Quantum Information and Paradoxes of Physics

Jay Lawrence

Colloquium, Univ. of Chicago, Jan. 2011

Einstein-Podolsky-Rosen (EPR) Paradox (1935)

- Realism and the incompleteness of quantum mechanics -

Schroedinger's Cat (1935)

- Entanglement and macro-realism -

Maxwell's Demon (1871)

- Thermodynamics of information processing -

main example:

EPR - From philosophy to practical application

Benjamin Schumacher and Michael Westmorland,

in “Quantum Processes, Measurement, and Information:”

> This book is written from the conviction that a modern student of physics needs a broader set of concepts than conventional quantum mechanics textbooks now provide. Unitary time evolution, quantum entanglement, density operator methods, open systems, thermodynamics, concepts of communication, and information processing - all of these are at least as essential to the meaning of quantum theory as is solving the time-independent Schroedinger equation.

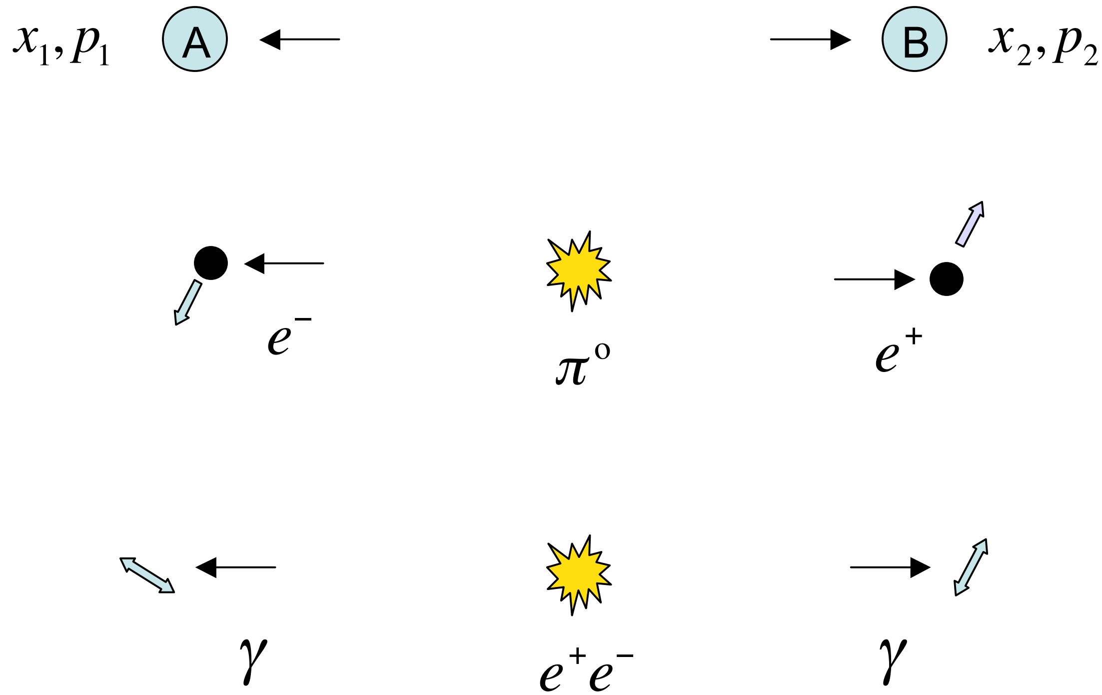
EPR - from philosophy to practical application

EPR (1935)

Aharonov-Bohm (1957)

Bell (1964)

CHSH* (1969)



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Bell States

singlet triplet

spins 1/2: $|\psi\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

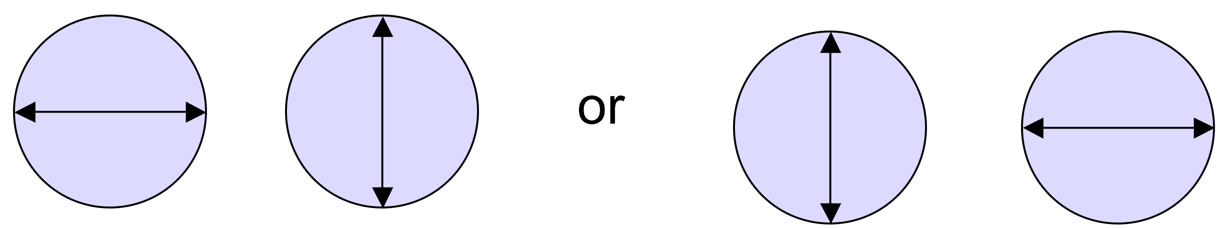
$$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$
$$|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$$
$$|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle$$

photons: $|\psi\rangle = |H,V\rangle - |V,H\rangle$

$$= |D,\overline{D}\rangle - |\overline{D},D\rangle$$

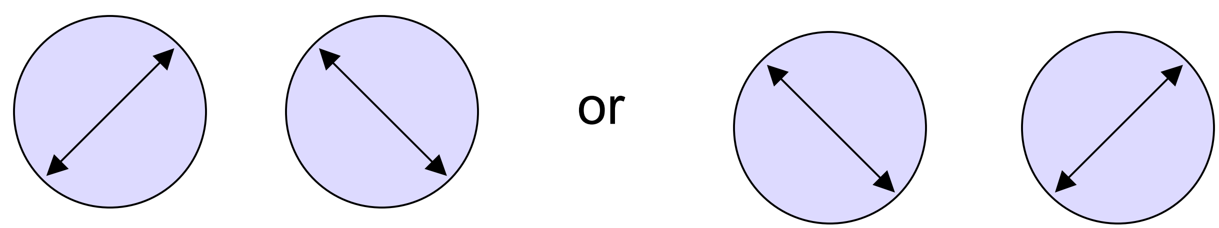
What Alice and Bob see with common detector settings:

(H,V):



A B

$(D,\overline{D})$:



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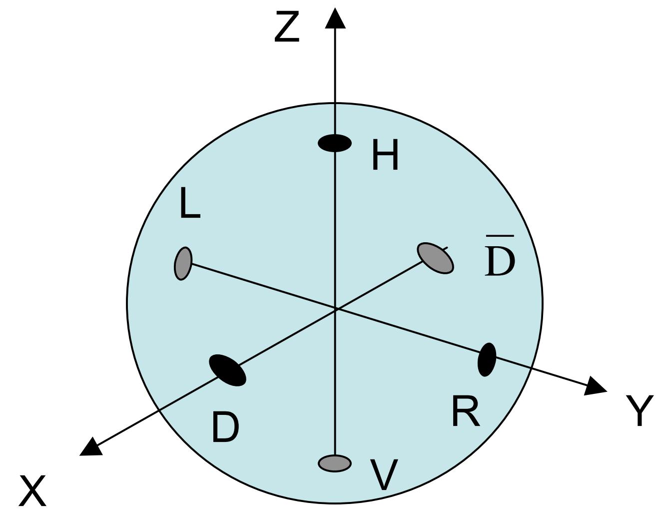

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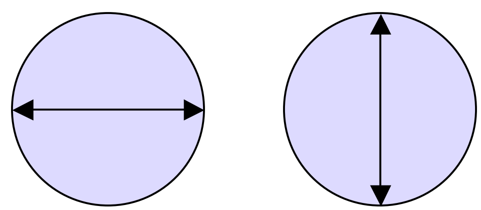
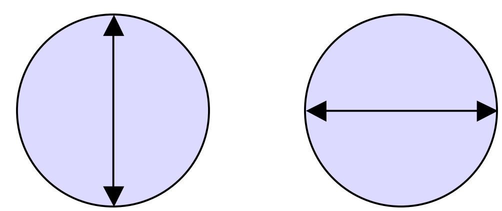
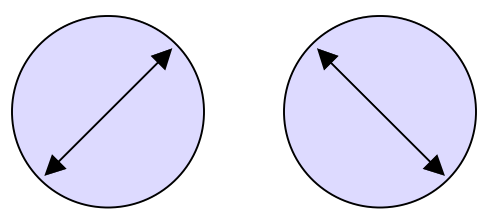


or



A B

$(D,\overline{D}):$



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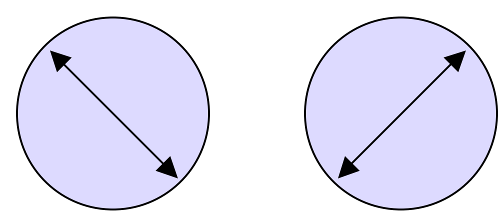
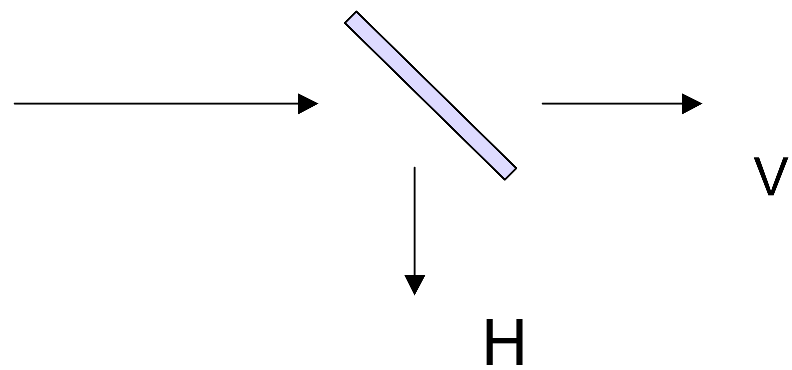
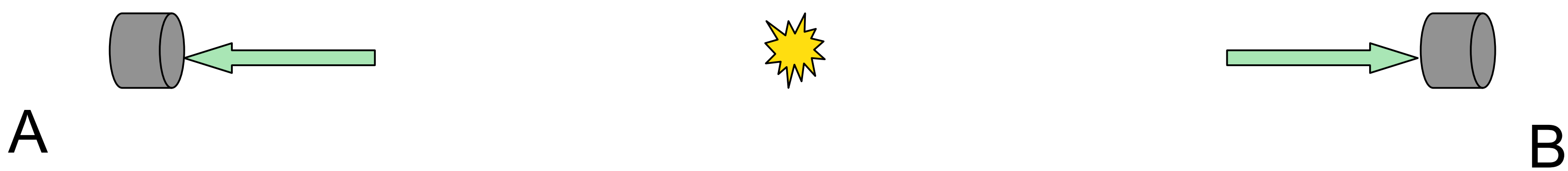
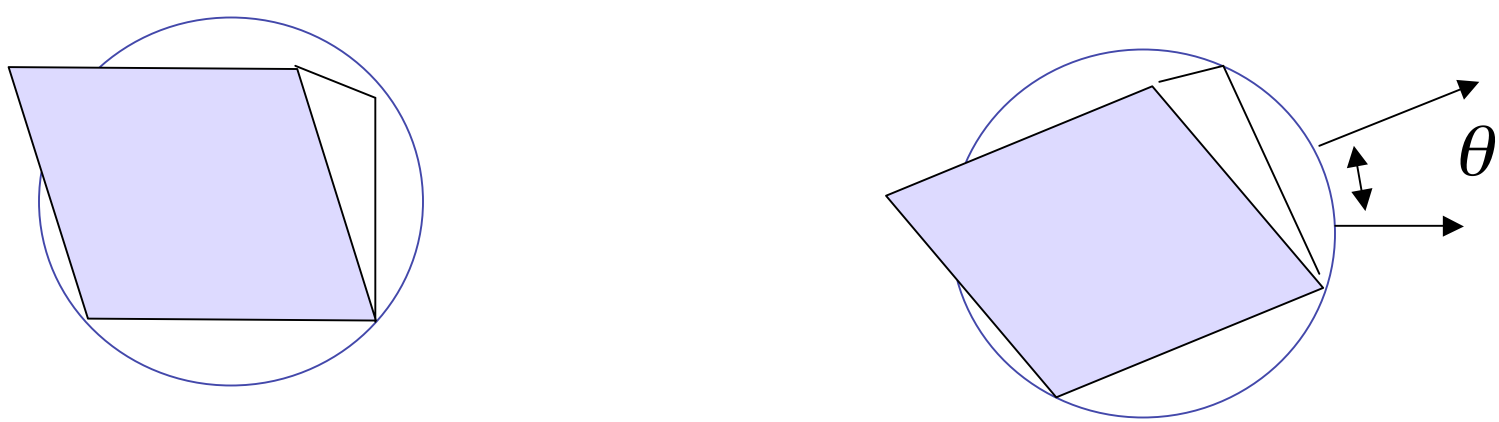


Choices of bases -

Alice and Bob have polarizing beam splitters

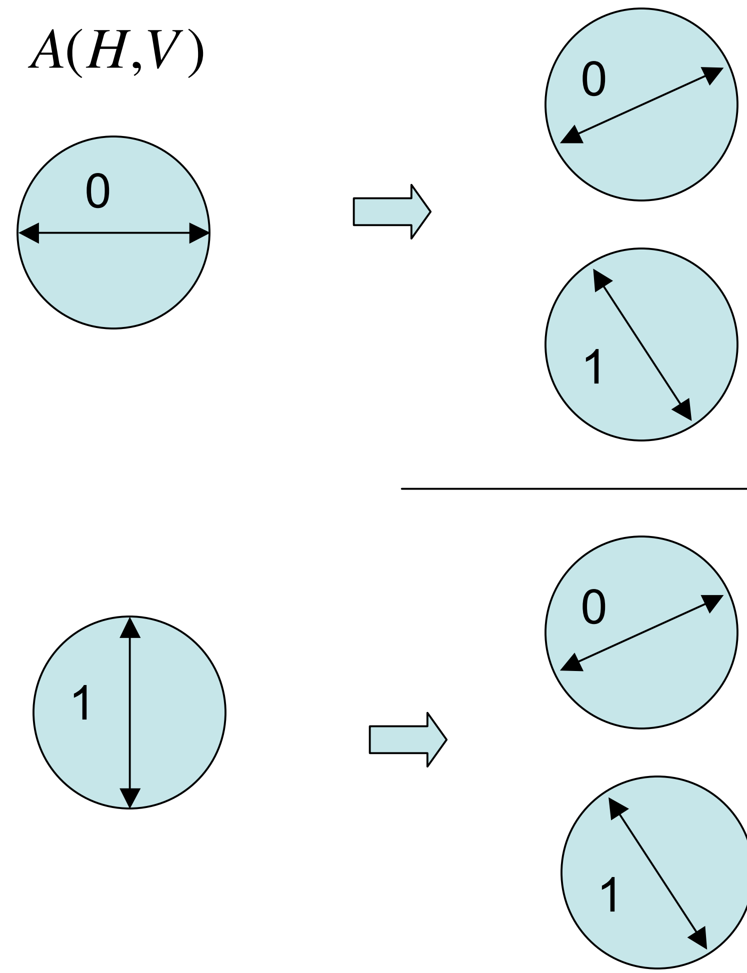






Probabilities with different polarization bases:

$A(H,V)$ $\qquad$ $B(\theta,\theta + 90^o)$



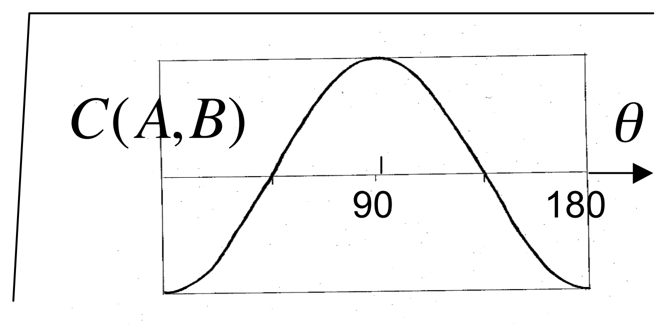
1/2 $\qquad$ $\sin^2\theta$ $\qquad$ $\cos^2\theta$

1/2 $\qquad$ $\cos^2\theta$ $\qquad$ $\sin^2\theta$

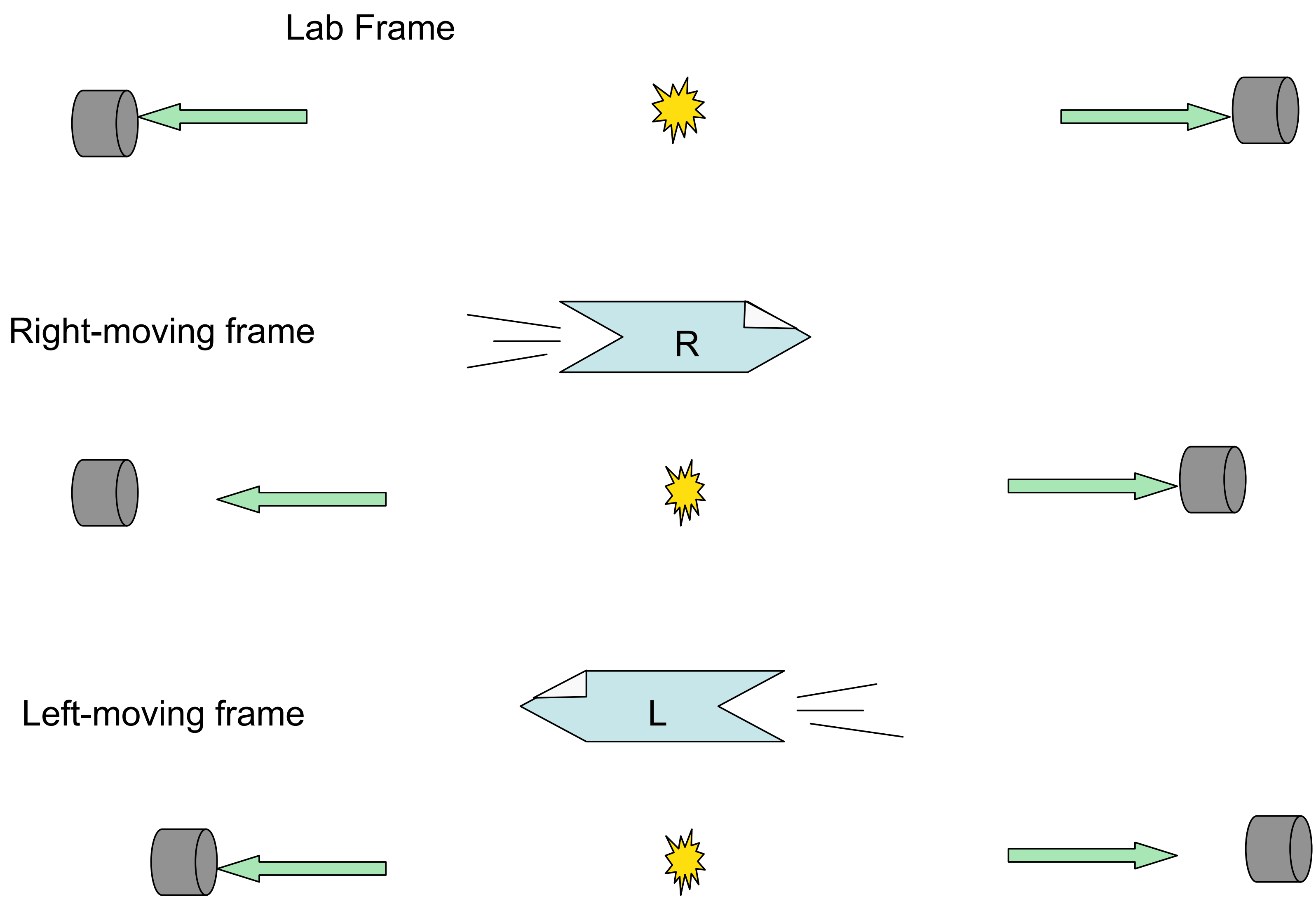
Correlation Function:

$$C(A,B) \equiv P(0,0) + P(1,1) - P(0,1) - P(1,0)$$

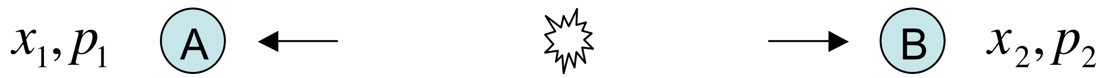
$$= \sin^2\theta - \cos^2\theta = -\cos 2\theta$$



No causal arrow in measurements



The EPR Paradox



Reality: If, without in any way disturbing a system, we can predict with certainty (ie, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Locality: On one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S2, is independent of what is done with the system S1, which is spatially separated from the former.

Since one can choose to measure either x or p of A, and thus predict with certainty the result of a measurement of x or p (respectively) of B, both x and p are elements of reality.

Since quantum mechanics doesn't admit real values for both x and p, it is incomplete. "We believe, however, that such a theory is possible."

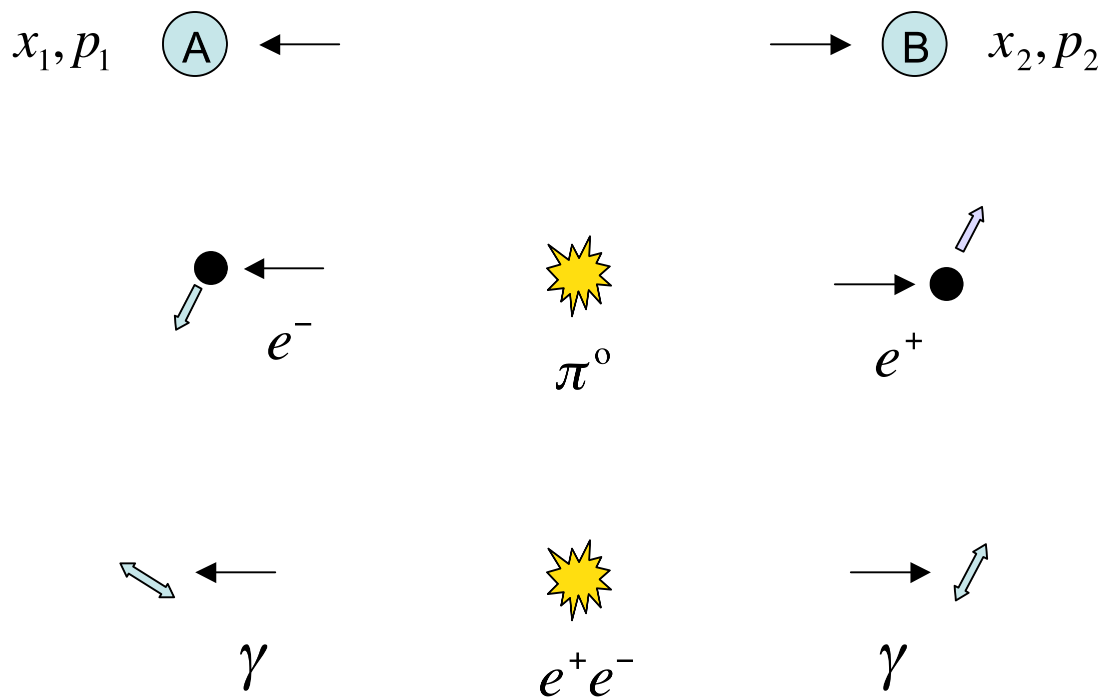
EPR - from philosophy to practical application

EPR (1935)

Aharonov-Bohm (1957)

Bell (1964)

CHSH* (1969)



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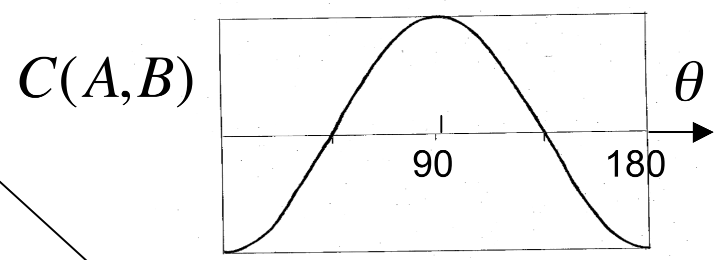
So the quantum mechanical prediction is:



Hidden variables argument of Bell - CHSH

Bell showed that a hidden variable model can reproduce perfect anticorrelations at $\theta = 0$ with random individual outputs. The correlation function is equal to

$$C(A,B) = \int d\lambda\ \rho(\lambda)\ a(A,\lambda)\ b(B,\lambda)$$

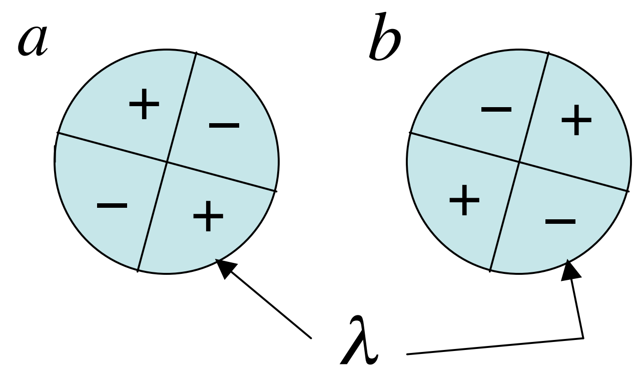
hidden variable — measurement outcomes,

$$a = \begin{cases} +1 \text{ for "0" state,} \\ -1 \text{ for "1" state.} \end{cases}$$

Here is a model: Let λ denote an angle (like A and B), set during the annihilation process, and let the ensemble be uniform, $\rho = 1/2\pi$.
Then we take

$$a(A,\lambda) = \mathrm{sgn}(45^o - |A - \lambda|),$$

$$b(B,\lambda) = \mathrm{sgn}(|B - \lambda| - 45^o).$$



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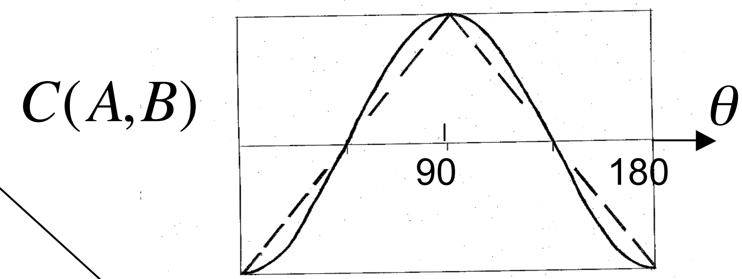

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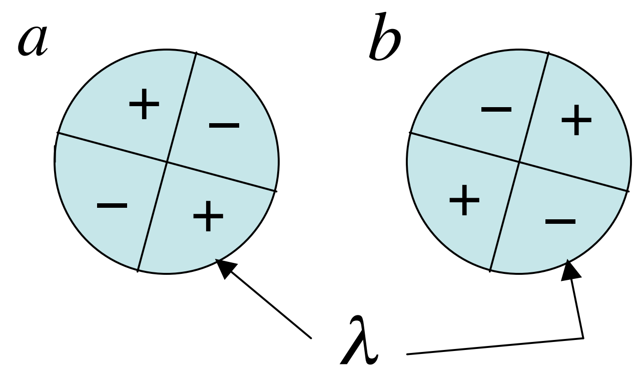
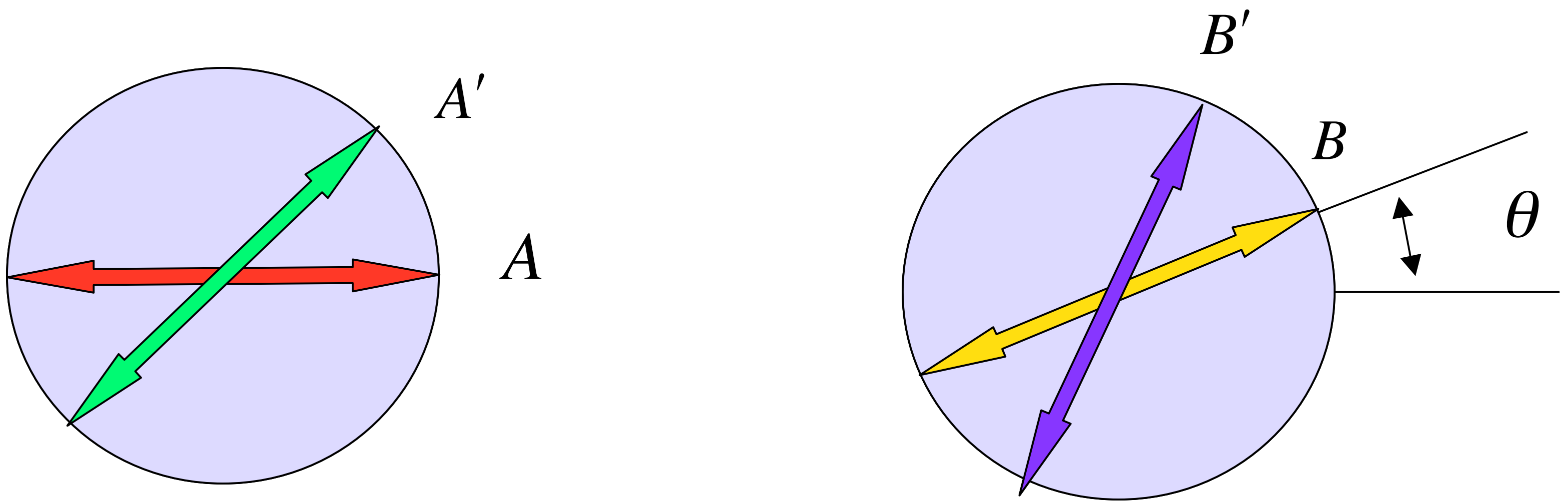


Hidden variables - more general

Show that there is NO local hidden variable model that reproduces the whole curve: Consider a combination of C(A,B) with maximum at 22.5.^{o}

$$S(A,B) = C(A,B) - C(A,B') + C(A',B) + C(A',B')$$

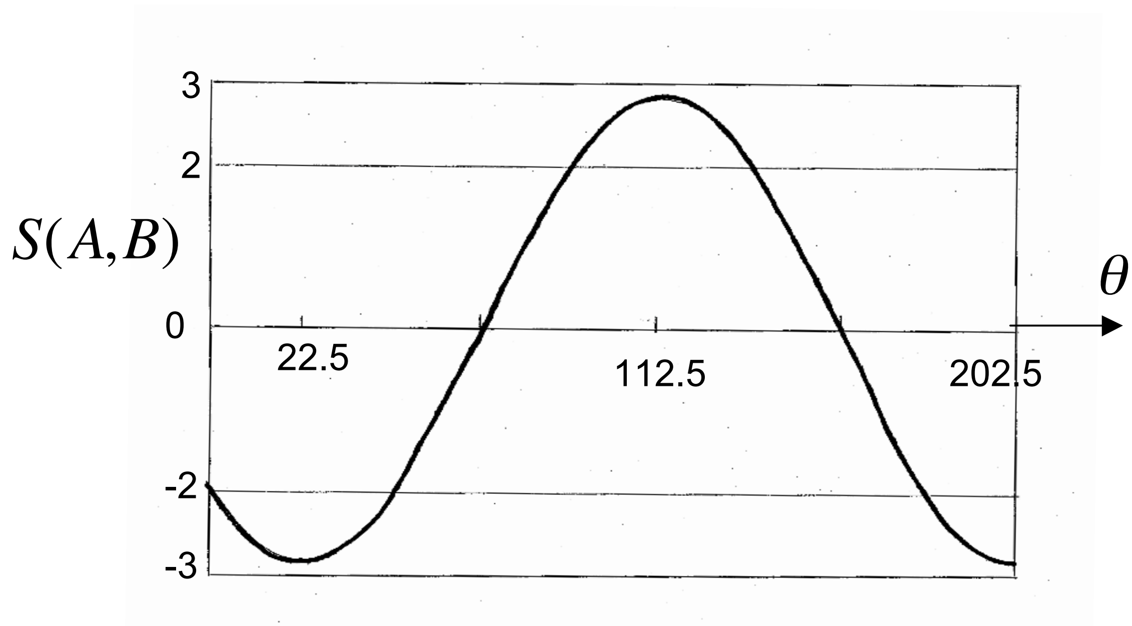


Hidden variable result is $S(A,B) = \int d\lambda\, \rho(\lambda) \begin{Bmatrix} a(A,\lambda)\ \left[b(B,\lambda) - b(B',\lambda)\right] \\ +a(A',\lambda)\ \left[b(B,\lambda) + b(B',\lambda)\right] \end{Bmatrix}.$

$$\Rightarrow \ \left|S(A,B)\right| \leq 2.$$

QM result is $S(A,B) = -2\cos 2\theta - 2\cos(\pi/2 - 2\theta) = -2\sqrt{2}\cos(2\theta - \pi/4).$

Quantum predictions and hidden variables bounds:



Experimental tests of quantum vs hidden variables predictions:

(1982): Aspect, Grangier, and Roger, Phys. Rev. Letters, vol. 49, 82;
Aspect, Dalibard, and Roger, Phys. Rev. Letters, vol. 49, 1804.

(1998): Weihs et. al., Phys. Rev. Letters, vol. 81, 5039.

Closes loophole by achieving spacelike separation of detectors with truly random switching between bases while photons are enroute.

(2007): Groblacher et. al., Nature, vol. 446, 871.

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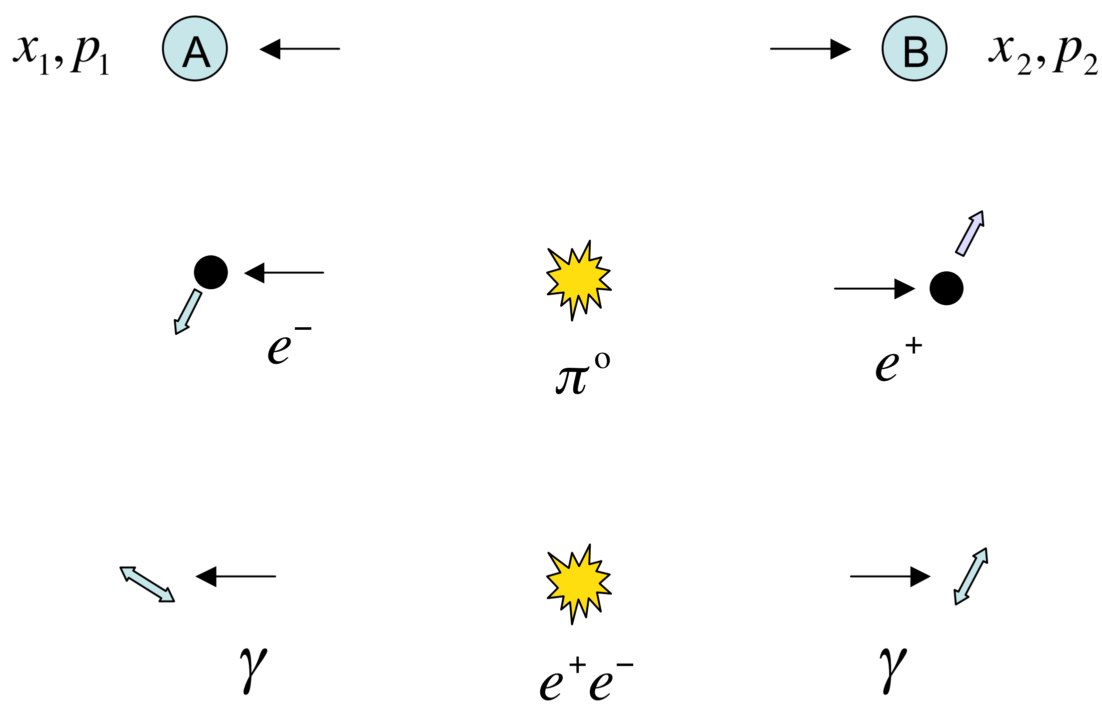

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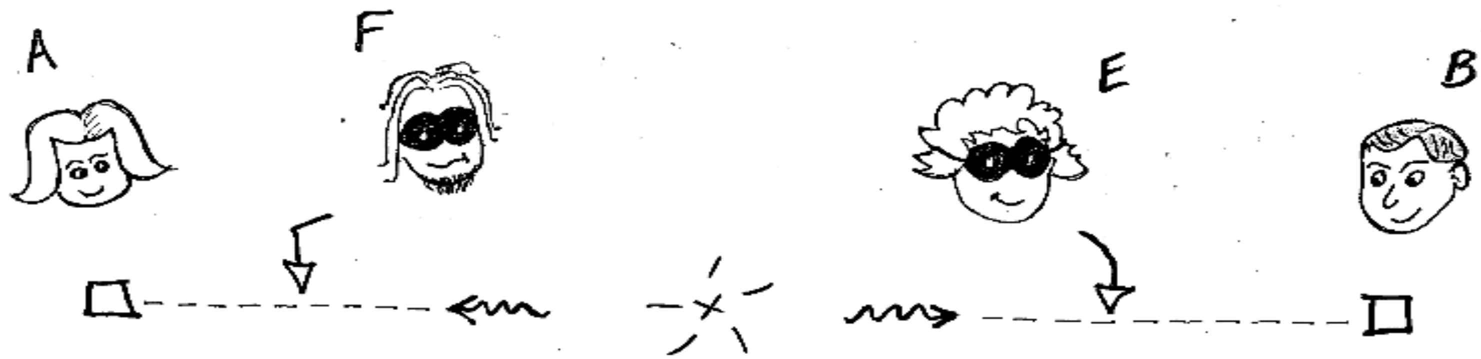
Alice and Bob with eavesdroppers



In the Ekert Protocol, A and B do measurements in 3 bases each .

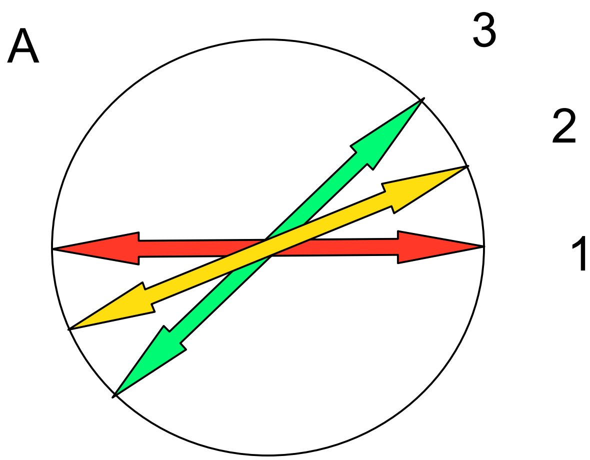
The Ekert Protocol (1991)

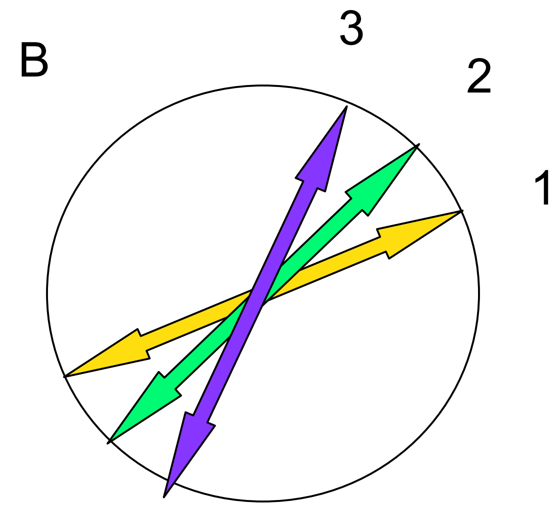
Alice's bases:

$$\theta = 0,\ \pi/8,\ \pi/4,$$

Bob's bases:

$$\theta = \pi/8,\ \pi/4,\ 3\pi/16.$$





(A_2, B_1) and (A_3, B_2) form secret key.

Eavesdroppers are detected by forming

$$S(A,B) = C(A_1,B_1) - C(A_1,B_3) + C(A_3,B_1) + C(A_3,B_3).$$

(E and F make elements of reality)

Ekert (1991):

“It is not a mathematical difficulty of a particular computation, but a fundamental physical law that protects the system, and as long as quantum theory is not refuted as a complete theory the system is secure.”

Jennewein et. al. (2000)*

Implement a variant of the Ekert protocol with users separated by 360 m, that generates raw keys at 400-800 bits/s at bit error rates of about 3%.

* Jennewein, Simon, Weinfurter, and Zeilinger, Phys. Rev. Letters, vol. 84, 4729 (2000).

Quantum Information and Paradoxes of Physics

Einstein-Podolsky-Rosen (EPR) Paradox (1935)

- Realism and the incompleteness of quantum mechanics -

Schroedinger's Cat (1935)

- Entanglement and macro-realism -

Maxwell's Demon (1871)

- Thermodynamics of information processing -

main example:

EPR - From philosophy to practical application

Schroedinger articles (1935 - 36):

1. (1935): Naturwissenschaften 23, pp. 807, 823, 844. (The present situation in quantum mechanics.)
2. (1935 -36): Proc. Cambridge Phil. Soc. 31, p. 555; ibid. 32, p. 446.

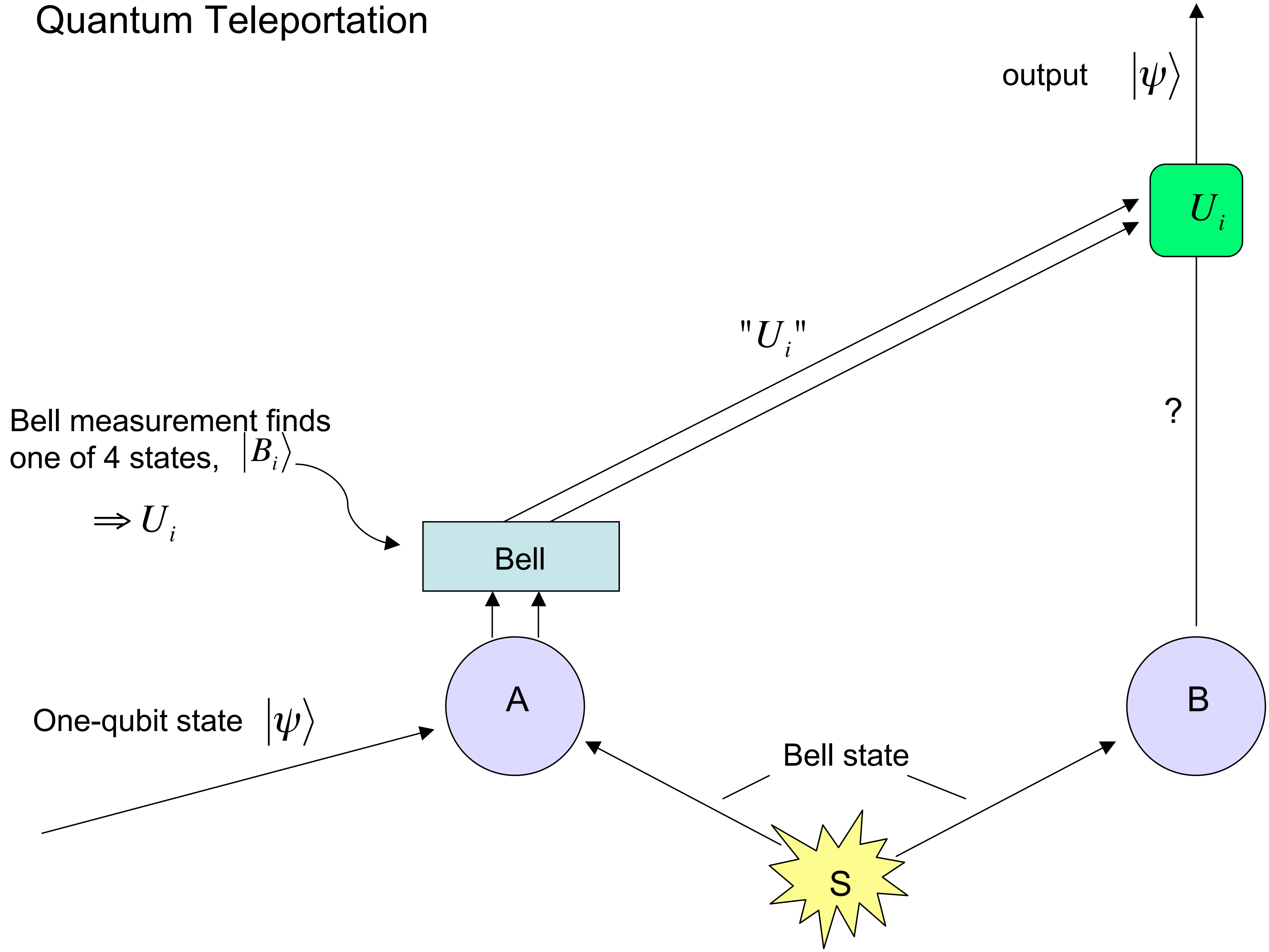
About entanglement and separation:

"…the best possible knolwledge of the whole does not necessarily include the best possible knowledge of all its parts, even though they may be entirely separate and therefore virtually capable of being 'best possibly known,'…

I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought."

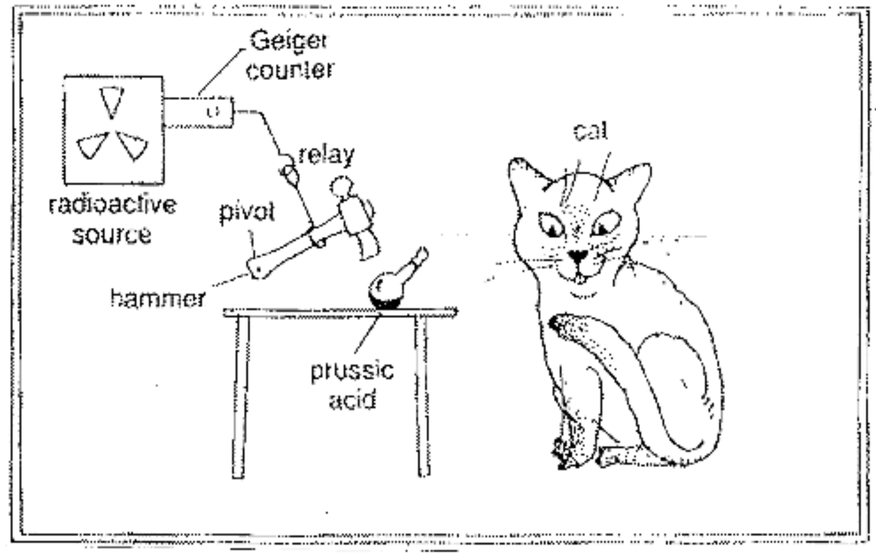
"Attention has recently been called to the obvious but very disconcerting fact that even though we restrict the disentangling measurements to one system, the representative obtained for the other system is by no means independent of the particular choice of observations which we select for that purpose and which by the way are entirely arbitrary. It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it."

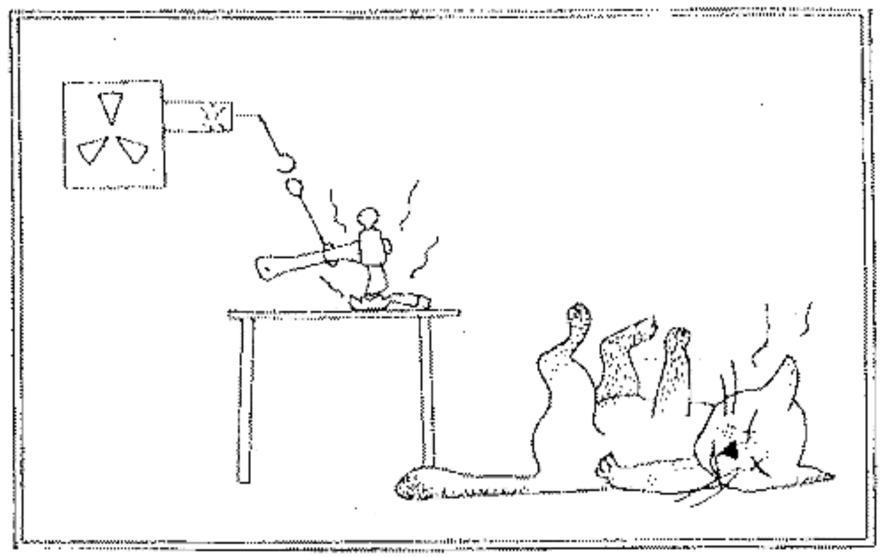
Quantum Teleportation



Birth of the Cat (1935)

“One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of ratioactive substance, so small, that perhaps in the course of an hour one atom decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left the entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.”





It is typical of these cases that an indeterminacy (in) the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation.

Einstein letter to Schroedinger (1950)

You are the only contemporary physicist, besides Laue, who sees that one cannot get around the assumption of reality, if only one is honest. Most of them simply do not see what sort of risky game they are playing with reality as sxomething independent of what is experimentally established. Their interpretation is, however, refuted most elegantly by your system of

radioactive atom + amplifier + charge of gunpowder + cat in a box,

in which the psi-function of the system contains both the cat alive and blown to bits. Nobody really doubts that the presence of absence of the cat is something independent of the act of observation.

Childhood of the Cat - related developments

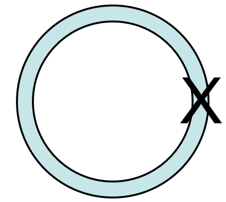
Wigner's friend (1961) in "The Scientist Speculates," ed. I.J. Good, p. 284.

H. Everett (1957), Rev. Mod. Phys., vol. 29, p.454.

Girardi, Rimini and Weber (1987), Phys. Rev. D36, p. 3287.

P. Pearle (1999), Phys. Rev. A59, p. 80.

Adulthood - making Schroedinger Cat states -



First observed superpositions in squid circuits:

Friedman et. al. (2000), Nature, vol. 406, p. 23.

C. Van der Wal et. al. (2000), Science,

Entanglement of three superconducting qubits:

Di Carlo et. al. (2010), Nature, vol. 467, p. 574.

Coupling a superconducting qubit to a mechanical resonator:

O'Connell (2010), Nature, vol.464, p.697.

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main example:

EPR - From philosophy to practical application

Birth of the Demon

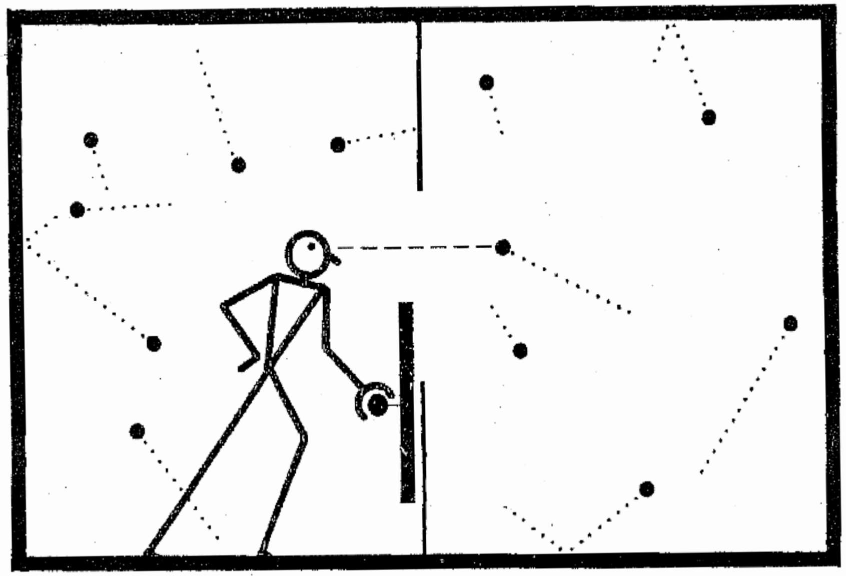
Clausius (1856) states the 2nd Law: Heat cannot flow from a colder body to a hotter body without an accompanying process (ie, work).

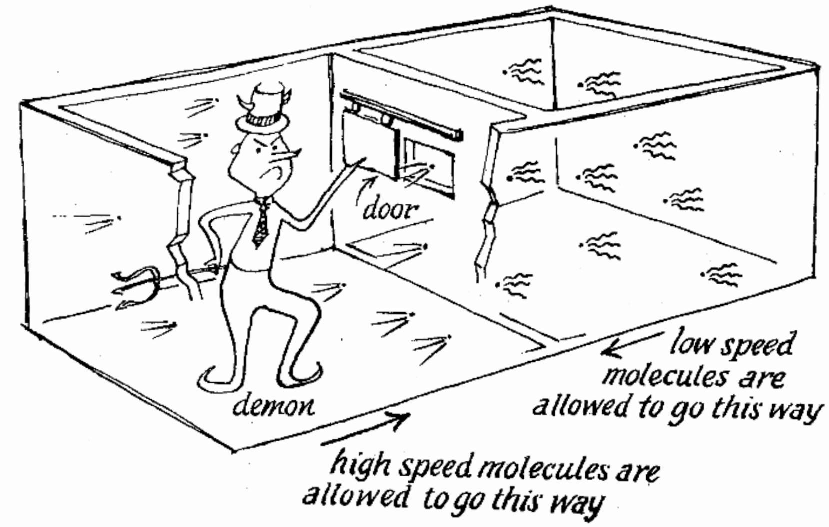
Boltzmann (1866) and Clausius (1871) both claim to derive the second law (as an exact law) from mechanics. Boltzmann claims priority (1871) and Clausius rebuts.

Maxwell knew that the 2nd law was valid only statistically. He commented on the above argument in a letter (1873) to Peter Guthrie Tait:

> “Hamiltonian’s Principle, the while, soars along in a region unvexed by statistical considerations …”

Maxwell had conceived of the demon in 1867, but introduced him publicly only in (1871), in his book “Theory of Heat.”





In "Theory of Heat" (1871):

> … This is the second law of thermodynamics, and it is undoubtedly true as long as we can deal with bodies only in mass, and have no power of perceiving or handling the separate molecules of which they are made up. But if we conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are as essentially finite as our own, would be able to do what is impossible to us… He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics.

The Life of the Demon

Birth: Maxwell (1871) - in “Theory of Heat.”

Childhood: Szilard’s Engine (1929) - entropy and measurement

Brillouin and Gabor (1951) - light signals

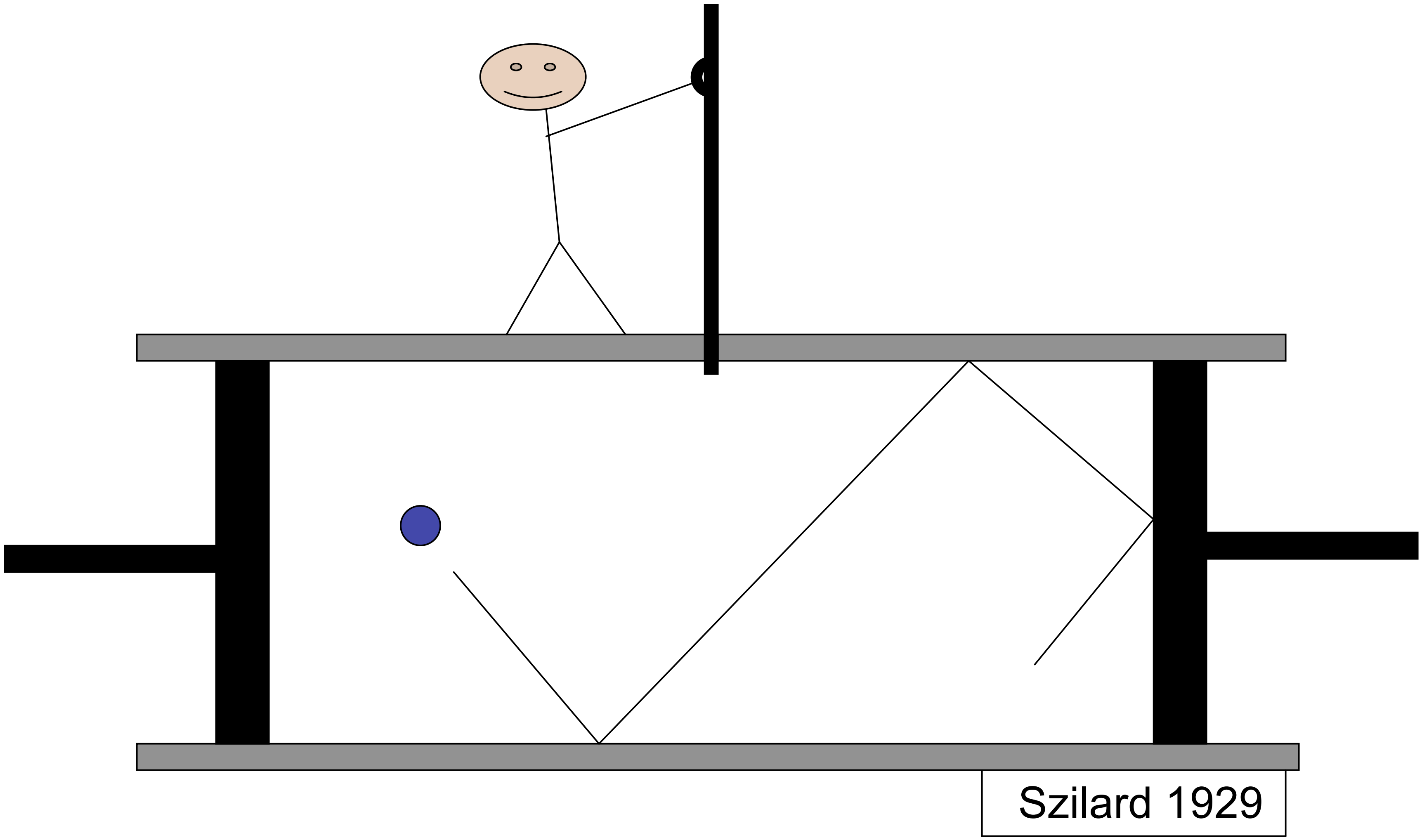
Adulthood: Landauer’s Principle (1961) - entropy and erasure

Bennett (1982) - resolution of the paradox

Feynman (1982): - “Quantum mechanical computers”

Piechocinska (2000) - proof of Landauer’s principle (C and Q)

Vedral (1999) - error correction with demon (C and Q)



Work extracted per (isothermal) cycle is $W = kT \ln 2.$

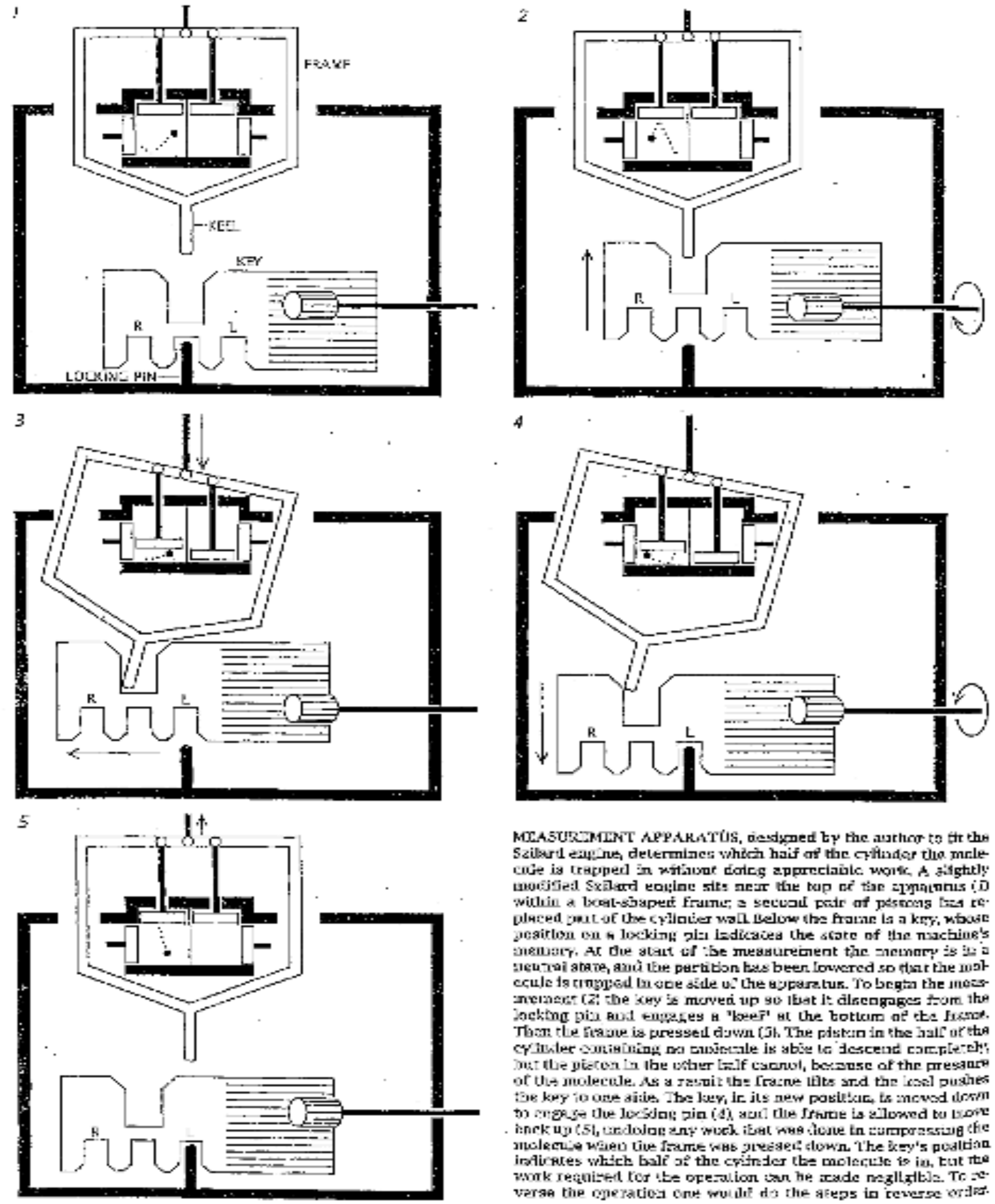
Bennett arguments

1) Measurements can be done reversibly.

2) Erasure is irreversible, and costs an entropy of at least

$$\Delta S = k \ln 2$$

per bit, on average.



MEASUREMENT APPARATUS, designed by the author to fit the Szilard engine, determines which half of the cylinder the molecule is trapped in without doing appreciable work. A slightly modified Szilard engine sits near the top of the apparatus (1) within a boat-shaped frame; a second pair of pistons has replaced part of the cylinder wall. Below the frame is a key, whose position on a locking pin indicates the state of the machine's memory. At the start of the measurement the memory is in a neutral state, and the partition has been lowered so that the molecule is trapped in one side of the apparatus. To begin the measurement (2) the key is moved up so that it disengages from the locking pin and engages a "keel" at the bottom of the frame. Then the frame is pressed down (3). The piston in the half of the cylinder containing no molecule is able to descend completely but the piston in the other half cannot, because of the pressure of the molecule. As a result the frame tilts and the keel pushes the key to one side. The key, in its new position, is moved down to engage the locking pin (4), and the frame is allowed to move back up (5), undoing any work that was done in compressing the molecule when the frame was pressed down. The key's position indicates which half of the cylinder the molecule is in, but the work required for the operation can be made negligible. To reverse the operation one would do the steps in reverse order.

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Conclusions:

Maxwell's Demon and Schroedinger's Cat are alive and thriving, and EPR remains one of the most important wrong papers ever written.

1) The 2nd Law is unthreatened by real or artificial intelligence; on the contrary, it is a useful tool in analyzing their processes.
2) There is no intrinsic size limit found in Schroedinger Cat states to date,
3) And no intrinsic distance limit for the persistence of entanglement.