Atomic Parity Violation

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APV proposes new physics beyond the standard model of elementary particles. APV is usually measured through the weak nuclear charge Q_w, quantifying the strength of the electroweak coupling between atomic electrons and quarks of the nucleus. This paper will be dealing with brief theoretical backgrounds of parity nonconserving (PNC), and coupled cluster approximation method to calculate removal energies of each state. Through the weak charge of ^{133}Cs nucleus, this paper will address one PNC experiment and compare with the standard model.

I. Introduction

Parity conservation cannot be addressed intuitively, but it deals with the most elementary symmetry properties, mirror symmetry. It is also true that the denomination of parity does not help to understand the property of the particle itself. However, it becomes prominent when one discovers parity violation or parity non-conservation (PNC) such as mirror symmetry breaking. Many objects are considered to be fall into two categories, each one being identical to the mirror image or different from it. And this can be determined through applying left or right hands as a way of representation of the state.

In PNC, interaction between the particles, which favor one orientation of the physical space with respect to the other, plays bigger role than the spatial arrangement of particles in deciding the symmetry property. For instance, if atom absorbs only the left-circularly polarized photons, then in a mirror, the same atom will absorb only the right-circularly polarized photons, which is different. This different due to PNC is usually tiny so that it was not until 1970s that physicists realized that PNC in atoms of a second interaction actually exists. Even though they were tiny, the existence of parity-violation effects was in complete contradiction with the laws in electrodynamics. This paper will go over the basic principles and history of atomic parity violation study and introduce the present approximation compared to the standard model theory.

II. PNC through the interaction

Recall parity operator, *P*. Eigen values of *P*, parity quantum number $\varepsilon_p = \pm 1$ characterizes quantum states, which coincide with their mirror images. For a non-relativistic particle in a central potential *V(r)*, the state vector $|\psi\rangle$ is expressed by a wave function $\psi(r)$, and for the mirror state, $|\tilde{\psi}\rangle = P|\psi\rangle$ is given by $\tilde{\psi}(r) = \psi(-r)$. For the case of a system of *N* independent electrons, Hartree-Fock atomic model is introduced where. $|\psi\rangle = \prod_{i=1}^{i=N} \psi_{n,l,m}(r)$. The parity quantum number of a system of *N* independent electrons will be the product of the *N* individual quantum numbers $\varepsilon_{p_i} = (-1)^{l_i}$.

Now, consider a system with a Hamiltonian, which has two parts, one even and the other odd under space reflection.

$$H = H^{even} + H^{odd} \tag{1}$$

Under the operator *P*, *H* transforms as: $P^{-1}HP = H^{even} - H^{odd} \neq H$ which results that $[P,H] \neq 0$. As a first result we get the non-commutation of *H* with the parity operator so that non-conservation of the parity quantum number between an initial and a final state: $\prod \varepsilon_i \neq \prod \varepsilon_f$. This non-conservation also can be applied to the time evolution operator $U(t) = \exp(-iHt/\hbar)$ such that $[P,U(t)] \neq 0$. Comparing a transition rate between two states $P_{A\to B} = \left|\langle B|U(t)|A\rangle\right|^2$ to the transition rate between their mirror states,

$$P_{\tilde{A}\to\tilde{B}} = \left|\left\langle \tilde{B}\middle|U(t)\middle|\tilde{A}\right\rangle\right|^2 = \left|\langle B|P^{-1}U(t)P|A\rangle\right|^2 \neq \left|\langle B|U(t)|A\rangle\right|^2 \tag{2}$$

This easily shows that PNC exists between the rates of transition between two states and their mirror states. Finally, when we connect to energies of states,

$$E_A = \langle A|H|A\rangle \tag{3}$$

$$E_{\tilde{A}} = \langle A|P^{\dagger}HP|A\rangle \neq \langle A|H|A\rangle \tag{4}$$

So the basic principle of most parity non-conservation experiments is to compare the rate of a given transition between two states, *A* and *B*, with the transition rate between their mirror states, $\tilde{A}$ and $\tilde{B}$. The outcome from the experiment is usually characterized by the A_{LR}, difference between these rates divided by their sum.

Another important symmetry is time reversal symmetry. It is important to know

that if a PNC interaction is invariant or not under *T*-reversal because the physical effect can manifest PNC present considerable differences. Because of the standard model, the weak interactions associated with Z_0 and $W^{\pm}$ exchanges are *T* conserving. As a result, the pseudoscalar quantities which can manifest their presence are necessarily even under the *T* operator. However, this rule is valid only if the transition rates can be calculated in the Born-approximation. If the experiment involves polarized atoms in the presence of an electric field *E*, the observation of a physical signal of pseudoscalar $E\cdot < J >$ would mean that both *P* and *T* are violated. It can be shown that *T*-reversal invariance hinders the existence of a static electric dipole moment for an atomic state if this state is stationary and non-degenerate. *T*-reversal invariance also does not forbid the existence of an electric dipole moment of transition between states of the same parity, for example the $nS_{1/2}$ states of caesium

$$E_1^{pv}(n'n) = \left\langle n'\tilde{S}_{1/2}m_s \middle| d_z \middle| n\tilde{S}_{1/2}m_s \right\rangle \tag{5}$$

where the tilde means that the states are perturbed by the parity-violating part of the Hamiltonian H_{pv} and m_s is the eigenvalue of J_z. The parity selection rule shows a magnetic dipole of transition between the same two states

$$M_1(n'n) = \left\langle n'\tilde{S}_{1/2}m_s \middle| \mu_z / c \middle| n\tilde{S}_{1/2}m_s \right\rangle \tag{6}$$

where μ_z is the magnetic moment operator. Most of the atomic PNC effects reported are manifestations of transition dipoles, which violate the parity selection rule.

III. Electroweak interaction and the weak charge

It is important to compare the electron-nucleus potentials, which describe the Coulomb interaction and the PNC interaction during Z_0 boson exchange. Coulomb potential can be written as

$$V_{Coulomb}(r_e) = \frac{Z\,e^2}{r_e} \tag{7}$$

where *Z* is the atomic number and PNC potential in non-relativistic limit

$$V_{pv}(r_e) = \frac{1}{2} Q_W g_{Z_0}^2 \frac{\exp(-M_{Z_0} c r_e / \hbar)}{r_e} \sigma_e \cdot \upsilon_e / c + HC \tag{8}$$

where g_{Z_0} is a coupling constant with the size of the order of e. Above two equations show close analogy existing between Ze^2 and $Q_w g_{Z_0}^2$, possible electroweak interactions between the electron and the nucleus. Replacing Yukawa potential by a Dirac distribution, the potential can be described as

$$V_{pv}(r_e) = \frac{Q_W G_F}{4\sqrt{2}} (\delta^3(r_e) \sigma_e \cdot \upsilon_e / c + HC) \tag{9}$$

The term, the weak charge of the nucleus Q_w is introduced through the analogy with the nuclear electric charge. Unlike the Coulomb potential, the chiral (different in mirror image) electron-nucleus interaction has a range shorter than the atomic size. As a result, its strength is given by the electron density near the nucleus times some constants. The weak charge Q_w is a fundamental constant of the atomic electroweak chiral interaction. Similar to the nuclear electric charge *Z*, the weak charge Q_w is the sum of the weak charges of all the constituents of the atomic nucleus. From the standard model, weak nuclear charge can be expressed as a simple formula

$$Q_W = -N + Z(1 - 4\sin^2\theta_W) \approx -N \tag{10}$$

where *N* is the number of neutrons, *Z* is the nuclear charge, and θ_W is the Weinberg angle ($\sin\theta_W \sim 0.23$).

In 1974 paper (Bouchiat and Bouchiat 1974a,b), they predicted that the electroweak effects in atoms should grow a little faster than the cube of the atomic number *Z*. The weak charge $Q_w \sim -N$ provides for the first *Z* factor because *N/Z* is ≥ 1 for most complex stable nuclei. The second factor *Z* comes from the vicinity of the nucleus the electronic density of penetrating valence orbitals grows like *Z*. Last *Z* factor can be traced back to the helicity factor $\sigma_e \cdot \upsilon_e / c$ in $V_{pv}(r_e)$.

IV. Excitation of highly forbidden transitions

The caesium appears the best compromise between a high value of *Z* and the simplicity of the atomic structure. QED strictly hinders the existence of electric dipole

amplitude between two atomic states having the same parity. For caesium, $6S_{1/2} \rightarrow 7S_{1/2}$ is highly forbidden. The Z^0 exchange interactions, however, breaks this rule and gives a small electric dipole amplitude $E_1^{pv}(\sim i \times 10^{-11} ea_0)$. Magnetic dipole amplitude is allowed by symmetry, but because the two states involved have different radial quantum numbers, M_1 $(\sim 0.4 \times 10^{-4} \mu_B / c)$ is reduced. The transition is still suppressed that at the atomic densities required to see a signal. To resolve this problem, 1975 paper (Bouchiat and Bouchiat 1975) proposed a static electric field $\boldsymbol{E}$ to create E_1 Stark effect to see the signal.

V. PNC experiment

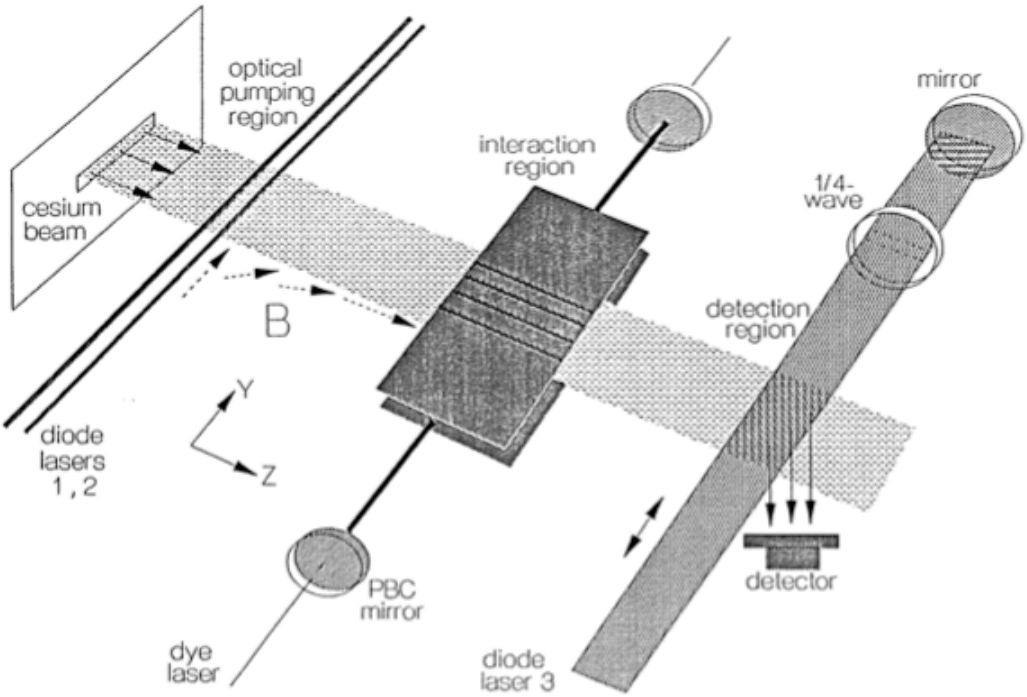

Figure 1. Boulder experiment in caesium, using a polarized atomic beam (Wood *et al* 1997)

This example experiment was done in 1997, improved version of 1985. The experiment depends on the search for the pseudoscalar quantity $\xi\vec{k} \cdot (\vec{E} \times \vec{B})$ in the absorption cross section of the excitation beam with angular momentum $\xi\vec{k}$. The atomic

beam is polarized. The beam goes through three different regions. First, all the atoms are collected in a single Zeeman sub-state, which will interact with the excitation laser. This interaction is done by hyperfine and Zeeman pumping in a low magnetic field with two diode laser beams, one of them is circularly polarized. Second, the atoms go through the interaction region and arrive at the detection region. In detection region, atoms from initially depopulated hyperfine state are detected well through resonant scattering of a large number of IR photons. Signals from this represent the modification of the hyperfine populations in the ground state generated by going through the interaction region. Its origin is $6S \rightarrow 7S$ transition. However, there should be further discrimination since $6S \rightarrow 7S$ transition may not be the only case. Also, the instantaneous signal contains the whole transition probability and not only its pseudoscalar part. The pseudoscalar part can be measured through making periodic reversals of all the parameters which change its sign which are electric field, the magnetic field in the pumping region, the laser polarization, the magnetic field in the interaction region, and the circular polarization of the optical pumping light.

The other part of the apparatus is a high-finesse (100 000) Fabry-Perot power buildup cavity (PBC) through the center where the beam of polarized atoms passes. A high-speed servosystem locks the green laser frequency to the resonance frequency of the cavity. The cavity is kept in resonance with the $6S \rightarrow 7S$ transition by another servosystem. The PBC enhances the transition rate and greatly suppresses the Stark-M_1 interference effect. The rear mirror birefringence is reversed by rotating it periodically so as to exchange the fast and slow axes. Laser intensity inside the cavity is 1 MW cm^{-2} at the antinodes, which means that the 6S-7S resonance frequency can be shifted by as much as 20 MHz at the peak intensity by Stark effect. This shift is spatially regulated so that even thought the atomic beam is highly collimated the atoms have a non-zero velocity along the wave vector $\boldsymbol{k}$. Therefore, different atoms going through the light beam with different paths will experience different Stark shifts; those which go through the largest shift corresponds the largest transition probability. The accuracy of this experiment was about 3.5×10^{-3}, which was close to all previous results (Table 1).

Table 1. Experiment summary in caesium

A—experiments*	B—atomic theory computations
Im $E_1^{pv}(10^{-11}\|e\|a_0)$	$(-N/Q_W)$Im $E_1^{pv}(10^{-11}\|e\|a_0)$
	First principles
$-0.803 \pm 0.095^{(a,b)}$ Paris 82–83	$-0.908 \pm 0.010^{(f)}$ Novosibirsk 89
$-0.867 \pm 0.069^{(c)}$ Boulder 85	$-0.905 \pm 0.009^{(g)}$ Notre Dame 90
$-0.828 \pm 0.020^{(d)}$ Boulder 88	
$-0.837 \pm 0.003_{exp} \pm 0.006^{(e)}_{theory}$ Boulder 97	
	Semi-empirical
*βa_0^3 from S.E. evaluations	$-0.935 \pm 0.02 \pm 0.03^{(j)}$ Paris 86
$27.19 \pm 0.4^{(h)}$	$-0.904 \pm 0.02^{(k)}$ Oxford 90
$27.17 \pm 0.35^{(i)}$	$-0.895 \pm 0.02^{(l)}$ Paris 91
βa_0^3 from first principles	
$27.00 \pm 0.20^{(m)}$	
$A/B \Longrightarrow Q_W^{exp} = -72.1 \pm 0.3_{exp} \pm 0.9_{theory}$	

(a) Bouchiat *et al* (1982); (b) Bouchiat *et al* (1984); (c) Gilbert *et al* (1985); (d) Noecker *et al* (1988); (e) Wood *et al* (1997); (f) Dzuba *et al* (1989b); (g) Blundell *et al* (1990); (h) Bouchiat and Piketty (1983); (i) Bouchiat and Guéna (1988); (j) Bouchiat and Piketty (1986); (k) Hartley and Sandars (1990); (l) Bouchiat (1991); (m) Blundell *et al* (1992).

We can write E_1^{pv} as the product of $(Q_w$ / -$N)$, $\varepsilon_1^{pv} = E_1^{pv}/(Q_W/-N)$. ε_1^{pv} can be considered to be an infinite sum over the intermediate P states mixed with the S states by the parity-violating interaction

$$\varepsilon_1^{pv} = \left(-\frac{N}{Q_W}\right)\sum_n \frac{\langle 7S_{1/2}|d_z|P_{1/2}\rangle\langle nP_{1/2}|V_{pv}|6S_{1/2}\rangle}{E(6S_{1/2}) - E(nP_{1/2})} + crossed \quad terms \tag{11}$$

The atomic orbitals and the valence-state energies are perturbed by many-body effects. There are two different approaches, one starting from first principles and the second semi-empirical. For the first case the Dirac equation is solved for a single electron in a Hartree-Fock potential and the many-body corrections are computed through perturbation theory. The second case is using numerous spectroscopic measurements of Cs, information regarding energies of the valence states, allowed-dipole matrix elements and hyperfine splittings of the $S_{1/2}$ and $P_{1/2}$ states.

From the table 1, it is clear that there still is a discrepancy between Q_W^{exp} and Q_W^{SM} (= -73.16(3)). This gap has later been diminished significantly through the Breit and QED corrections, based on Coupled-Cluster approximation, which referes to coupled-cluster single, double, and valence triple (CCSDvT). Through this correction, the recent paper

calculated $Q_W^{\exp}(Cs) = -73.16 \pm (0.29)_{\exp} \pm (0.20)_{th}$.

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