SPECIAL TOPICS IN ELEMENTARY PARTICLE PHYSICS

Physics 481

Problem Set #7 - Due Thursday, November 29, 2007.

1. In supersymmetric quantum mechanics two "partner" Hamiltionians $H_\pm = -(d^2/dx^2) + V_\pm(x)$ are factored into products of first-order operators:

$$H_+ = A^\dagger A \ , \quad H_- = AA^\dagger \ , \quad A = -\frac{d}{dx} + f(x) \ , \quad A^\dagger = \frac{d}{dx} + f(x) \ . \tag{1}$$

If we choose the zero of energy such that the ground state of H_+ has energy $E = 0$, then H_+ and H_- have identical spectra except for the ground state of H_+, which has no partner for H_-. Now consider a constant potential $V_-(x) = \kappa^2$. What is its partner potential $V_+(x)$ with precisely one bound state at zero energy? You may assume $V(x) = V(-x)$.

2. Estimate the total amount of dark matter in the halo of the Milky Way contained within a radius of 50 kpc. Express your answer in number of solar masses. Assume that the galactic rotation curve approaches a virial velocity of $\langle v^2 \rangle^{1/2} = 220$ km/s and stays constant out to at least 50 kpc.

3. Can you justify the estimate of the dark matter density in our galactic halo, $\rho_X \sim (1/3) m_p/\mathrm{cm}^3$, given by Hao Huan in his posted report, based on any observations you can think of? If the density is constant at this value out to a radius of 8 kpc (roughly the Sun's distance from the center of the Milky Way), what should the rotation velocity of the Sun be in the absence of other matter in the Galaxy?

4. If the virial velocity of galaxies at a distance of 1 Mpc from the center of a rich cluster is measured to be 1000 km/s, what is the number of solar masses contained in the halo?