SPECIAL TOPICS IN ELEMENTARY PARTICLE PHYSICS

Physics 481

Problem Set #5 - Due Thursday, November 8, 2007.

A standard Higgs doublet of $SU(2)_L$,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \tag{1}$$

has a vacuum expectation value

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} . \tag{2}$$

It leads to the mass relation $M_W = M_Z \cos\theta$, where θ is the Weinberg angle ($\sin^2\theta \simeq 0.23$). Electroweak corrections as well as impurities due to higher-dimensional Higgs representations of $SU(2)_L$ can affect this relation. These effects can be parametrized by expressing $M_W^2 = M_Z^2 \rho \cos^2\theta$, where $\rho = 1$ corresponds to the pure-Higgs-doublet case in lowest electroweak order. Calculate the effect on ρ when, in addition to the standard Higgs vacuum expectation value $v = 246$ GeV, there are small contributions from the following two cases of Higgs triplets:

(a) A triplet of the form

$$\Phi_a = \begin{pmatrix} \Phi_a^{++} \\ \Phi_a^{+} \\ \Phi_a^{0} \end{pmatrix} \text{ with } \langle \Phi_a^0 \rangle = V_{1,-1}/\sqrt{2} ; \tag{3}$$

(b) A triplet of the form

$$\Phi_b = \begin{pmatrix} \Phi_b^{+} \\ \Phi_b^{0} \\ \Phi_b^{-} \end{pmatrix} \text{ with } \langle \Phi_b^0 \rangle = V_{1,0}/\sqrt{2} . \tag{4}$$

You may express the correction $\Delta\rho \equiv \rho - 1$ to first order in $V_{1,-1}/v$ or $V_{1,0}/v$.

Answers: In calculating $|\mathbf{D}_\mu \Phi|^2$ we will need the triplet representation for weak isospin:

$$I_3 = \begin{bmatrix} 1 & & \\ & 0 & \\ & & -1 \end{bmatrix} \quad , \quad I_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad , \quad I_2 = \frac{i}{\sqrt{2}} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ o & 1 & 0 \end{bmatrix} \quad . \tag{5}$$

The result, if $\langle\Phi^0\rangle = V_{1,-1}/\sqrt{2}$, is that

$$\langle|\mathbf{D}_\mu\Phi|^2\rangle = \frac{V_{1,-1}^2}{2}\left\{\frac{g^2}{2}[(W^1)^2+(W^2)^2]+(-gW^3+g'B)^2\right\} \quad . \tag{6}$$

The same combination of W^3 and B gets a mass as in the case of one or more Higgs doublets, simply because we assumed that it was a neutral Higgs field which acquired a vacuum expectation value. Electromagnetic gauge invariance remains valid; the photon does not acquire a mass. However, the ratio of W and Z masses is altered. In the presence of doublets and this type of triplet, we find

$$M_W^2 = \frac{g^2}{4}(v^2+2V_{1,-1}^2) \quad , \quad M_Z^2 = \left(\frac{g^2+g'^2}{4}\right)(v^2+4V_{1,-1}^2) \quad , \tag{7}$$

so the ratio $\rho = (M_W/M_Z\cos\theta)^2$ is no longer 1, but becomes

$$\rho = \frac{v^2+2V_{1,-1}^2}{v^2+4V_{1,-1}^2} \quad . \tag{8}$$

This type of Higgs boson thus leads to $\rho < 1$.

The complex triplet Φ_b in Eq. (4) is characterized by $Y = 0$. If we let $\langle\Phi^0\rangle = V_{1,0}/\sqrt{2}$, we find by an entirely similar calculation, that

$$M_W^2 = \frac{g^2}{4}(v^2+4V_{1,0}^2) \quad , \quad M_Z^2 = \left(\frac{g^2+g'^2}{4}\right)v^2 \quad . \tag{9}$$

Here we predict

$$\rho = 1 + \frac{4V_{1,0}^2}{v^2} \quad , \tag{10}$$

so this type of Higgs boson leads to $\rho > 1$.