ELEMENTARY PARTICLE PHYSICS

Physics 363 - Spring Quarter, 2007 - University of Chicago

PROBLEMS DUE TUESDAY, MAY 15

A π^- (mass $m_\pi = 139.57$ MeV, 4-momentum q) decays to a μ^- (mass $m_\mu = 105.66$ MeV, 4-momentum p_1) and a $\bar{\nu}_\mu$ (assumed massless, 4-momentum p_2). We are taking $c = 1$. The Feynman matrix element may be written

$$\mathcal{M}_{fi} = q^\mu f_\pi \frac{G_F}{\sqrt{2}} \bar{u}(p_1)\gamma_\mu(1-\gamma^5)v(p_2)V_{ud} \ , \tag{1}$$

where $f_\pi = 131$ MeV is the pion decay constant, $G_F = 1.16637 \times 10^{-5}$ GeV^{-2} is the Fermi decay constant, and $V_{ud} \simeq 0.974$ is a Cabibbo-Kobayashi-Maskawa (CKM) matrix element.

(1) Calculate the pion lifetime in seconds. A convenient conversion factor is $\hbar = 6.528 \times 10^{-25}$ (GeV $\cdot$ seconds). Assume the pion decays 100% of the time to $\mu\bar{\nu}$.

(2) Calculate the ratio of decay rates $\Gamma(\pi \to e\bar{\nu}_e)/\Gamma(\pi \to \mu\bar{\nu}_\mu)$.

(3) A bound state of the charmed quark c and an anti-strange quark $\bar{s}$, the D_s^+, has a mass of 1968.2 MeV and can decay to $e^+\nu_e$, $\mu^+\nu_\mu$, and $\tau^+\nu_\tau$, where $m_e = 0.511$ MeV and $m_\tau = 1777$ MeV. The lifetime of the D_s^+ is measured to be 0.5 ps $= 0.5 \times 10^{-12}$ s. Using the Feynman matrix element (1) but with f_π replaced by f_{D_s} and V_{ud} replaced by $V_{cs} = V_{ud}$, calculate the branching ratios for $D_s \to (e^+\nu_e,\ \mu^+\nu_\mu,\ \tau^+\nu_\tau)$, expressed in terms of the ratio $f_{D_s}/(275 \text{ MeV})$. [The constant f_{D_s} has recently been measured by the CLEO Collaboration at Cornell to be $274 \pm 13 \pm 7$ MeV by observing the decays $D_s \to (\mu^+\nu_\mu\ ,\ \tau^+\nu_\tau)$.]

ANSWERS

(1) The sum over lepton polarizations is

$$\begin{aligned} T_{\mu\nu} &\equiv \sum_{\text{pol}} [\bar{u}(p_1)\gamma_\mu(1-\gamma^5)v(p_2)][\bar{u}(p_1)\gamma_\nu(1-\gamma^5)v(p_2)]^* \\ &= \text{Tr}[(\not{p}_1 + m_\mu)\gamma_\mu(1-\gamma^5)\ \not{p}_2(1+\gamma^5)\gamma_\nu] = 2\text{Tr}(\not{p}_1\gamma_\mu\ \not{p}_2\gamma_\nu)\ , \end{aligned} \tag{2}$$

where we have used the identities $\{\not{p}_2, \gamma^5\} = 0$ and $(1-\gamma^5)^2 = 2(1-\gamma^5)$. Then, noting that $q = p_1 + p_2$,

$$q^\mu q^\nu T_{\mu\nu} = 2\mathrm{Tr}[\not{p}_1(\not{p}_1 + \not{p}_2)\not{p}_2(\not{p}_1 + \not{p}_2)] = 8m_\mu^2\, p_1 \cdot p_2 \ . \tag{3}$$

Note the proportionality of the spin-averaged squared matrix element to the square of the lepton mass. Since $(p_1+p_2)^2 = m_\pi^2$, we have $8p_1 \cdot p_2 = 4(m_\pi^2 - m_\mu^2)$. Then

$$\Gamma(\pi \to \mu\bar{\nu}) = \frac{G_F^2}{2}\frac{1}{2m_\pi}\frac{p^*}{4\pi m_\pi}4(m_\pi^2 - m_\mu^2)m_\mu^2 f_\pi^2 |V_{ud}|^2 \ . \tag{4}$$

Now we use the expression for the magnitude p^* of the final c.m. 3-momentum in the case of one massless particle, $p^* = (m_\pi^2 - m_\mu^2)/(2m_\pi)$ to obtain the final result

$$\Gamma(\pi \to \mu\bar{\nu}) = G_F^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 \frac{m_\mu^2 m_\pi f_\pi^2 |V_{ud}|^2}{8\pi} \ . \tag{5}$$

With the values of the parameters given in the problem, we find $\Gamma(\pi \to \mu\bar{\nu}) = 2.46 \times 10^{-17}$ GeV, or $\tau_\pi = \hbar/\Gamma = 26.7$ ns, where we have used the conversion factor $\hbar = 6.582 \times 10^{-25}$ GeV $\cdot$ s. The actual lifetime quoted by the Particle Data Group is 26.0 ns.

(2) The ratio of decay rates is

$$\frac{\Gamma(\pi \to e\bar{\nu}_e)}{\Gamma(\pi \to \mu\bar{\nu})} = \frac{m_e^2}{m_\mu^2}\frac{\left(1 - \frac{m_e^2}{m_\pi^2}\right)^2}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} = 1.31 \times 10^{-4} \ , \tag{6}$$

where we have used $m_e = 0.511$ MeV. The observed branching ratio $\mathcal{B}(\pi \to e\bar{\nu}_e) = (1.230 \pm 0.004) \times 10^{-4}$ is slightly lower as a result of radiative corrections.

(3) With the parameters shown, one finds $\Gamma[D_s^+ \to (e^+\nu_e,\ \mu^+\nu_\mu,\ \tau^+\nu_\tau)] = (2.00 \times 10^{-19},\ 8.48 \times 10^{-15},\ 8.25 \times 10^{-14})$ GeV, or with $\mathcal{B} = \Gamma\tau_{D_s}/\hbar$, one finds $\mathcal{B}[D_s \to (e^+\nu_e,\ \mu^+\nu_\mu,\ \tau^+\nu_\tau)] = (1.52 \times 10^{-7}\ , 6.44 \times 10^{-3},\ 6.27 \times 10^{-2})$, all for $f_{D_s} = 275$ MeV and scaling as $(f_{D_s}/275\ \mathrm{MeV})^2$. The CLEO Collaboration measures $\mathcal{B}(D_s^+ \to \mu^+\nu_\mu) = (5.94 \pm 0.66 \pm 0.31) \times 10^{-3}$ and $\mathcal{B}(D_s^+ \to \tau^+\nu_\tau) = (8.0 \pm 1.3 \pm 0.4) \times 10^{-2}$.