ELEMENTARY PARTICLE PHYSICS

Physics 363 - Spring Quarter, 2007 - University of Chicago

PROBLEMS DUE THURSDAY, APRIL 5

(1) Taking account just of effective quark masses and hyperfine (spin–spin) interactions, one may write the following mass formulae for the lowest-mass spin-1/2 baryons:

$$\begin{aligned} M(N) &= 3m_0 - 3\frac{a}{m_0^2}\ , \qquad M(\Lambda) = 2m_0 + m_s - 3\frac{a}{m_0^2}\ , \\ M(\Sigma) &= 2m_0 + m_s + \frac{a}{m_0^2} - 4\frac{a}{m_0 m_s}\ , \ M(\Xi) = m_0 + 2m_s + \frac{a}{m_s^2} - 4\frac{a}{m_0 m_s}\ , \end{aligned} \quad (1)$$

where the choices $m_0 \equiv m_{u,d} = 363$ MeV and $m_s = 538$ MeV give a reasonable fit to the observed masses. [See, e.g., S. Gasiorowicz and J. L. Rosner, Am. J. Phys. **49**, 954 (1981).] Here $M(N)$ denotes the nucleon mass (average of neutron and proton masses). Show that the Gell-Mann – Okubo combination

$$\Delta_{\rm GMO} \equiv \frac{M(N) + M(\Xi)}{2} - \frac{3M(\Lambda) + M(\Sigma)}{4} \quad (2)$$

is of second order in $m_s - m_0$ (i.e., second order in SU(3) breaking). Calculate its value. For up-to-date baryon masses, see the Particle Data Group, `http://pdg.lbl.gov/`. Click on Summary Tables and then go to Baryons. You will find p on p. 1, n on p. 4, Λ on p. 15, the Σs starting on p. 20, and the Ξs starting on p. 25. For masses you may take averages over an isospin multiplet, e.g., $M(N) = [M(p) + M(n)]/2$.

(2) Calculate the magnetic moments of the proton, neutron, Λ, charged Σs, and Ξs in terms of quark magnetic moments $\mu_q = eQ_q/2m_q$. Using the masses m_q mentioned above, predict the values of these magnetic moments in terms of nuclear magnetons $e/2m_N$. Compare with experimental values. These may be found in the Particle Data Group tables, as noted in Problem 1.

ANSWERS

(1) With the expressions (1) for the baryon masses, one has

$$\Delta_{\rm GMO} = \frac{a}{2}\left(\frac{1}{m_s} - \frac{1}{m_0}\right)^2 = \frac{a}{2}\frac{(m_0 - m_s)^2}{m_s^2 m_0^2} \quad (3)$$

which is second-order in the SU(3)-breaking difference $m_s - m_0$. Numerically $M(p) =$ 938.272 MeV, $M(n) = 939.565$ MeV, $M(N) = 938.92$ MeV; $M(\Lambda) = 1115.68$ MeV; $M(\Sigma^+) = 1189.37$ MeV, $M(\Sigma^0) = 1192.64$ MeV, $M(\Sigma^-) = 1197.45$ MeV, $M(\Sigma) \equiv [M(\Sigma^+) + M(\Sigma^0) + M(\Sigma^-)]/3 = 1193.15$ MeV; $M(\Xi^0) = 1314.83$ MeV, $M(\Xi^-) =$ 1321.31 MeV, $M(\Xi) \equiv [M(\Xi^0) + M(\Xi^-)]/2 = 1318.1$ MeV, so $\Delta_{\rm GMO} = -6.55$ MeV. This is small compared with splitting among members of the baryon octet.

(2) For octet baryons of the form $B = q_1 q_1 q_2$, we showed in class that

$$\mu(B) = \frac{4}{3}\mu_1 - \frac{1}{3}\mu_2 \quad , \quad \mu_i \equiv \frac{e}{2m_i} Q_i \quad , \tag{4}$$

while for the Λ, all the spin is carried by the s quark, so $\mu(\Lambda) = \mu_s$. The magnetic moments of the quarks for $m_0 = 363$ MeV, $m_s = 538$ MeV are

$$\mu_u = \frac{2}{3}\frac{e}{2m_0} = 1.724\mu_N \ , \quad \mu_d = -\frac{1}{3}\frac{e}{2m_0} = -0.862\mu_N \ , \quad \mu_s = -\frac{1}{3}\frac{e}{2m_s} = -0.582\mu_N \ , \tag{5}$$

where one nuclear magneton (n.m.) is $\mu_N = e/[2M(N)]$. The predicted and observed magnetic moments, in units of μ_N, are compared in the Table.

Particle	Magnetic moment (n.m.)	
	Predicted	Observed
p	2.587	2.793
n	-1.724	-1.913
Λ	-0.582	-0.613 ± 0.004
Σ^+	2.493	2.458 ± 0.010
Σ^-	-0.956	-1.160 ± 0.025
Ξ^0	-1.350	-1.250 ± 0.014
Ξ^-	-0.488	-0.6507 ± 0.0025