Cold Dark Matter Candidates

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Abstract

According to astrophysical and cosmological observations, about 20 percent of the total energy in the universe is contributed by dark matter, which, within present observational limits, only exhibits gravitational interaction. Meanwhile, the Standard Model of elementary particles don’t contain such weakly-interacting constituents. To solve this mystery several schemes extending the Standard Model have been suggested and in this lecture we will talk about several of them: their theoretical backgrounds, correspondence with the dark matter scenario, and experimental detections.

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I. INTRODUCTION

In recent years physicists and astronomers have come to an agreement that the total energy density of the universe consists of three parts: the "usual" atomic or baryonic matter, the dark matter sector, and the dark energy component which behaves like a cosmological constant. Up till now, the most accurate data about the fraction of these components come from the power spectrum of CMB anisotropy, which gives the fractions of atomic and dark matter in the total energy of the universe, \( \Omega_i = \rho_i / \rho_{tot} \)

\[
\Omega_{at} = 0.042 \pm 0.003 \quad (1)
\]

\[
\Omega_{DM} = 0.20 \pm 0.02 \quad (2)
\]

where \( \Omega_{at} \) is consistent with the result obtained from Big-Bang Nucleosynthesis. Therefore the dark matter component contained in \( \Omega_{DM} \) should be non-baryonic matter. Moreover, because dark matter is only observed in the gravitational effects, it should be only weakly interacting with other particles.

Apart from being non-baryonic and weakly interacting, the dark matter we are discussing here should be also be "cold", in the sense that it should be moving quite slowly at the time of structure formation, so as not to erase out fluctuations at small scales. In fact, because we are getting information about dark matter from the CMB anisotropy, the dark matter candidate particles should be non-relativistic at the time of matter-radiation decoupling \( T \sim 0.1 \) to 1 eV, otherwise the anisotropy will be smeared out. This naively rules out very light particles as candidates for the cold dark matter, say, \( m < 1 \)eV, like the light neutrinos appearing in the see-saw mechanism of Majorana mass. There might be exceptions, though, which we will discuss later.

Having roughly analyzed the required properties of the dark matter particles, we can turn our eyes to the various models in particle physics and try to find a few candidates satisfying these requirements. Another prerequisite for candidate particles is that they should be stable enough to survive the cosmological time. In the following sections I will present the possibilities that attract most attention, including various kinds of Weakly Interactive Massive Particles (WIMPs) and also axions related to strong CP violation which affect the QCD vacuum.
II. WIMP DARK MATTER

A. General Dynamics

The idea of WIMP is quite straightforward. Assume there exists a heavy elementary particle $\chi$, which hasn’t been produced by current experiments due to the inadequate energy and luminosity, i.e. $m_\chi > 100\text{GeV}$. If the particle has a small interacting cross section with other known particles, it may manifest as the cold dark matter.

To get more information about the mass scale and the interaction of WIMP, we have to examine its cosmological history. In the early period of the universe, i.e. $T > m_\chi$, radiation was dominating and WIMPs were created in thermal equilibrium with other particles. After the temperature $T$ drops below $m_\chi$, the production of WIMPS stopped and if they are stable, the major process concerning WIMPs was the annihilation of two $\chi$’s into ”mundane” lighter particles. As the universe expanded the density of $\chi$ got smaller and smaller and finally it became so difficult to find another WIMP that the number of WIMPS was fixed. With such a simple picture we can estimate the cosmological abundance of WIMPs. The universe expansion rate is given by the Hubble constant, which in radiation dominated universe is

$$H = \frac{\dot{a}}{a} = g_*^{1/2} \frac{T^2}{M_{Pl}} \left(\frac{\pi^2}{90}\right)^{1/2}$$

where $g_* \sim 80$ is the total degrees of freedom of the hot thermal gas composed by quarks, gluons, leptons, and photons, and $M_{Pl} = 1/(8\pi G_N)^{1/2}$ is the reduced Planck scale.

Given the thermally averaged annihilation cross section of WIMPs $\langle \sigma_{ann} v \rangle$, and the number density of WIMPs $n_\chi$, we can get the annihilation rate of a WIMP

$$\Gamma = \langle \sigma_{ann} v \rangle n_\chi$$

The ”freeze-out” of WIMPs happens when the expansion rate of the universe is at the same order of the annihilation rate. Introducing the ”freeze-out temperature” $T_f$, there is

$$g_*^{1/2} \frac{T_f^2}{M_{Pl}} \left(\frac{\pi^2}{90}\right)^{1/2} \sim \langle \sigma_{ann} v \rangle n_\chi(T_f)$$

that is,

$$n_\chi(T_f) \sim g_*^{1/2} \frac{T_f^2}{\langle \sigma_{ann} v \rangle M_{Pl}} \left(\frac{\pi^2}{90}\right)^{1/2}$$

$$3$$
As the universe expands approximately adiabatically in standard cosmology, it is convenient to normalize the density of WIMPs with the entropy density of the universe, which is proportional to both the total degrees of freedom \( g_* \) and \( a^{-3} \)

\[
s = g_* T^3 (2\pi^2/45)^{1/2}
\]  

Let’s define the ”yield” of WIMPs to be the normalized number density \( Y_\chi = n_\chi/s \), then at the ”freeze-out” temperature, the yield is

\[
Y_\chi \sim \frac{g_*^{1/2}}{2\langle \sigma_{\text{ann}} v \rangle T_f M_{Pl}} = \frac{g_*^{1/2} x_f}{2\langle \sigma_{\text{ann}} v \rangle m_\chi M_{Pl}}
\]

where the dimensionless parameter \( x_f \) is defined as \( x_f = m_\chi / T_f \), i.e. the ratio between the decoupling temperature and the ”freeze-out” temperature. Therefore \( x_f \) should reflect the characteristic of the annihilation process, described by the Boltzmann kinetic equation. After freezing-out, the number density of WIMPs and the entropy density of the universe should both evolve as \( a^{-3} \), and the yield of \( \chi \) would be a constant. Hence the abundance of \( \chi \) in the current universe can be calculated using the yield multiplied by the current entropy density

\[
n_\chi = Y_\chi s \sim \frac{g_*^{1/2} T^3 x_f}{\langle \sigma_{\text{ann}} v \rangle m_\chi M_{Pl}} (\pi^2/90)^{1/2} \]

That is, the ratio of the energy density of WIMPs to the critical density

\[
\rho_c = \frac{3H^2}{8\pi G_N} = \frac{\pi^2}{30} g_* T^4
\]

is directly

\[
\Omega_\chi = \frac{m_\chi n_\chi}{\rho_c} \sim \frac{1}{6} \left(\frac{\pi^2/90}{\langle \sigma_{\text{ann}} v \rangle T M_{Pl}}\right)^{-1/2} \frac{g_*^{1/2} x_f}{\langle \sigma_{\text{ann}} v \rangle T M_{Pl}}
\]

In order for the calculated energy density to be consistent with the observed result Eq.2, plugging in the numerical values, with \( x_f \sim 20 \) as a numerical solution of the related Boltzmann equation, we obtain

\[
\langle \sigma_{\text{ann}} v \rangle \sim 10^{-9}\text{GeV}^{-2}
\]

For a particle with mass \( m_\chi \) and coupling strength \( \alpha \), the typical annihilation cross section at tree level is

\[
\sigma_{\text{ann}} v \sim \frac{\pi \alpha^2}{m_\chi^2}
\]
Assuming the interaction between WIMPs to be relatively weak with $\alpha \sim \alpha_{EM}$, then the tree approximation is justified and we get

$$m_\chi \sim 300\text{GeV} \quad (14)$$

This result is quite interesting. With the observed facts about dark matter we find the plausible WIMP mass to be just beyond the current investigated energy scale and on the other hand, within the reach of the next generation accelerators, e.g. LHC, ILC, etc. In fact, the cross section in Eq.12 is just the typical size of the production cross sections expected for new particles at the LHC. In this way dark matter physic is closely related to the frontiers in particle physics and there is hope that the WIMP model of dark matter can be thoroughly tested in the foreseeable future.

A few more words about $x_f$: it is clear that $x_f$ depends on the values of $m_\chi$ and $\langle \sigma_{\text{ann}} v \rangle$. Nonetheless in the Boltzmann equation the temperature is on exponentials; therefore by hand-waving argument $x_f = m_\chi/T_f$ should have only logarithmic dependence on the dimensionfull parameters. This is why it appears to be a constant and varies only slightly with $m_\chi$ and $\langle \sigma_{\text{ann}} v \rangle$ as well.

B. Particle Candidates

Having specified the approximate annihilation cross section and mass of the dark matter particle, the next step is to look for certain candidates. The most popular WIMPs come from supersymmetric models, as superpartners of the photon and Z, and the two neutral Higgs bosons. These four "neutralino" states mix among each other in the electroweak $SU(2) \times U(1)$ symmetry breaking and the lightest one is a favorite candidate for WIMP dark matter. To clarify this idea, we notice that in supersymmetric extensions of the Standard Model there will naturally arise vertices involving the up quark, down quark, and the superpartner of the strange quark $\tilde{s}$, which is a scalar particle. Similarly the superpartner of the strange quark can couple to a positron and an anti-up quark. Combining these two vertices together we can get the process $ud \to e^+\bar{u}$, which leads to proton decay. The decay width is proportional to $|T|^2$, i.e. $m_{\tilde{s}}^{-4}$, therefore by dimensional analysis the lifetime of the proton will be in the order of $m_{\tilde{s}}^4/m_p^5 \sim 10^{-12}s$, which is way too short according to the results from SuperKamioKande, etc.
To solve this problem the mentioned vertices should be eliminated from the Lagrangian. The most convenient and natural way is to impose a $\mathbb{Z}_2$ symmetry called “R-parity”

$$R = (-1)^{3B+L+2s}$$

(15)

where $s$ is the spin of the particle. In effect this definition assigns even parity to all the particles in the original Standard Model, and odd parity to the superpartners, thus prohibiting proton decay. Now the lightest supersymmetric particle (LSP) has odd R-parity so it cannot decay to lighter particles in the Standard Model. The lightest neutralino mentioned above is often such an LSP and remains stable throughout the universal time. Furthermore, by adjusting the parameters in sub-TeV supersymmetric theory we can find those parameter regions which lead to the required LSP properties, i.e. the mass, the annihilation cross section, etc. This is why we welcome neutralinos as dark matter candidate particles. In fact, sub-TeV supersymmetry was motivated to solve the hierarchy problem. Therefore it is possible for us to solve both these two problems altogether, which is just to the favor of some physicists.

Besides supersymmetry, other extensions of the Standard Model also give new particles which behave like WIMPs. For instance, in the technicolor model the lightest technibaryon may have similar properties with WIMPs, and is stable, like the proton in QCD. Nonetheless it is difficult to generate sufficient mass for quarks due to the experimental limits on the compositeness of quarks. The "little Higgs" model applies symmetries to protect the Higgs boson mass much lower than the compositeness scale, in a similar way leading the pion mass much smaller than that of a proton, i.e. generating Higgs as pseudo Nambu-Goldstone bosons. Hence even a very high compositeness scale, say, 10 TeV, may lead to a Higgs mass of a few hundred GeV inducing reasonable quark masses. Besides, as in supersymmetry, tree-level exchange of new particles, e.g. techniquarks, etc., tend to cause tension with precision electroweak constraints like flavor changing neutral currents. In order for the new states to be not so heavy so as to solve the hierarchy problem, a ”T-parity” is introduced. Then the new particles can not appear at tree-level, removing the constraints on the masses of the new states. The lightest particle with an odd T-parity (LTP) thus qualifies for a candidate of dark matter.

Another interesting possibility is to introduce WIMPs via extra dimensions. There are a bunch of extra dimension models including possible WIMP candidates. In a warped
extra dimension model like the Randall-Sandrum model, new particles are excited in the warped fifth dimension and the Planck scale varies from $10^{19}$ GeV to TeV along the fifth dimension. Therefore in contrast from the large extra dimension model like AKK, the standard model particles can live in the extra dimension and will interact with the new states. This characteristic will also bring up questions about proton longevity and a $Z_3$ symmetry should be placed. The lightest $Z_3$-charged particle (LZP) is stable and may play a role like dark matter.

I have to mention here that it is not necessary to arrive at the stability of the WIMP candidate by solving theoretical-experimental "conflicts" like the proton decay; instead, it can arise quite naturally. Consider a flat fifth dimension $y$, the Dirac equation is

$$\left(i\gamma^\mu \partial_\mu + i\gamma^5 y\right)\psi(x, y) = 0$$  \hspace{1cm} (16)$$

where $x$ denotes the ordinary four dimensions. Assuming the fifth dimension to be a circle with radius $R$, the wave function can be expanded in terms of Kaluza-Klein modes $\psi_n(x)e^{-iny/R}$, which satisfies

$$\left(i\gamma^\mu \partial_\mu + \gamma^5 n/R\right)\psi_n(x) = 0$$  \hspace{1cm} (17)$$

Replacing $\gamma^5$ with $-1$ for left-handed fermions and 1 for right-handed fermions, there are

$$\left(i\gamma^\mu \partial_\mu - n/R\right)\psi^l_n(x) = 0$$  \hspace{1cm} (18)$$

$$\left(i\gamma^\mu \partial_\mu + n/R\right)\psi^r_n(x) = 0$$  \hspace{1cm} (19)$$

However, as $n$ is any integer, the theory is naturally degenerate for left-handed and right-handed fermions, not consistent with the Standard Model. To include chirality we can identify the points $y$ and $-y$ on the circle, which is equivalent to setting $n$ always non-negative. Now there are only half of the states left and we can get chiral fermions. The system obtains a $Z_2$ symmetry, called the KK parity. At tree-level, all first Kaluza-Klein modes are degenerate $m_1 = 1/R$. Radiative corrections split their masses and typically the first Kaluza-Klein excitation of the $U(1)_Y$ boson is the lightest KK state (LKP) which is stable according to the KK parity. This LKP plays a role like LSP in supersymmetry and their collider phenomena very much resemble each other’s. It is a WIMP candidate, of course.

To sum up, in the various extensions of the Standard Model, new particles carrying a conserved quantum number will be introduced to solve the hierarchy problem. The lightest
state with that quantum number will be stable, and, in order not to invoke new hierarchy, have a sub-TeV mass. Moreover, the interactions between the state and Standard Model particles will be suppressed by the supersymmetry breaking scale. Therefore it satisfies the three conditions for WIMP.

C. Experimental Detections

To detect the WIMP and confirm it as the dark matter component, the task is two fold. On one hand, we can run experiments on accelerators to find the particles with predicted mass and other properties; on the other hand, it is possible to detect the dark matter particle directly and compare it with what we may find on accelerators. I am now going into more details.

WIMP is weakly interacting so it cannot be seen directly by a detector, like the neutrinos. To find the trace of WIMP it is necessary to know its characteristic interaction with other particles, which is model-dependent. Taking the supersymmetric model for an example, other supersymmetric partners with an odd R-parity will be heavier than the WIMP as LSP and decay to WIMP, emitting Standard Model particles like quarks and gluons. The resulting hadron jets will have several characteristics, among which a loss of four-momentum. The current data from Tevatron set a lower limit of about 300GeV on the masses of superpartners. On the LHC we expect to see collisions with total energy above 2000GeV at a significant rate, covering nearly all the parameter space for the SUSY WIMP model. There is substantial hope for testing other models of WIMPs, too.

In detecting galactic dark matter particles, the difficulties lie in the low density and slow speed of the galactic halo. For our galaxy, the halo has a typical speed of \( v \sim 220 \text{km/s} \). A WIMP with mass \( m_\chi \sim 100 \text{GeV} \) then has kinetic energy \( E_k = \frac{1}{2}m_\chi v^2 \sim 50 \text{keV} \). This tiny energy deposit is far smaller than the background from natural radioactivity which has a typical scale of MeV. Therefore the detectors should be extremely clean and placed deep underground to shield it from cosmic ray backgrounds. Then with a local halo density \( \rho_\chi^{\text{halo}} \sim 0.3 \text{GeV/cm}^3 \), the flux of WIMPs

\[
\dot{f}_\chi = v n_\chi = v \rho_\chi^{\text{halo}} / m_\chi
\]

If the target is made up of atoms with mass number \( A \) and the elastic cross section of WIMP
on the nucleus is $\sigma_A$, the expected event rate is

$$ R = f_\chi \sigma_A \frac{m_{\text{target}}}{m_A} $$

The elastic cross section of WIMP on neutron or proton may be spin-dependent or spin-independent. For the spin-independent case, the scattering amplitude goes as $A$; in most phenomenological cases we assume $\sigma_A = \sigma_p A^2$, then

$$ R = v \rho_{\chi}^{\text{halo}} / m_\chi \frac{m_{\text{target}}}{m_A} \sigma_p A^2 \sim \frac{10}{\text{year}} \frac{100\GeV}{m_\chi} \frac{m_{\text{target}}}{100\text{kg}} \frac{A}{56} \frac{\sigma_p}{10^{-42}\text{cm}^2} $$

(22)

We can see the event rate is quite slow.

There are a few currently undergoing experiments to detect WIMPs directly, measuring both the event rate and the energy deposit. Due to its astrophysical origin, there are two signatures of WIMP signal. One is the daily forward/backward asymmetry of the nuclear recoil direction caused by the alternate sweeping of the WIMP cloud by the rotating earth. The other is the few percent annual modulation of the recoil rate due to the Earth speed adding to or subtracting from the speed of the sun. The DAMA experiment operating 100kg of NaI(Tl) in Gran Sasso has observed an annually modulated signal with the expected phase with a statistical significance of $6.3\sigma$, over a period of 7 years with a total exposure of around 100 000kg·d, in the 2 to 6 keV energy interval. If explained with the standard halo model described above, it requires a WIMP with $m_\chi \sim 50\GeV$ and $\sigma_p \sim 7 \cdot 10^{-6}\text{pb}$.

Complementary to the direct detection is the indirect direction of galactic WIMPs, namely trying to detect the annihilation products of WIMPs instead of WIMPs themselves. Dark matter needs to accumulate for annihilation to occur, that is, at galactic centers, in the galactic halo, in the center of the sun, etc. Annihilations inside the sun produce extra neutrinos, and annihilations in the halo lead to continuous gamma rays and anti-protons. Among the many possibilities, the solar neutrino signal is of special interest: as the direct detection sensitivity goes down as $m_\chi^{-1}$ due to the factor of number density, the neutrino signal remains somewhat flat for heavy WIMPs, because the neutrino cross section grows as $E_\nu \sim m_\chi$. Also up till now the results from the solar neutrinos are the only competitive ones to the direct detections.
III. AXIONS

A. Theoretical Background

In the $SU(3)_c$ QCD, there can be an extra term $\frac{\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\text{Tr}(G_{\mu\nu}G_{\rho\sigma})$ added, where $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor. This term obeys the gauge invariance and can cause CP violation due to the presence of the Levi-Civita tensor. It seems to be a four-derivative and should vanish in the action, but the asymptotically free property of QCD tells that it may have non-vanishing effects. Nonetheless current experimental limits on the strong CP violation puts an upper limit on $\theta$ as $10^{-10} \ll 1$, which is very unnatural. In fact, the $\theta$ term can be absorbed into the constituent quark masses in the chiral perturbation theory, but leaving over a phase angle, i.e. $\theta$ is periodic under the transformation $\theta \rightarrow \theta + 2\pi$. Therefore a ”natural” value of $\theta$ should be of order unity, inconsistent with the experimental limit.

A plausible solution is to introduce a dynamical field which effectively sets $\theta$ at zero at the minimum potential. The dynamical field, denoted by $a$, is called ”axion”. It effectively couples to the gluons with

$$\mathcal{L} = (\theta + \frac{a}{f_a})\epsilon^{\mu\nu\rho\sigma}\text{Tr}(G_{\mu\nu}G_{\rho\sigma})$$

(23)

where $\theta$ is the ”natural” value with order unity, and $f_a$ is called axion decay constant with the dimension of energy. At its minimum potential, the axion field should settle to $a = -\theta f_a$, so we can construct the potential as

$$V \sim m^2_\pi f^2_\pi (1 - \cos(\theta + \frac{a}{f_a}))$$

(24)

which preserves the periodicity. The factor of $m^2_\pi f^2_\pi$ is introduced to render the axion condensation at the QCD scale. Let’s write $a = \tilde{a} - \theta f_a$, then

$$V \sim m^2_\pi f^2_\pi (1 - \cos(\frac{\tilde{a}}{f_a})) \sim \frac{1}{2} m^2_\pi f^2_\pi \tilde{a}^2 \frac{f^2_a}{f^2_\pi}$$

(25)

hence the mass of the axion is

$$m_a \sim \frac{m_\pi f_\pi}{f_a} \sim 6\mu\text{eV} \frac{10^{12}\text{GeV}}{f_a}$$

(26)

Thus the axion mass only depends on the decay constant $f_a$, which constraints its coupling to gluons. A too small $f_a$ means strong coupling, therefore permitting axions to carry away
energy from stars, cooling them too quickly. Current astrophysical observations require $f_a \gtrsim 10^{10}\text{GeV}$, so the mass $m_a \lesssim 0.6\text{meV}$. Models of axions with such light masses have two popular versions, KSVZ and DFSZ. Turning to the cosmological implications of axions, I will not talk about these detailed models in the article.

B. Cosmological Evolution

Although with such a small mass, axions can be a candidate for dark matter. To see this clearly we follow the evolution of the axion field in the cosmological time. In the early universe when $T \gg \text{TeV}$, the axion potential looks flat and it may lie at any state, most likely not at the minimum. As the universe cools down, the dynamics of axions can be derived from the Lagrangian

$$L \sim a^3 \left( \frac{1}{2} \dot{\phi}^2 - V(\phi) \right)$$

(27)

where we use $\phi$ to denote the axion field and $a$ for the characteristic scale of the universe. The spatial variation has been neglected with only the time dependence alone, because of the flatness of the universe. The derived Euler-Lagrange equation is

$$\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0$$

(28)

where $H = \dot{a}/a$. Because of the large $f_a$, $V(\phi)$ can be approximated by the quadratic mass term $\frac{1}{2}m^2\phi^2$. The Euler-Lagrange equation is thus homogeneous

$$\ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0$$

(29)

While in the real radiation-dominated universe $H(t) = 1/2t$, it is useful to see the characteristic of the solutions with a constant $H$ first. Taking the Fourier mode $\phi(t) \sim e^{-i\omega t}$, we have

$$-\omega^2 - 3iH + m^2 = 0$$

(30)

with the solutions for $\omega$

$$\omega_\pm = \frac{1}{2} (-3iH \pm \sqrt{-9H^2 + 4m^2})$$

(31)

In the early universe and even after the photon decoupling time, due to the tiny mass of the axion, we have $H \gg m$, with the solutions $\omega_+ = -3iH$ and $\omega_- = -im^2/3H$. The former mode damps quickly as $\phi_+ \sim e^{-3Ht}$, while the latter one is nearly stationary.
\[ \phi_+ \sim e^{-(m^2/3H)t}. \] Therefore after substantial time the solution is nearly all \( \phi_+ \), which is "stuck" by the friction term \(-3H\dot{\phi}\). On the other hand, when \( H \ll m \), the solutions are \( \omega_+ = m - i3H/2 \), \( \omega_- = -m - i3H/2 \), that is, the axion field oscillates around the minimum with the expected frequency \( \pm m \) as it decays slowly in \( e^{-3Ht/2} \).

The qualitative aspects of the solution remains unchanged for \( H(t) = 1/2t \). Using adiabatic approximation, we replace \( e^{-i\omega_\pm t} \) with \( e^{-i \int_0^t dt' \omega_\pm(t')} \). For \( H \gg m \), we get

\[ \phi_+ = \phi_0 e^{-3 \int_0^t H(t') dt'} = \phi_0 (t/t_0)^{-3/2} = \phi_0 (a/a_0)^{-3} \tag{32} \]
\[ \phi_- = \phi_0 e^{-m^2 \int_0^t dt' / 2H(t')} = \phi_0 e^{-m^2 (t^2 - t_0^2)/3} = \phi_0 e^{-1/12(m^2/H(t)^2 - m^2/H(t_0)^2)} \sim \phi_0 \tag{33} \]

with \( \phi_+ \) decaying quickly in the early-universe rapid expansion, leaving over only \( \phi_- \) which is stuck by friction. For \( H \ll m \)

\[ \phi_\pm = \phi_0 e^{\pm imt} e^{-2 \int_0^t H(t') dt'} = \phi_0 e^{\pm imt} (t/t_0)^{-3/4} = \phi_0 e^{\pm imt} (a/a_0)^{-3/2} \tag{34} \]

that is, after decoupling the axion field oscillates around its minimum potential and dilutes in the same way as non-relativistic matter. Meanwhile, in the early universe, including the matter-radiation decoupling period, it can sit on the potential and doesn’t roll down, because of the large friction term. Hence such a light scalar field like axion can make a candidate for the cold dark matter. The effect is caused by the "friction" in the expansion and I don’t have any intuitive way to explain it yet.

For light fermions, however, there is no bilinear term of \( \dot{\phi} \) in the Lagrangian, so the Euler-Lagrange equation will only contain the first-order derivative of the field and only the "normal" solution \( \phi_+ \) will arise. Therefore light fermions like neutrinos cannot satisfy the imposed conditions in Sec.I.

C. Experimental Search

In addition to the required coupling to QCD gluons, most axion models predict its coupling to photons with the similar order of magnitude. At present two experiments are going on searching for axionic dark matter. They both employ high quality cavities to detect the conversion of an axion to a photon. The quality factor \( Q \) enhances the conversion rate when the mass of the axion coincides with the resonance frequency \( \omega_0 \), i.e. \( \hbar \omega_0 = m_a c^2 \).

By changing the resonant frequency in steps we can scan over a range of axion mass. The
current result from LLNL has ruled out axions with mass between 1.9 and 3.3 µeV, corresponding to \( f_a \sim 4 \times 10^4 \) GeV, as a major part of the dark halo in our galaxy, if the axion-photon coupling \( g_{a\gamma\gamma} \) is near the upper end of the theoretically expected range. The smaller CARRACK experiment in Kyoto, Japan gets the preliminary results which exclude axions with mass in a narrow range around 10 eV as a major component of the dark halo for a plausible range of \( g_{a\gamma\gamma} \) values.

These mentioned results have just been applied to the KSVZ axion model. An upgrade of CARRACK to CARRACK II is in the way to probe the axion mass range between 2 and 50 µeV, sensitive to both KSVZ and KFSZ models, if axions make up most of the dark matter.

Besides the astrophysical observation of "axion-like" components of the dark matter, we would wish to produce axions directly in particle colliders. However, the severe problem related to this idea is the high scale of \( f_a \). To directly observed substantial effects of the axion field we should get up to that scale which is simply beyond our reach in a not-too-short time. This high scale also prohibits anomalous effects in the precision low-energy measurements. It is also a large hierarchy here, and particle theorists may introduce other new mechanisms to reduce the hierarchy, when the astrophysical detections come to an end in vain.