ELEMENTARY PARTICLE PHYSICS

Physics 363 - Spring Quarter, 2007 - University of Chicago

PROBLEMS DUE TUESDAY, MAY 8

The Feynman rule for the electromagnetic interaction of a scalar (spinless) particle ϕ with charge e, incoming 4-momentum p, and outgoing momentum p' in lowest order of perturbation theory is $-ie(p+p')^\mu$, where μ is the photon polarization Lorentz index. For production of a pair $\phi^+(p')\phi^-(p)$ it is then $-ie(p'-p)^\mu$.

(1) Calculate the differential cross section $d\sigma/d\Omega$ per unit of solid angle for the process $e^+e^- \to \phi^+\phi^-$. Neglect the electron mass but assume the scalar particle has mass m and charge e. Express your answer in terms of the square of the center-of-mass energy $s = E^2_{\text{c.m.}}$, the fine-structure constant $\alpha = e^2/4\pi$, and either m or the c.m. velocity of $\phi^\pm$, $\beta = (1-4m^2/s)^{1/2}$.

(2) Neglecting m, compare your result for the distribution in the cosine of the c.m. polar angle, $z \equiv \cos\theta^*$, with that for $e^+e^- \to \mu^+\mu^-$, where the muon is also assumed massless.

(3) Performing the integral over solid angle, compare the behavior near threshold for $e^+e^- \to f\bar{f}$ (where f is a spin-1/2 fermion like the muon) with that of $e^+e^- \to \phi^+\phi^-$, where ϕ is a spinless particle.

ANSWERS

(1) The expression for the differential cross section for $e^-(p)e^+(p')$ annihilation is

$$\frac{d\sigma}{d\Omega} = \frac{1}{4F}\frac{1}{4}\sum_{\text{pol}}|\mathcal{M}_{fi}|^2\frac{p^*}{16\pi^2 W} \quad , \tag{1}$$

where $F = \sqrt{(p\cdot p')^2 - p^2(p')^2}$, p^* is the magnitude of the final c.m. 3-momentum, and $W = \sqrt{s}$ is the total energy. For a final state of two scalars $\phi^-(k)\ \phi^+(k')$, with $k + k' = p + p' \equiv q$, the polarization-averaged square of the covariant Feynman matrix element $\mathcal{M}_{fi}$ is

$$\frac{1}{4}\sum_{\text{pol}}|\mathcal{M}_{fi}|^2 = \frac{1}{4}\left(\frac{e^2}{q^2}\right)E_{\mu\nu}(k-k')^\mu(k-k')^\nu \quad . \tag{2}$$

Here, as in $e^-e^+ \to f\bar{f}$, where f is a fermion,

$$E_{\mu\nu} = \text{Tr}[(\not{p}+m_e)\gamma_\mu(\not{p}'-m_e)\gamma_\nu] = 4[p_\mu p'_\nu + p_\nu p'_\mu - g_{\mu\nu}(p\cdot p' + m_e^2)] \quad . \tag{3}$$

In what follows we shall neglect m_e, denoting the final particle's mass by m, its c.m. velocity by β, and its c.m. energy by $E = W/2 = \sqrt{s}/2$. The various Lorentz-invatiant quantities may be expressed, as in the $e^-e^+ \to f\bar{f}$ calculation done in class, as

$$k \cdot p = k' \cdot p' = E^2(1-\beta z) \ , \quad k \cdot p' = k' \cdot p = E^2(1+\beta z) \ ,$$

$$k^2 = (k')^2 = m^2 = E^2(1-\beta^2) \ , \quad p \cdot p' = 2E^2 = s/2 \ , \quad k \cdot k' = E^2(1+\beta^2) \quad , \tag{4}$$

where $z = \cos\theta^*$ is the cosine of the c.m. scattering angle. Then the quantity (2) simplifies to

$$\frac{1}{4}\sum_{\rm pol} |\mathcal{M}_{fi}|^2 = (e^2/q^2)^2 \left[2p\cdot(k-k')p'\cdot(k-k') - (k-k')^2 p\cdot p'\right] = (e^2)^2\beta^2(1-z^2)/2 \ . \tag{5}$$

Using $4F = 2s$ and $p^*/W = \beta/2$, we finally find

$$\frac{d\sigma}{d\Omega}(e^-e^+ \to \phi^-\phi^+) = \frac{\alpha^2}{8s}\beta^3(1-z^2) = \frac{\alpha^2}{8s}\beta^3\sin^2\theta^* \ . \tag{6}$$

(2) The angular distribution for $e^-e^+ \to \phi^-\phi^+$ is proportional to $\sin^2\theta^*$ whether or not the final-state mass is neglected. For $e^-e^+ \to f\bar{f}$ we found in class that the differential cross section (neglecting m_e) was proportional to $1 + \beta^2 z^2 + (1-\beta^2)$, or to $1+\cos^2\theta^*$ in the high-energy limit $\beta \to 1$.

(3) Integrating over solid angle, we find

$$\sigma(e^-e^+ \to \phi^-\phi^+) = \frac{\pi\alpha^2}{3s}\beta^3 = \frac{\pi\alpha^2}{3s}\left(1 - \frac{4m^2}{s}\right)^{3/2} \quad , \tag{7}$$

which in the limit $s \to \infty$, $\beta \to 1$ goes to 1/4 the cross section for a spin-1/2 fermion of unit charge. The corresponding integral for spin-1/2 fermion pair production gives

$$\sigma(e^-e^+ \to f\bar{f}) = \frac{2\pi\alpha^2}{3s}\beta(3-\beta^2) = \frac{4\pi\alpha^2}{3s}\left(1-\frac{4m^2}{s}\right)^{1/2}\left(1+\frac{2m^2}{s}\right) \quad . \tag{8}$$

The fermion pair production cross section, being proprtional to the first power of β, rises much more rapidly above threshold $s = 4m^2$ than the scalar pair production cross section, which is proportional to β^3.