SPECIAL TOPICS IN ELEMENTARY PARTICLE PHYSICS

Physics 481 - Problem Set #2 - Due Tuesday, October 9, 2007

(1) The time-dependent decay rate for an initial B^0 ($\bar{B}^0$) to decay to $J/\psi K_S$ is

$$\frac{d\Gamma(t)}{dt} \left\{ \begin{array}{c} B^0 \\ \bar{B}^0 \end{array} \right\} = e^{-\Gamma t}(1 \mp \sin 2\beta \sin \Delta m t) \ . \tag{1}$$

Find the time-integrated asymmetry

$$A_{CP} = \frac{\Gamma(\bar{B}^0 \to J/\psi K_S) - \Gamma(B^0 \to J/\psi K_S)}{\Gamma(\bar{B}^0 \to J/\psi K_S) + \Gamma(B^0 \to J/\psi K_S)} \tag{2}$$

as a function of $x \equiv \Delta m/\Gamma$ and calculate its ratio with respect to its maximum possible value for any x.

Answer: The relevant integrals are

$$\int_0^\infty dt e^{-\Gamma t} = \frac{1}{\Gamma} \ ; \quad \int_0^\infty dt e^{-\Gamma t} \sin \Delta m t = \frac{1}{2i}\left(\frac{1}{\Gamma - i\Delta m} - \frac{1}{\Gamma + i\Delta m}\right) = \frac{\Delta m}{\Gamma^2 + (\Delta m)^2} \tag{3}$$

so the asymmetry is

$$A_{CP} = \frac{x}{1+x^2} \sin 2\beta \ . \tag{4}$$

The function $x/(1+x^2)$ reaches its maximum of $1/2$ at $x = 1$. For the observed value of $x_d = 0.776 \pm 0.008$ [1], its value is 0.484 ± 0.001, or about 97% of the maximum.

(2) Suppose there exists a fourth family (t', b') of quarks, so that the CKM matrix is expanded to:

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{bmatrix} \ . \tag{5}$$

Using the *measured* values of $|V_{ij}|$ quoted in Ref. [1] (not those based on the assumption of three-family unitarity), find the upper limits on the magnitudes of elements involving the fourth family.

Answer: The handling of errors is tricky here. I will quote 1σ bounds, though for 90% confidence level bounds one should really use 1.28σ.

The Particle Data Group [1] quotes $|V_{ud}| = 0.97377 \pm 0.00027$, $|V_{us}| = 0.2257 \pm 0.0021$, $|V_{cd}| = 0.230 \pm 0.011$, $|V_{cs}| = 0.957 \pm 0.017 \pm 0.093 = 0.957 \pm 0.095$, $|V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}$, $|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$, and $|V_{tb}| > 0.78$. This last comes from the fraction of t quark decays which contain a b quark, as reported by CDF and D0. We shall try to get away without using information on V_{td} and V_{ts} as these are not directly measured.

The main constraints come from the 2×2 Cabibbo-GIM submatrix. Considering its rows:

$$|V_{ud}|^2 + |V_{us}|^2 = 0.9991 \pm 0.0005 \pm 0.0009 = 0.9991 \pm 0.0011 > 0.9980 \;, \tag{6}$$

where we have neglected the very small contribution of $|V_{ub}|^2$, so that
$|V_{ub'}| \leq \sqrt{1 - 0.9980} = 0.044$ which is ten times as large as the central value of $|V_{ub}|$! We will see that other bounds are similarly poor. For example,

$$|V_{cd}|^2 + |V_{cs}|^2 = 0.9687 \pm 0.0051 \pm 0.181 = 0.9687 \pm 0.181 > 0.788 \;, \tag{7}$$

where we have neglected the very small contribution of $|V_{cb}|^2$, so that
$|V_{cb'}| \leq \sqrt{1 - 0.788} = 0.46$, again a large number in comparison with $|V_{cb}|$. Now considering the columns of the Cabibbo-GIM submatrix, we have

$$|V_{ud}|^2 + |V_{cd}|^2 = 1.0011 \pm 0.0005 \pm 0.0051 = 1.0011 \pm 0.0051 > 0.9960 \;, \tag{8}$$

where we have neglected a possible small contribution of $|V_{td}|^2$, so that
$|V_{t'd}| \leq \sqrt{1 - 0.9960} = 0.063$, and

$$|V_{us}|^2 + |V_{cs}|^2 = 0.9668 \pm 0.0009 \pm 0.181 = 0.9668 \pm 0.181 > 0.786 \;, \tag{9}$$

where we have neglected a possible small contribution of $|V_{ts}|^2$, so that
$|V_{t's}| \leq \sqrt{1 - 0.786} = 0.46$. Finally, the bound $V_{tb} > 0.78$ can be considered just in the 2×2 submatrix connecting (b, b') with (t, t'), so $|V_{tb'}|$ and $|V_{t'b}|$ are bounded from above by $\sqrt{1 - (0.78)^2} = 0.63$ and $V_{t'b'} > 0.78$. The net result is that CKM unitarity provides rather weak constraints on couplings to a fourth family.

References

[1] W.-M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006). See `http://pdg.lbl.gov/` for updated particle listings.