SPECIAL TOPICS IN ELEMENTARY PARTICLE PHYSICS

Physics 481 - Problem Set #1 - Due Tuesday, October 2, 2007

1. (a) The $\Delta(1232)$ resonance has isotopic spin $I = 3/2$. Here 1232 denotes its mass in MeV. Compare the cross sections $\sigma(\pi^+ p \to \pi^+ p)$, $\sigma(\pi^- p \to \pi^- p)$, and $\sigma(\pi^- p \to \pi^0 n)$ at the resonance peak. (b) Compare these same cross sections at the peak of an isospin $I = 1/2$ resonance N.

Answers: Clebsch-Gordan coefficients may be found in the Particle Data Group compilation, Sec. 35 (p. 299 in the latest booklet). We need the coefficients $(j_1 j_2 m_1 m_2 | JM)$ for $j_1 = 1$, $j_2 = 1/2$:

$$\begin{aligned} \pi^+ p \ : \quad & \left(1 \tfrac{1}{2}\ 1 \tfrac{1}{2} \middle| \tfrac{1}{2} \tfrac{3}{2}\right) = 0 \ , \quad \left(1 \tfrac{1}{2}\ 1 \tfrac{1}{2} \middle| \tfrac{3}{2} \tfrac{3}{2}\right) = 1 \ , & (1) \\ \pi^- p \ : \quad & \left(1 \tfrac{1}{2}\ {-1} \tfrac{1}{2} \middle| \tfrac{1}{2}\ {-\tfrac{1}{2}}\right) = \sqrt{\tfrac{2}{3}} \ , \quad \left(1 \tfrac{1}{2}\ {-1} \tfrac{1}{2} \middle| \tfrac{3}{2}\ {-\tfrac{1}{2}}\right) = \sqrt{\tfrac{1}{3}} \ , & (2) \\ \pi^0 n \ : \quad & \left(1 \tfrac{1}{2}\ 0\ {-\tfrac{1}{2}} \middle| \tfrac{1}{2}\ {-\tfrac{1}{2}}\right) = -\sqrt{\tfrac{1}{3}} \ , \quad \left(1 \tfrac{1}{2}\ 0\ {-\tfrac{1}{2}} \middle| \tfrac{3}{2}\ {-\tfrac{1}{2}}\right) = \sqrt{\tfrac{2}{3}} \ . & (3) \end{aligned}$$

(a) The relative amplitudes are

$$A(\pi^+ p \to \Delta \to \pi^+ p) \ : \ A(\pi^- p \to \Delta \to \pi^- p) \ : \ A(\pi^- p \to \Delta \to \pi^0 n)$$

$$= 1 \cdot 1 \ : \ \sqrt{\frac{1}{3}} \cdot \sqrt{\frac{1}{3}} \ : \ \sqrt{\frac{1}{3}} \cdot \sqrt{\frac{2}{3}} = 1 \ : \ \frac{1}{3} \ : \ \frac{\sqrt{2}}{3} \ . \tag{4}$$

so the relative cross sections are in the ratio $1 \ : \ \frac{1}{9} \ : \ \frac{2}{9}$.

(b) The relative amplitudes are

$$A(\pi^+ p \to N \to \pi^+ p) \ : \ A(\pi^- p \to N \to \pi^- p) \ : \ A(\pi^- p \to N \to \pi^0 n)$$

$$= 0 \cdot 0 \ : \ \sqrt{\frac{2}{3}} \cdot \sqrt{\frac{2}{3}} \ : \ \sqrt{\frac{2}{3}} \cdot -\sqrt{\frac{1}{3}} = 0 \ : \ \frac{2}{3} \ : \ -\frac{\sqrt{2}}{3} \ . \tag{5}$$

so the relative cross sections are in the ratio $0 \ : \ \frac{4}{9} \ : \ \frac{2}{9}$.

2. In the 4-component quark basis $[u, c, d, s]$ the charge-raising weak charge $Q^{(+)}_{\rm weak}$ may be written

$$Q^{(+)}_{\rm weak} = \begin{bmatrix} 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \ , \tag{6}$$

with $Q^{(-)}_{\rm weak} = [Q^{(+)}_{\rm weak}]^\dagger$. Show that $Q^{(3)}_{\rm weak} \equiv \frac{1}{2}[Q^{(+)}_{\rm weak}, Q^{(-)}_{\rm weak}]$ contains no flavor-changing neutral currents. Show that the algebra closes, i.e., that $[Q^{(3)}_{\rm weak}, Q^{(\pm)}_{\rm weak}] = \pm Q^{(\pm)}_{\rm weak}$.

Answers: Taking the Hermitian conjugate, we find

$$Q^{(-)}_{\rm weak} = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \end{array} \right] ~, \tag{7}$$

and then taking the commutator with $Q^{(+)}_{\rm weak}$ we find $Q^{(3)}_{\rm weak} = \frac{1}{2}{\rm diag}(1,1,-1,-1)$, so $Q^{(3)}_{\rm weak}$ contains no flavor-changing neutral currents.

Closure (thanks to Tom Weisgarber for this general method): Working with 2×2 submatrices A and B, and defining $Q_3 \equiv Q^{(3)}_{\rm weak}$, one can show

$$Q_3 \left(\begin{array}{cc} 0 & A \\ B & 0 \end{array} \right) = \frac{1}{2} \left(\begin{array}{cc} 0 & A \\ -B & 0 \end{array} \right) ~, \quad \left(\begin{array}{cc} 0 & A \\ B & 0 \end{array} \right) Q_3 = \frac{1}{2} \left(\begin{array}{cc} 0 & -A \\ B & 0 \end{array} \right) ~, \tag{8}$$

so that $[Q_3, Q^{(\pm)}] = \pm Q^{(\pm)}$.

References

[1] W.-M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006). See `http://pdg.lbl.gov/` for updated particle listings.