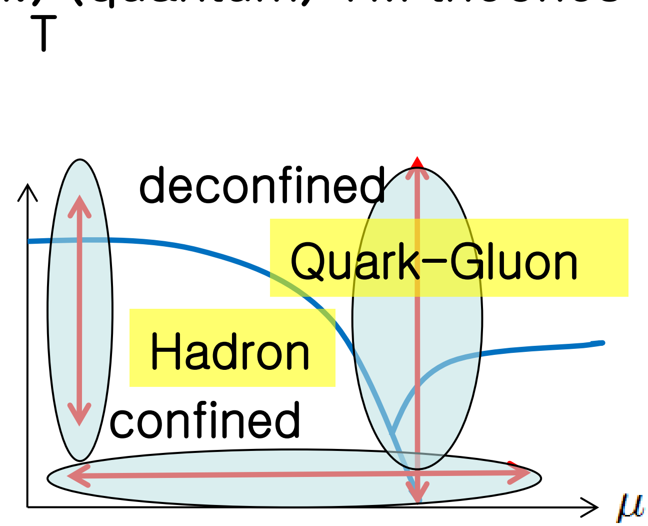
IV. Summary

- Holographic Principles (through the D-brane configuration)

(d+1 dim.) (classical) Sugra ↔ (d dim.) (quantum) YM theories

- SUGRA w/ BH ↔ Finite Temperature
- Constructing the dual geometry :

Top-down & Bottom-up



- Holographic QCD

– w/o chemical potential –

phase : confined phase ↔ deconfined phase transition

Geometry : thermal AdS ↔ AdS BH

Hawking-Page transition

– in dense matter – (U(1) chemical potential → baryon density)

deconfined phase by RNAdS BH ↔ hadronic phase by tcAdS

Hawking-Page phase transition

IV. Summary – continued

- AdS/CMT
- Holographic principle can also be applied to the nonequilibrium physics
- Holographic Principle may provide a new methodology for the strongly interacting phenomena!
- String Theory may provide the new paradigm for the theoretical physics in 21st Century

Thank You !