

SPECIAL TOPICS IN ELEMENTARY PARTICLE PHYSICS

Physics 481

Problem Set #3 - Due Thursday, October 18, 2007.

1. Show that one can measure a neutrino mass via a “missing-mass” technique (e.g., in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$), or an oscillation technique, but not both. [This is a nice application of the Uncertainty Principle. See, e.g., B. Kayser, *Phys. Rev.* **D24**, 110 (1981).]

ANSWER: In order to see oscillations, one must specify $\Delta m^2 t / (4p\hbar) = \Delta m^2 L / (4p\hbar)$ to an accuracy much less than 1, where L is the path length. (Here we have set $c = 1$.) That is, we need

$$\frac{\Delta m^2 \Delta L}{4\hbar p} \ll 1 \quad , \quad \text{or} \quad \frac{\Delta m^2}{4p} \ll \frac{\hbar}{\Delta L} < \Delta p \quad , \quad (1)$$

where in the last step we have used the uncertainty principle. That is, the need to specify the path length has introduced an uncertainty in the momentum. But the last equation implies

$$\Delta m^2 \ll 2\Delta(p^2) \quad , \quad (2)$$

i.e., the uncertainty in p^2 is larger than the squared mass difference which we want to measure. This then prevents us from determining the squared mass difference from the kinematic relation $m^2 = E^2 - p^2$ (assuming a known value of E). A more elegant discussion of this result can be found in the article by Kayser mentioned above.

2. It has been proposed to build an “off-axis” neutrino beam in order to study $\nu_\mu \rightarrow \nu_e$ oscillations. When a pion of laboratory energy E_π decays to muon and a neutrino which makes an angle θ in the laboratory with respect to the pion, the energy E_ν of the neutrino is a function of E_π and θ . Calculate $E_\nu(E_\pi, \theta)$ in the limit $\theta \ll 1$ and $E_\pi \gg m_\pi$ and show that for a fixed value of θ , E_ν attains a maximum for some value of E_π . (This means that the neutrino energy spectrum from a broad-band neutrino beam will show a peak at its maximum value, since E_ν is least sensitive to E_π there.) Illustrate this behavior for the cases $\theta = 15, 30$ mr with a crude sketch and give the corresponding values of E_π and E_ν^{\max} .

ANSWER: Use 4-momentum conservation: $p_\pi - p_\nu = p_\mu$ and take the invariant square:

$$(p_\pi - p_\nu)^2 = p_\mu^2 = m_\mu^2 = m_\pi^2 - 2E_\pi E_\nu(1 - \beta_\pi \cos \theta) \quad , \quad (3)$$

where E_π and E_ν are the pion and neutrino lab energies, and β_π is the pion velocity in the lab. Solving for E_ν , one finds

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2E_\pi(1 - \beta_\pi \cos \theta)} \quad . \quad (4)$$

Now one makes the small- θ , large-pion-energy approximation:

$$\beta_\pi = \sqrt{1 - \gamma_\pi^{-2}} \simeq 1 - \frac{m_\pi^2}{2E_\pi^2}, \quad \cos \theta \simeq 1 - \frac{\theta^2}{2} \quad (5)$$

and keeps leading terms in

$$1 - \beta_\pi \cos \theta \simeq \frac{m_\pi^2}{2E_\pi^2} + \frac{\theta^2}{2} \quad (6)$$

so that

$$E_\nu \simeq \frac{m_\pi^2 - m_\mu^2}{m_\pi^2/E_\pi + \theta^2 E_\pi} \quad (7)$$

This reaches a maximum for any θ when $E_\pi = m_\pi/\theta$.

The behavior is illustrated in the Figure. If $\theta = 15$ mr (solid), the maximum E_ν is 1.97 GeV, attained for $E_\pi = 9.3$ GeV, while for $\theta = 30$ mr (dashed), the maximum E_ν is 0.99 GeV, attained for $E_\pi = 4.7$ GeV. For small E_π the neutrino energy is $E_\nu \simeq E_\pi[1 - (m_\mu/m_\pi)^2]$ (dotted line).

