ELEMENTARY PARTICLE PHYSICS

Physics 363 - Spring Quarter, 2007 - University of Chicago

PROBLEMS DUE TUESDAY, MAY 22

The effective Lagrangian for the interaction of a Z with a fermion f is

$$\mathcal{L}_{\rm Zf\bar{f}} = \sqrt{g^2 + g'^2} \bar{\psi}_f \gamma^\mu (I_{3L} - Q_f \sin^2\theta) \psi_f Z_\mu \quad , \tag{1}$$

where Q_f is the fermion's electric charge in units of $|e|$, and the weak mixing angle satisfies $\sin^2\theta \simeq 0.231$. A useful relation is $G_F/\sqrt{2} = (g^2 + g'^2)/(8M_Z^2)$.

(1) Calculate the partial width $Z \to f\bar{f}$ for each species of fermion: three neutrinos, three charged leptons, and the five quarks u, d, s, c, b. Neglect the masses of all fermions except for b, where you may take $m_b = 4.8$ GeV. For quarks apply a QCD factor $3(1 + [\alpha_S/\pi])$, where 3 is the number of quark colors and $\alpha_S(M_Z) \simeq 0.12$.

(2) Sum the partial widths to get a total Z width. (Optional: Compare your answer with the experimental value quoted by the Particle Data Group.) Use this total width to calculate branching ratios for each species of fermion.

ANSWERS

(1) The Feynman matrix element for $Z(P) \to f(p_1)\bar{f}(p_2)$ can be written

$$\mathcal{M}_{fi} = \sqrt{g^2 + g'^2} \bar{u}(p_1) \gamma^\mu (g_V + g_A\gamma^5) v(p_2)\ \epsilon_\mu(P), \tag{2}$$

where

$$g_V = \frac{I_{3L}}{2} - Q\ x\ , \quad g_A = -\frac{I_{3L}}{2}\ , \quad x \equiv \sin^2\theta\ , \tag{3}$$

and $\epsilon_\mu(P)$ is the polarization vector of the Z. When taking the squared absolute value of this matrix element, summing over final lepton spins and averaging over Z polarizations, the result is

$$\frac{1}{3}\sum_{\rm pol} |\mathcal{M}_{fi}|^2 = \frac{1}{3}\left(-g_{\mu\nu} + \frac{P_\mu P_\nu}{M_Z^2}\right)(g^2 + g'^2)(g_V^2 T_+^{\mu\nu} + g_A^2 T_-^{\mu\nu})\ , \tag{4}$$

where for a pair of fermions of mass m in the final state,

$$T_\pm^{\mu\nu} = {\rm Tr}[(\not{p}_1 + m)\gamma^\mu(\not{p}_2 \mp m)\gamma^\nu]\ . \tag{5}$$

The sign flip in $T_-^{\mu\nu}$ comes from anticommuting a γ^5 through an odd number of γ matrices. Then

$$T_\pm^{\mu\nu} = 4[p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - g^{\mu\nu}(p_1 \cdot p_2 \pm m^2)]\ ; \tag{6}$$

$$\frac{1}{3}\sum_{\rm pol}|\mathcal{M}_{fi}|^2 = \frac{4}{3}(g^2+g'^2)M_Z^2\left[g_V^2\left(1+\frac{2m^2}{M_Z^2}\right)+g_A^2\left(1-\frac{4m^2}{M_Z^2}\right)\right] \ . \tag{7}$$

Using the expression for the partial width

$$\Gamma(Z\to f\bar{f}) = \frac{1}{2M_Z}\frac{1}{3}\sum_{\rm pol}|\mathcal{M}_{fi}|^2\frac{p^*}{4\pi M_Z} \ , \tag{8}$$

where p^* is the magnitude of the final center-of-mass 3-momentum, and the relation $G_F/\sqrt{2} = (g^2+g'^2)/(8M_Z^2)$, one finds

$$\Gamma(Z\to f\bar{f}) = \frac{2G_F M_Z^3}{3\pi\sqrt{2}}\left(1-\frac{4m^2}{M_Z^2}\right)^{1/2}\left[g_V^2\left(1+\frac{2m^2}{M_Z^2}\right)+g_A^2\left(1-\frac{4m^2}{M_Z^2}\right)\right] \ . \tag{9}$$

Note that the $m\neq 0$ kinematic factor accompanying g_V^2 is the same appearing in $e^+e^-\to f\bar{f}$, while that accompanying g_A^2 is the $(p^*)^3$ factor characteristic of P-wave $f\bar{f}$ production. The expressions for g_V and g_A, the corresponding partial widths (in MeV), and the predicted branching ratios are shown in the Table. Here we have applied a QCD correction factor $3(1+[\alpha_S/\pi])$ with $\alpha_S(M_Z) = 0.12$ to the partial widths for $Z\to q\bar{q}$. Masses have been neglected except in the case of $Z\to b\bar{b}$, where they lead to a net correction factor of 0.9888.

f	g_V	g_A	$\Gamma(Z\to f\bar{f})$ (MeV)	$\mathcal{B}(Z\to f\bar{f})$ (%)
ν	$\frac{1}{4}$	$-\frac{1}{4}$	165.9	6.68
ℓ^-	$-\frac{1}{4}+x$	$\frac{1}{4}$	83.4	3.36
u, c	$\frac{1}{4}-\frac{2x}{3}$	$-\frac{1}{4}$	296.2	11.94
d, s	$-\frac{1}{4}+\frac{x}{3}$	$\frac{1}{4}$	381.9	15.39
b	$-\frac{1}{4}+\frac{x}{3}$	$\frac{1}{4}$	377.6	15.22
Total			2482	100.00

The actual value of Γ_Z quoted in the Particle Data group is $\Gamma(Z) = (2495.2\pm 2.3)$ MeV; the difference between this and the calculated value reflects the importance of higher-order radiative corrections.