Physics 342 Final Project:
Entangled Photons and Quantum Teleportation

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Abstract

This paper contains an overview of the concept of photon entanglement and discusses generating EPR photon pairs and performing Bell state measurements on them. The procedure of quantum teleportation, or transferring the quantum state of a particle onto another particle, is described in detail. The experimental progress in performing such operations is reviewed in relationship to the developing field of quantum communication.

1 Entangled photons

1.1 Bell states

Consider the polarization of a single photon. Choose as basis kets the states where the photon is horizontally ($|\leftrightarrow\rangle$) or vertically ($|\updownarrow\rangle$) polarized. The polarization state will be a linear superposition of these two base kets:

$$|\Psi\rangle = \alpha\,|\leftrightarrow\rangle + \beta\,|\updownarrow\rangle\,, \tag{1}$$

with α and β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$.

In quantum computing, such a quantum state describing a two-state system is referred to as a qubit, or a unit of quantum information. The basis of quantum computing lies in the fact that, unlike a classical bit which can take a value of either 0 or 1, the qubit can be any superposition of the two base kets. The following discussion is valid not just for a photon, but for any two-state system, for instance a spin-$\frac{1}{2}$ particle, by replacing $|\leftrightarrow\rangle$ and $|\updownarrow\rangle$ with the two appropriate states for the problem.

An essential feature of qubits is that they can become entangled. A Bell state is a quantum state of two maximally entangled qubits. There exist 4 such states:

$$\begin{aligned} \left|\Phi^{\pm}\right\rangle_{12} &= \frac{1}{\sqrt{2}} \left(|{\leftrightarrow}\rangle_1 \, |{\leftrightarrow}\rangle_2 \pm |{\updownarrow}\rangle_1 \, |{\updownarrow}\rangle_2 \right) \\ \left|\Psi^{\pm}\right\rangle_{12} &= \frac{1}{\sqrt{2}} \left(|{\leftrightarrow}\rangle_1 \, |{\updownarrow}\rangle_2 \pm |{\updownarrow}\rangle_1 \, |{\leftrightarrow}\rangle_2 \right). \end{aligned} \tag{2}$$

An Einstein-Podolsky-Rosen (EPR) pair is a pair of qubits in a Bell state. The entangled state contains no information about the individual states, but only on the relationship between the two. A measurement of the polarization state of one photon instantaneously determines the polarization of the other, no matter how far apart the two are brought. Alternatively, two observers, each performing a polarization measurement on one photon of the pair, would obtain results that prove 100% correlated upon comparison. Each observer's measurement of his own photon has a 50% chance of yielding $|{\leftrightarrow}\rangle$ or $|{\updownarrow}\rangle$. However, if the first observer measures $|{\leftrightarrow}\rangle_1$ for his photon and the system is known to be in one of the $|\Phi^{\pm}\rangle_{12}$ states, then the second observer will necessarily also get $|{\leftrightarrow}\rangle_2$ for his photon. For the $|\Psi^{\pm}\rangle_{12}$ states, observer 2 will measure the opposite polarization from 1, in this case $|{\updownarrow}\rangle_2$.

Such a puzzling, instantaneous interaction at a distance violates Einstein's locality principle. The issue was first raised in a paper written in 1935 by Einstein, Podolsky and Rosen [6] and is known as the EPR paradox. The locality principle can be employed to derive Bell's inequalities, which have been proven not to hold in numerous experiments, the first of which was done in 1972. A possible explanation for this paradox is the incompleteness of the probabilistic quantum mechanical theory, and one resolution assumes the existence of some yet undefined, nonlocal hidden variables.

It must be mentioned that entanglement does not imply superluminal communication between observers, because they need to share information about their measurement outcomes through a classical communication channel in order to note the correlation.

1.2 Creation of entangled photons

Polarization entangled photons can be produced through spontaneous parametric down-conversion (SPDC) [10]. This procedure makes use of a nonlinear crystal in which the polarization vector is not linearly proportional to the electric field of the light; rather, the material has a second order nonlinear susceptibility, such that

$$P(t) = \epsilon_0 \left(\chi^{(1)} E(t) + \chi^{(2)} E^2(t) \right) \tag{3}$$

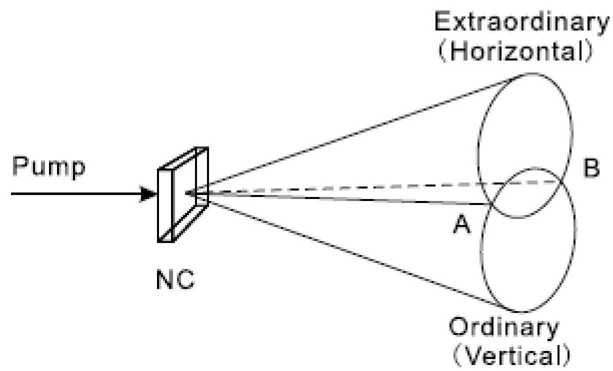

Figure 1: Production of entangled photon pairs through a nonlinear crystal (from [12]).

and the index of refraction depends on the polarization of the incoming light. A pump photon incident on a β-barium-borate (BBO) birefringent crystal can spontaneously decay into two a pair of lower energy photons, conserving energy and momentum. The two can be considered to be generated simultaneously up to less than 1 picosecond. If a beam of ultraviolet light is sent through a Type-II configured crystal, two separate but overlapping light cones will emerge, one consisting of horizontally and the other of vertically polarized photons (see Fig. 1). The photons emitted at the intersection of the two cones will be entangled in a $|\Psi\rangle^{\pm}$ state. The $|\Phi^{\pm}\rangle$ states can be obtained by suitably passing one of the photons through waveplates. Pulsing the beam makes it possible to distinguish individual pairs of photons in the process [8].

Polarization entanglement is preferentially used due to its relative ease of experimental implementation and higher precision, but one can also have entanglement in momentum, time, orbital angular momentum or even a combination of these degrees of freedom (hyper-entanglement).

Entangled photons are essential in quantum information processing, allowing for the development of fields such as quantum cryptography, also called quantum key distribution (QKD), and quantum communication. The first QKD protocol, BB84, developed by Bennett and Brassard in 1984, can be used to transfer information through the polarization state of single photons, but this method is vulnerable to attacks and the photon signal will attenuate and decohere over large distances (hundreds of kilometers). Later on, QKD protocols based on entangled photons were developed, such as

Ekert91. More recent possibilities, quantum secret sharing and third-man quantum cryptography, allow more than two users share a message through the production of entangled states of 3 up to 6 photons; such methods are still in their early stages.

1.3 Bell-state measurements

A Bell state measurement (BSM) is a measurement performed on two qubits, which serves to determine which of the four Bell states the two qubits are in. It is an entangling operation because it projects an arbitrary state of two particles onto the basis of the 4 Bell states, even if the qubits were not in a Bell state before the measurement. The measurement gives no information on the individual particles, only on their joint state.

This can be realized experimentally with the use of a beam splitter (BS), a half-silvered mirror that reflects half the incident light and transmits the other half. The two photons subject to the measurement are directed towards the beam splitter from opposite sides, say one going upwards and one going downwards. There are three possible outcomes: both photons can emerge going outwards, both going downwards, or one going upwards and one going downwards. In the last case, the photons cannot be told apart, as they could have either both been reflected or both been transmitted through the BS, and thus become entangled. If two detectors placed on each side of the BS go off simultaneously, this means that the pair has been projected onto the $|\Psi^-\rangle_{12}$ Bell state.

1.4 Experimental status

Entanglement has so far been experimentally preserved at a distance of 144 km in an experiment by Ursin et al. [11], where high-quality pairs of polarization-entangled photons are produced on the Canary Island of La Palma. One of the photons in the pair is directed to a transmitter telescope and sent through a 144 km free-space optical link to the 1 m Ritchey−Chrétien reflecting telescope of the Optical Ground Station of the European Space Agency in Tenerife. Beam drifts are managed through a closed-loop tracking system which automatically control the alignment of the transmitter telescope; beam attenuation due to diffraction, absorption and turbulence reduces the efficiency of the link, but doesn't affect polarization. The group obtained a violation of the CHSH Bell-type inequality by more than 13 standard deviations, proving entaglement between the separated

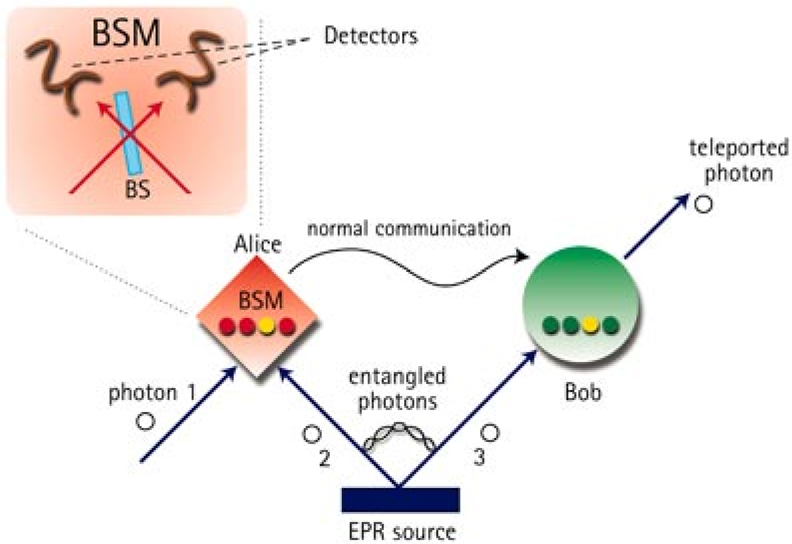

Figure 2: Sketch of the quantum teleportation procedure (from [5]).

photons. This is an important step towards a future global scale satellite-based network for quantum communication.

2 Quantum teleportation

Quantum teleportation refers to transferring a qubit from an emitter, Alice, to a receiver in another location, Bob, without actually transferring the particle carrying the state. This can be achieved with the condition that neither of the two observers obtain information about the properties of the state during the process, and the initial state in Alice's possession is destroyed as a result of the procedure. Teleportation only transfers information, not energy or matter, and this information is never communicated faster than the speed of light.

Charles Bennett et al. first proposed this idea in 1993. [1] The following section describes their suggested implementation.

2.1 Method

Initially, Alice is in possession of a particle in an unknown quantum state $|\Psi\rangle_1$. The purpose of teleportation is to have Bob obtain a particle in the same state. Physically sending the particle itself is impractical, as this could take too long and, more importantly, would likely destroy the fragile quantum state. Also, attempting to perform measurements on the system in order obtain enough information to be able to reconstruct it is useless: any measurement on a system found in a superposition of states will make it collapse into one of these states, losing the initial state. The solution is to have Alice and Bob share an EPR pair of particles, then have Alice perform a joint Bell-state measurement on the initial particle and her EPR particle, which causes them to become entangled. This projects Bob's particle onto a state related to the initial state Alice wanted to transmit, and he can use the result of Alice's measurement to recover her state through a suitably chosen unitary transformation.

An important property of this method is that the information transfer is accomplished through two separate channels, one entirely classical and one entirely non-classical.

To clarify how this works, suppose Alice starts out with a photon in an unknown polarization state $|\Psi\rangle_1$, which she wishes to teleport:

$$|\Psi\rangle_1 = \alpha\,|{\leftrightarrow}\rangle_1 + \beta\,|{\updownarrow}\rangle_1\,. \tag{4}$$

A pair of photons (labeled 2 and 3) are prepared in an EPR singlet state through the SPDC procedure previously described:

$$\left|\Psi^-\right\rangle_{23} = \frac{1}{\sqrt{2}}\left(|{\leftrightarrow}\rangle_2\,|{\updownarrow}\rangle_3 - |{\updownarrow}\rangle_2\,|{\leftrightarrow}\rangle_3\right). \tag{5}$$

Alice takes particle 2 and Bob takes particle 3. The 3 particles can then be described by the state

$$\begin{aligned}|\Psi\rangle_{123} = |\Psi\rangle_1 \otimes \left|\Psi^-\right\rangle_{23} &= \left(\alpha\,|{\leftrightarrow}\rangle_1 + \beta\,|{\updownarrow}\rangle_1\right) \otimes \frac{1}{\sqrt{2}}\left(|{\leftrightarrow}\rangle_2\,|{\updownarrow}\rangle_3 - |{\updownarrow}\rangle_2\,|{\leftrightarrow}\rangle_3\right)\\ &= \frac{\alpha}{\sqrt{2}}(|{\leftrightarrow}\rangle_1\,|{\leftrightarrow}\rangle_2\,|{\updownarrow}\rangle_3 - |{\leftrightarrow}\rangle_1\,|{\updownarrow}\rangle_2\,|{\leftrightarrow}\rangle_3)\\ &+ \frac{\beta}{\sqrt{2}}(|{\updownarrow}\rangle_1\,|{\leftrightarrow}\rangle_2\,|{\updownarrow}\rangle_3 - |{\updownarrow}\rangle_1\,|{\updownarrow}\rangle_2\,|{\leftrightarrow}\rangle_3).\end{aligned} \tag{6}$$

This can be re-expressed in the Bell state basis of photons 1 and 2 ($|\Phi^{\pm}\rangle_{12}$,

$|\Psi^{\pm}\rangle_{12}$) using

$$
\begin{aligned}
|{\leftrightarrow}\rangle_1 |{\leftrightarrow}\rangle_2 &= \frac{1}{\sqrt{2}}(\left|\Phi^+\right\rangle_{12} + \left|\Phi^-\right\rangle_{12}) \\
|{\leftrightarrow}\rangle_1 |{\updownarrow}\rangle_2 &= \frac{1}{\sqrt{2}}(\left|\Psi^+\right\rangle_{12} + \left|\Psi^-\right\rangle_{12}) \\
|{\updownarrow}\rangle_1 |{\leftrightarrow}\rangle_2 &= \frac{1}{\sqrt{2}}(\left|\Psi^+\right\rangle_{12} - \left|\Psi^-\right\rangle_{12}) \\
|{\updownarrow}\rangle_1 |{\updownarrow}\rangle_2 &= \frac{1}{\sqrt{2}}(\left|\Phi^+\right\rangle_{12} - \left|\Phi^-\right\rangle_{12}),
\end{aligned}
\tag{7}
$$

which gives

$$
\begin{aligned}
|\Psi\rangle_{123} = \frac{1}{2}\,\big[&\left|\Psi^-\right\rangle_{12}(-\alpha\,|{\leftrightarrow}\rangle_3 - \beta\,|{\updownarrow}\rangle_3) + \left|\Psi^+\right\rangle_{12}(-\alpha\,|{\leftrightarrow}\rangle_3 + \beta\,|{\updownarrow}\rangle_3) \\
&+ \left|\Phi^-\right\rangle_{12}(\alpha\,|{\updownarrow}\rangle_3 + \beta\,|{\leftrightarrow}\rangle_3) + \left|\Phi^+\right\rangle_{12}(\alpha\,|{\updownarrow}\rangle_3 - \beta\,|{\leftrightarrow}\rangle_3)\big]
\end{aligned}
\tag{8}
$$

Next, Alice performs a Bell state measurement on her particles, 1 and 2, thus projecting them into one of the 4 Bell states. Each of these states ($|\Phi^{\pm}\rangle_{12}$, $|\Psi^{\pm}\rangle_{12}$) has the same outcome probability (25%). The essential point is that, during this measurement, Bob's photon is also projected in the particular state that is the coefficient of the measured Bell state among the superposition in equation (8). So, for instance, if Alice obtains $|\Psi^-\rangle_{12}$, Bob's particle is then known to be in the state $|\Phi\rangle_3 = -\alpha\,|{\leftrightarrow}\rangle_3 - \beta\,|{\updownarrow}\rangle_3$ and so on. Each of the four possible outcomes for particle 3 is related to Alice's initial $|\Psi\rangle_1 = \alpha\,|{\leftrightarrow}\rangle_1 + \beta\,|{\updownarrow}\rangle_1$ state at most through a unitary transformation:

$$
\begin{aligned}
-\alpha\,|{\leftrightarrow}\rangle_3 - \beta\,|{\updownarrow}\rangle_3 &\to -\,|\Psi\rangle_1 \\
-\alpha\,|{\leftrightarrow}\rangle_3 + \beta\,|{\updownarrow}\rangle_3 &\to \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} |\Psi\rangle_1 \\
\alpha\,|{\updownarrow}\rangle_3 + \beta\,|{\leftrightarrow}\rangle_3 &\to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} |\Psi\rangle_1 \\
\alpha\,|{\updownarrow}\rangle_3 - \beta\,|{\leftrightarrow}\rangle_3 &\to \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} |\Psi\rangle_1
\end{aligned}
\tag{9}
$$

It can be noted that the matrices in the last 3 cases are related to the Pauli matrices: they are $-\sigma_3$, σ_1 and $-i\sigma_2$ respectively. Now, in order for Bob to reconstruct $|\Psi\rangle_1$, it is necessary for him to receive the outcome of Alice's measurement through a classical communication channel. Each teleportation requires the transfer of 4 classical bits of information, encoding which of the 4 Bell states was obtained. If Alice broadcasts that she obtained

$|\Psi^-\rangle_{12}$ in her measurement, then Bob's photon already is in state $|\Psi\rangle_1$ up to a phase factor. For the other 3 cases, he needs to apply the corresponding unitary operator to get Alice's initial state. This can be achieved by passing the photon through the right combination of half-wave plates. Either way, the net result is that Bob ends up with a photon in Alice's initial polarization state 1, while Alice loses all information about $|\Psi\rangle_1$ and is left with an EPR pair of photons.

Teleportation is in accord with the no-cloning theorem, which states that one cannot produce copies of a quantum state, because the initial qubit is destroyed. The procedure is secure: an eavesdropper intercepting the classical message will only know the Bell state of photons 1 and 2 and thus the unitary transformation needed for recovery of the desired state, but the information is useless if he doesn't have access to Bob's particle.

2.2 Experimental realizations

The first experimental confirmation of teleportation was achieved in 1997 by Bouwmeester et al. in Innsbruck [3]. They only used the case in which photons 1 and 2 are projected in the $|\Psi^-\rangle_{12}$ Bell state, which occurs 25% of the time. To check whether teleportation works, a basis consisting of the states linearly polarized at 45° and −45° was chosen, so that a superposition of $|\leftrightarrow\rangle$ and $|\updownarrow\rangle$ was tested. Photon 1 was prepared with either 45° or −45° polarization. The state of photon 3 was studied by placing detectors D1 and D2 at the two outputs of a polarizing beam splitter that separates 45°/−45° polarization. Detection is made by registering a three-fold coincidence between the two detectors used in performing the Bell-state measurement and only one of D1 and D2. For instance, to teleport a photon at 45°, one needs to register a coincidence between the Bell detectors and D1 *and* a lack of coincidence between the Bell detectors and D2. They obtained teleportation visibilities of 63%-64%. The authors extended their tests by also trying out photons linearly polarized at 0° (66% visibility) and 90° (61%) and circularly polarized (57%). In 1998, Boschi et al. [2] in Rome managed to implement teleportation of linearly and elliptically polarized photons identifying all four Bell states.

Recently, in May 2010, teleportation was achieved over a distance of 16 km in China [7]. A free-space channel was used to overcome photon losses and decoherence effects in optical fibre, which limit transmission; atmospheric absorption was minimized by selecting an appropriate wavelength range. The group managed to obtain an 89% teleportation fidelity by optimizing their telescopes and developing an active feed-forward technique for real-time information transfer.

2.3 Extensions and outlook

Teleportation doesn't only work with pure states, but can also be performed on mixed or even entangled states. Alice's photon may itself be part of an entangled pair with another remote particle 0, in which case performing the teleportation leaves photons 0 and 3 in a Bell state (this is called entanglement swapping). Also, the discussed formalism can be extended for particles characterized by a superposition of N>2 orthogonal states.

Furthermore, teleportation was demonstrated not only between pairs of photons, but also between atom pairs and even photon-atom pairs. In 2004, Riebe et al. [9] managed to perform quantum state teleportation between trapped $^{40}Ca^{+}$ ions (using asource ion and an entangled ion pair) with a fidelity of 75%. In 2008, Chen et al. [4] teleported the polarization state of a photon over 7 meters onto an atomic qubit with 73% - 86% fidelity, creating a readable quantum memory that can store the state for up to 8 μs, after which decoherence occurs. However, the low probability of success of the procedure (need many experimental runs to get a successful one) and the short memory span signal the need for improvement in order to get efficient long-distance quantum communication.

Open destination teleportation, in which an unknown qubit is teleported onto a superposition of N particles and can subsequently be read out at any of these particles, is another emerging protocol that has so far been proven to work for N=3 [14]. To achieve this, the authors manage to produce a five-photon entangled state, the minimum number necessary for universal quantum error correction (protecting quantum information from decoherence and noise errors). Moreover, a step towards composite system teleportation was taken with the teleportation of a two-qubit system consisting of the arbitrary polarization state of two photons [13] using a six-photon interferometer.

To conclude, quantum communication based on entanglement is a young and promising field with ample room for theoretical and experimental progress. Advances in long distance teleportation combined with perfecting quantum memories are key ingredients for developing a global quantum communication network.

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