SPECIAL TOPICS IN ELEMENTARY PARTICLE PHYSICS

Physics 481

Problem Set #6 - Due Thursday, November 15, 2007.

An upper bound on the Higgs mass may be obtained by considering the scattering of w^+w^-, zz, and HH, where $w^\pm$ and z denote longitudinal W and z. In the limit in which the SU(2) $\times$ U(1) gauge couplings g and g' vanish, the interactions of these bosons are derivable from the Higgs potential

$$V(\Phi) = \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 - \frac{\mu^2}{2}\Phi^\dagger\Phi \ , \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1}$$

where minimization of the Higgs potential leads to a vacuum expectation value $\phi^0 \;\; v/\sqrt{2}$. After shifting the real part of ϕ^0 and defining $\phi = (w^+, z, w^-)$ or $\phi = (w_1, w_2, w_3 = z)$ and $M_H = \mu$, we have [1]

$$V(\Phi) = \frac{M_H^2}{8v^2}(\phi^2 + H^2)^2 + \frac{M^2}{2v}H(\phi^2 + H^2) + \frac{1}{2}M_H^2 H^2 \ , \tag{2}$$

from which one may derive the following scattering amplitudes in the basis $(w^+w^-, zz/\sqrt{2}, HH/\sqrt{2})$ in the limit of $s/M_H^2 \to \infty$ [2]:

$$T = -\frac{G_F M_H^2}{4\pi\sqrt{2}} \begin{bmatrix} 1 & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} \\ \sqrt{\frac{1}{8}} & \frac{3}{4} & \frac{1}{4} \\ \sqrt{\frac{1}{8}} & \frac{1}{4} & \frac{3}{4} \end{bmatrix} \ . \tag{3}$$

Find the eigenchannel with the largest eigenvalue and use the partial-wave expansion

$$T = 16\pi \sum_{J=0}^{\infty} (2J+1) t_J P_J(\cos\theta) \tag{4}$$

and the unitarity condition $|t_J| \le 1$ for $J = 0$ to place an upper limit on the Higgs boson mass. Hint: It should be more stringent than the bound placed using the elastic scattering process $w^+w^- \to w^+w^-$, which is $M_H^2 \le 4\pi\sqrt{2}/G_F$. Find the other two eigenchannels. What do you notice about them?

Answers: We have to solve the characteristic equation Det $M(\lambda) = 0$, where

$$M(\lambda) \equiv \begin{bmatrix} 1-\lambda & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} \\ \sqrt{\frac{1}{8}} & \frac{3}{4}-\lambda & \frac{1}{4} \\ \sqrt{\frac{1}{8}} & \frac{1}{4} & \frac{3}{4}-\lambda \end{bmatrix} . \tag{5}$$

A little algebra yields

$$\text{Det } M(\lambda) = (1-\lambda)\left(\frac{1}{2} - \frac{3}{2}\lambda + \lambda^2\right) - \frac{1}{4}\left(\frac{1}{2} - \lambda\right) \tag{6}$$

$$= \left[(1-\lambda)^2 - \frac{1}{4}\right]\left(\frac{1}{2} - \lambda\right) = \left(\frac{3}{2} - \lambda\right)\left(\frac{1}{2} - \lambda\right)^2 . \tag{7}$$

The largest eigenvalue is $\lambda = 3/2$, so the largest eigenvalue of $|T|$ is 3/2 times the coefficient of the matrix in Eq. (3). Thus the bound on M_H^2 is 2/3 that based on $w^+w^- \to w^+w^-$ (using the first row and column of (3)), or

$$M_H^2 \leq 8\pi\sqrt{2}/(3G_F) = (1.008 \text{ GeV})^2 . \tag{8}$$

The corresponding eigenvector $V^+ = (V_1^+, V_2^+, V_3^+)$ must satisfy

$$V_1^+ = \sqrt{\frac{1}{2}}\left(V_2^+ + V_3^+\right) , \quad \sqrt{\frac{1}{8}}V_1^+ + \frac{1}{4}V_3^+ = \frac{3}{4}V_2^+ , \quad \sqrt{\frac{1}{8}}V_1^+ + \frac{1}{4}V_2^+ = \frac{3}{4}V_3^+ , \tag{9}$$

so $V_1^+ \sim (1, 1/s, 1/s)$ and the corresponding eigenchannel is proportional to $2w^+w^- + zz + HH$).

The other two eigenvalues are degenerate, $\lambda = 1/2$, so the corresponding eigenchannels are any linear combination of the two eigenvectors $(1, -\sqrt{2}, 0)$ and $(1, 0, -\sqrt{2})$ orthogonal to V_1^+.

References

[1] M. Chanowitz and M. K. Gaillard, Phys. Lett. **142B**, 85 (1984); Nucl. Phys. **B261**, 379 (1985).

[2] B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. Lett. **38**, 883 (1977); Phys. Rev. D **16**, 1519 (1977).