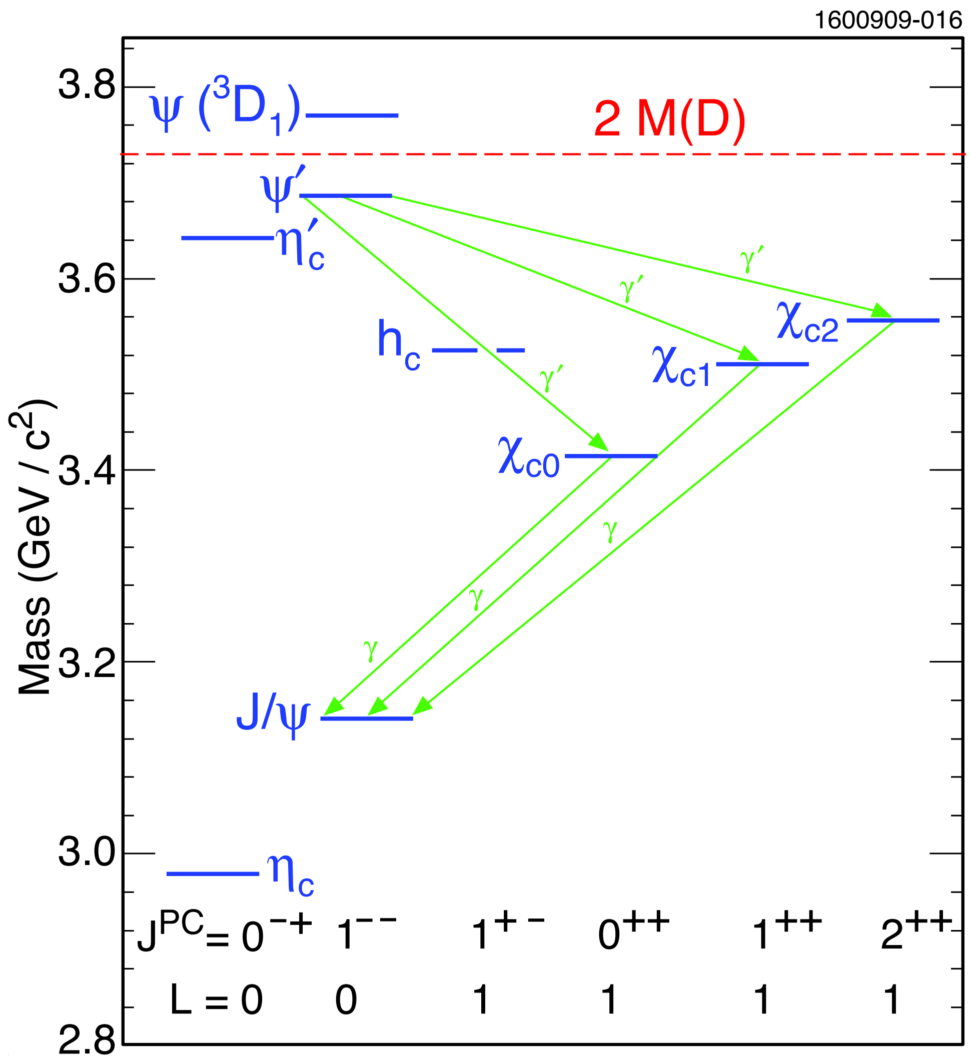
HIGHER MULTIPOLES IN CHARMONIUM RADIATIVE TRANSITIONS

Analysis of CLEO data by J. Ledoux and R. S. Galik; JLR proxy for RSG

M. Artuso *et al.*, arXiv:0910.0046 ⇒ PRD



Transitions $\psi' \to \gamma\chi_{cJ} \to \gamma\gamma J/\psi$ dominantly electric dipole (E1)

Higher multipoles allowed:
M2 for $J_\chi = 1$; M2 and E3 for $J_\chi = 2$;
No E3 for $S \leftrightarrow P$ single-quark trans.

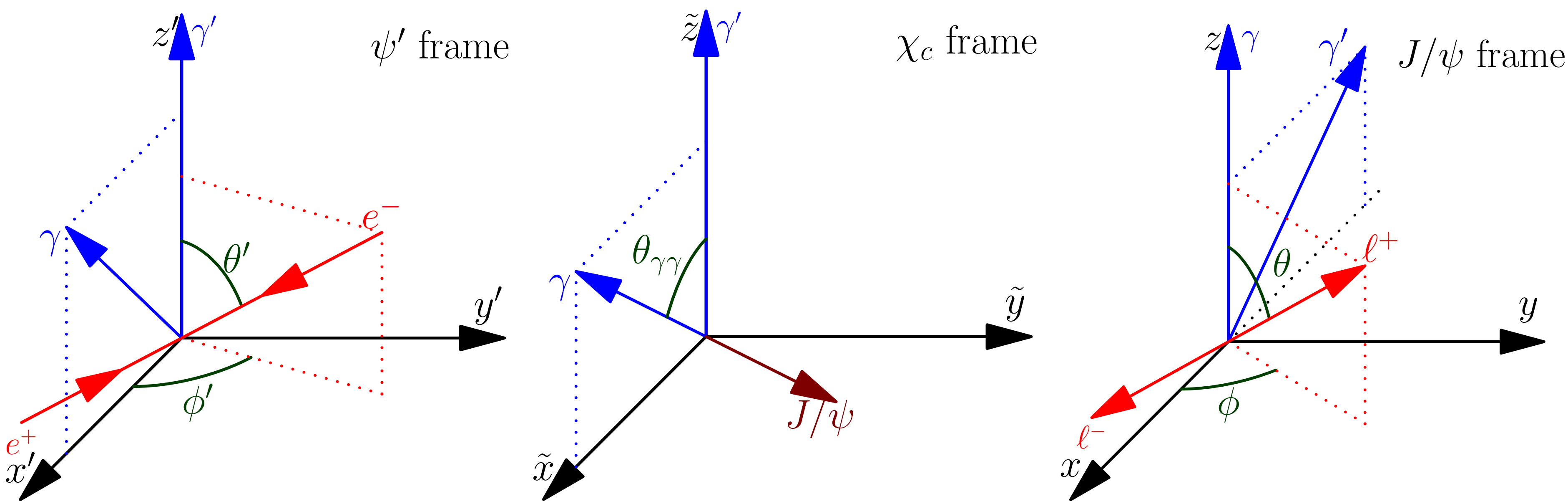
Angular distributions of photons are sensitive to higher multipoles

M2 transition measures charmed quark's total magnetic moment

Provides a check of measurements in M1 transitions such as $J/\psi \to \gamma\eta_c$

ANGLE DEFINITIONS

Reaction $e^+e^- \to \psi' \to \gamma'\chi_{cJ} \to \gamma'\gamma J/\psi \to \gamma'\gamma\ell^+\ell^-$ ($\ell = e$ or μ)



Primed angles: initial lepton pair orientation relative to γ' in $\psi' \to \gamma'\chi_{cJ}$

Unprimed angles: final lepton pair orientation relative to γ in $\chi_{cJ} \to \gamma J/\psi$

Angle $\theta_{\gamma\gamma'}$ between γ and γ' in χ_{cJ} rest frame

Angular dist. $W(\cos\theta', \phi', \cos\theta_{\gamma\gamma'}, \cos\theta, \phi)$ depends on helicity amps. B'_ν or A_ν:

$\psi'(\lambda') \to \gamma'(\mu') + \chi(\nu')$ (helicity amps. B'_ν)

$\chi(\nu) \to \gamma(\mu) + J/\psi(\lambda)$ (helicity amps. A_ν)

HELICITY AMPLITUDES AND MULTIPOLES

Parity relates amplitudes for $\nu, \nu' < 0$ to ones with $\nu, \nu' > 0$

Transform between normalized helicity amps. A (or B) and multipole amps. $a^{J_\chi}_{J_\gamma}$:

$$\begin{pmatrix} A_0^{J=1} \\ A_1^{J=1} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} a_1^{J=1} \\ a_2^{J=1} \end{pmatrix} ,$$

$$\begin{pmatrix} A_0^{J=2} \\ A_1^{J=2} \\ A_2^{J=2} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{10}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{2}{5}} \\ \sqrt{\frac{3}{10}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{8}{15}} \\ \sqrt{\frac{3}{5}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{15}} \end{pmatrix} \begin{pmatrix} a_1^{J=2} \\ a_2^{J=2} \\ a_3^{J=2} \end{pmatrix} .$$

M2 amplitudes related to anomalous moment κ_c of charm quark (potl. indep.!):

$$a_2^{J=1} \equiv \frac{M2}{\sqrt{E1^2 + M2^2}} = -\frac{E_\gamma}{4m_c}(1+\kappa_c)$$

$$a_2^{J=2} \equiv \frac{M2}{\sqrt{E1^2 + M2^2 + E3^2}} = -\frac{3}{\sqrt{5}}\frac{E_\gamma}{4m_c}(1+\kappa_c) ;$$

similarly for b amplitudes with sign change, $E_\gamma \to E_{\gamma'}$. These follow from

$$H_I = -\frac{e_c}{2m_c}(\vec{A}^* \cdot \vec{p} + \vec{p} \cdot \vec{A}^*) - \mu\vec{\sigma} \cdot \vec{H}^* + (\mathrm{spin-orbit\ term})$$

$e_c \equiv \frac{2}{3}|e|$; $\mu \equiv (e_c/2m_c)(1+\kappa_c)$; $\vec{A}^*$ and $\vec{H}^* \equiv \nabla \times \vec{A}^*$ refer to emitted photon

RATIOS OF MULTIPOLE AMPLITUDES

Ratios of predicted multipole amplitudes independent of m_c and κ_c to $\mathcal{O}(E_{\gamma^{(\prime)}}/m_c)$

Uncertainties $\mathcal{O}(E_\gamma/m_c)^2$ assigned to each amplitude [$m_c = 1.5$ GeV/c^2]

$$\left(\frac{a_2^{J=1}}{a_2^{J=2}}\right)_{\rm th} = \frac{E_\gamma^{J=1}}{E_\gamma^{J=2}}\frac{\sqrt{5}}{3} = 0.676 \pm 0.071 \ [\mathrm{Expt.}: 0.02^{+0.17}_{-0.16}] \ ,$$

$$\left(\frac{a_2^{J=1}}{b_2^{J=1}}\right)_{\rm th} = -\frac{E_\gamma^{J=1}}{E_{\gamma'}^{J=1}} = -2.27 \pm 0.16 \ [\mathrm{Expt.}: -0.02^{+0.30}_{-0.32}] \ ,$$

$$\left(\frac{b_2^{J=2}}{b_2^{J=1}}\right)_{\rm th} = \frac{E_{\gamma'}^{J=2}}{E_{\gamma'}^{J=1}}\frac{3}{\sqrt{5}} = 1.000 \pm 0.015 \ [\mathrm{Expt.}: 1.5^{+2.2}_{-1.1}] \ ,$$

$$\left(\frac{b_2^{J=2}}{a_2^{J=2}}\right)_{\rm th} = -\frac{E_{\gamma'}^{J=2}}{E_\gamma^{J=2}} = -0.297 \pm 0.025 \ [\mathrm{Expt.}: -1.01^{+0.60}_{-0.93}] \ .$$

"Expt." refers to *previous* experiments (next slide). Only $b_2^{J=2}/b_2^{J=1}$ is consistent with prediction.

PREVIOUS RESULTS

$J_\chi = 1$:

Experiment	$a_2^{J=1}$	$b_2^{J=1}$	Signal Evts.
SPEAR Crystal Ball	$-0.002^{+0.008}_{-0.020}$	$0.077^{+0.050}_{-0.045}$	921
Fermilab E-835	$0.002 \pm 0.032 \pm 0.004$		2090
Theory ($m_c = 1.5$ GeV)	$-0.065(1+\kappa_c)$	$0.029(1+\kappa_c)$	

$J_\chi = 2$:

Experiment	$a_2^{J=2}$	$b_2^{J=2}$	Signal Evts.
SPEAR Crystal Ball	$-0.333^{+0.116}_{-0.292}$	$0.132^{+0.098}_{-0.075}$	441
Fermilab E-760	-0.14 ± 0.06		1904
Fermilab E-835	$-0.093^{+0.039}_{-0.041} \pm 0.006$		5908
BES-II		$-0.051^{+0.054}_{-0.036}$	731
Theory ($m_c = 1.5$ GeV)	$-0.096(1+\kappa_c)$	$0.029(1+\kappa_c)$	

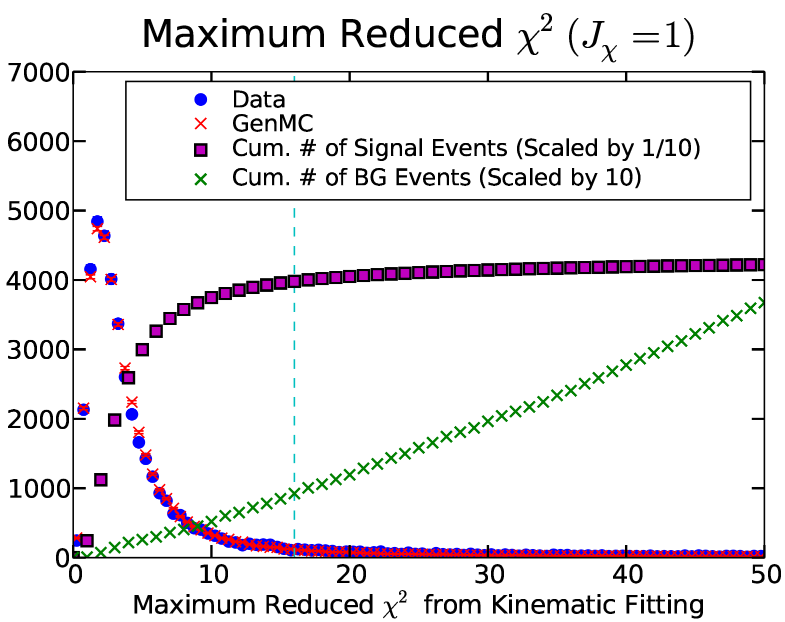
CLEO has $\sim$ (40,20)K events of $J_\chi = (1,2)$ after cuts

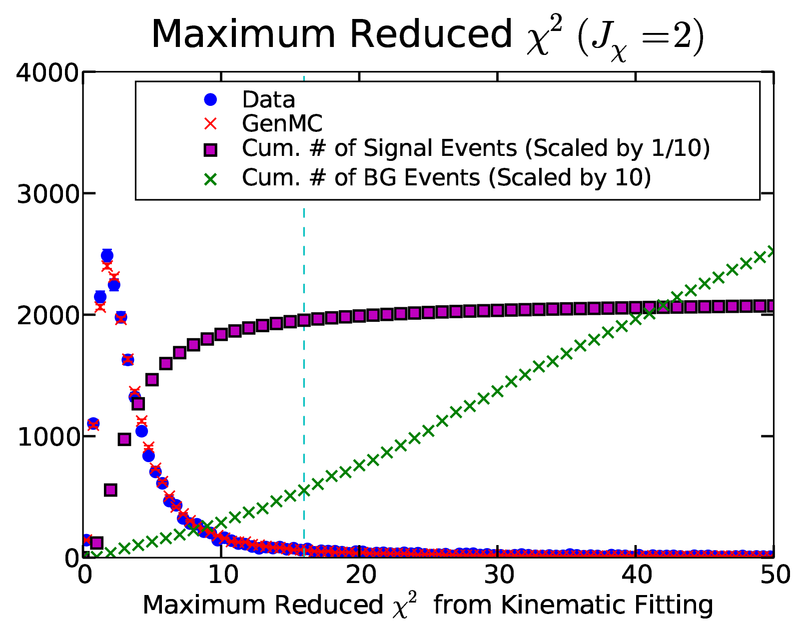
DATA AND MONTE CARLO

Data sample used: $\int \mathcal{L}dt = 48.07/\text{pb}$, $24.45 \pm 0.49) \times 10^6$ ψ' events, implying production of $(91900 \pm 6600, 48200 \pm 3600)$ $\chi_{c1,2}$, respectively

CLEO-c detector configuration, showers in $|\cos\theta| \leq 0.79$ or $0.85 \leq |\cos\theta| \leq 0.93$, require $\geq$ 2 tracks and 2 showers; choose showers and tracks with greatest energies

Kinematic fits: (1) 1C fit to $M(J/\psi)$ (including any bremsstrahlung photons within 100 mr of lepton tracks); (2) 4C fit to ψ' 4-vector; require both to have $\chi^2_R < 16$





Generic MC: 120M events, scaled to number of events in data histogram

SUMMARY OF CRITERIA

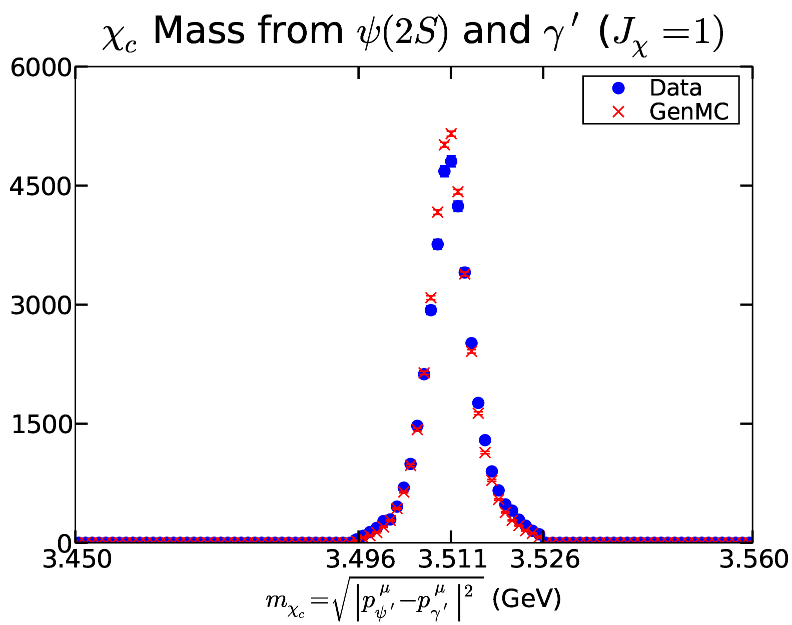
All reduced χ^2 from kinematic fits ≤ 16 (including vertex fits)

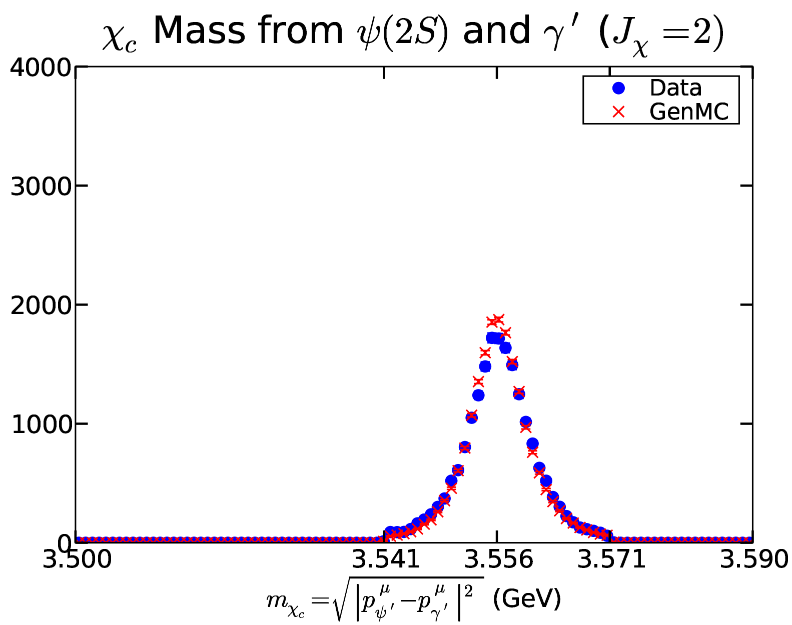
$J/\psi \to \mu^+\mu^-$: $(E/p)_{\text{larger}} < 0.5$, $(E/p)_{\text{smaller}} < 0.25$;
$J/\psi \to e^+e^-$: $(E/p)_{\text{larger}} > 0.85$ and $(E/p)_{\text{smaller}} > 0.5$

$m(\chi_{cJ}) \equiv \sqrt{|p^\mu_{J/\psi} + p^\mu_\gamma|^2}$ within 0.015 GeV of $m_{\chi_{c(1,2)}} = (3.511, 3.556)$ GeV

J/ψ momentum between 0.24 GeV$/c$ and 0.51 GeV$/c$;
Maximum energy of the third most energetic shower < 30 MeV
(suppresses backgrounds from $\psi' \to XJ/\psi,\ X = (\pi^0\pi^0, \eta, \pi^0)$)

Standard requirements for "good" tracks and showers





FITTING THE DATA

Probability density function (PDF) $W(\Omega;\mathcal{A})$ gives probability for event with angles Ω to occur given a set of multipole amplitudes $\mathcal{A} \equiv (a_i, b_j)$

Find parameters (a_i, b_j) maximizing likelihood $\mathcal{L}_W(\mathcal{A}) \equiv \prod_{d=1}^{N_d} W(\Omega_d;\mathcal{A})$

Detector efficiency $\epsilon(\Omega)$ from phase space MC uniform in $(\cos\theta', \phi', \cos\theta_{\gamma\gamma'}, \cos\theta, \phi)$

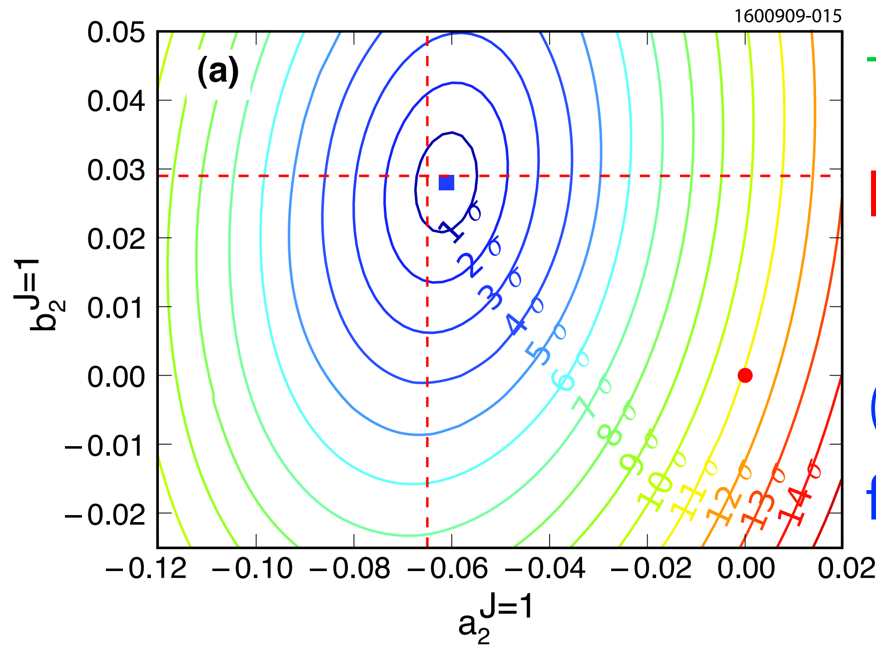
$J_\chi = 1$: Best fit to 39363 events is $\sigma_{\rm E1} \equiv \sqrt{2\Delta\log\mathcal{L}} = 11.1\sigma$ from pure E1:

Fit	$a_2^{J=1}$ 10^{-2}	σ_{a_2} 10^{-2}	$b_2^{J=1}$ 10^{-2}	σ_{b_2} 10^{-2}	$\sigma_{\rm E1}$
2-parameter	-6.11	0.63	2.81	0.73	11.1
1-param. ($\frac{a_2}{b_2}$ fixed)	-6.15	0.55	2.71	0.24	11.1
Theory (m_c=1.5 GeV)	$-6.5(1+\kappa_c)$		$2.9(1+\kappa_c)$		

$J_\chi = 2$: Best fits to 19755 events are at least 6σ from pure E1:

Fit	$a_2^{J=2}$ 10^{-2}	σ_{a_2} 10^{-2}	$b_2^{J=2}$ 10^{-2}	σ_{b_2} 10^{-2}	$a_3^{J=2}$ 10^{-2}	σ_{a_3} 10^{-2}	$b_3^{J=2}$ 10^{-2}	σ_{b_3} 10^{-2}	$\sigma_{\rm E1}$
2-parameter	-9.3	1.6	1.0	1.3	0	-	0	-	6.2
3-parameter	-9.3	1.6	0.7	1.3	0	-	-0.8	1.2	6.3
2-param. ($\frac{a_2}{b_2}$ fixed)	-9.2	1.6	2.7	0.5	0	-	-0.1	1.1	6.1
4-parameter	-7.9	1.9	0.2	1.4	1.7	1.4	-0.8	1.2	6.4
Th. (m_c=1.5 GeV)	$-9.6(1+\kappa_c)$		$2.9(1+\kappa_c)$		0		Model dep.		

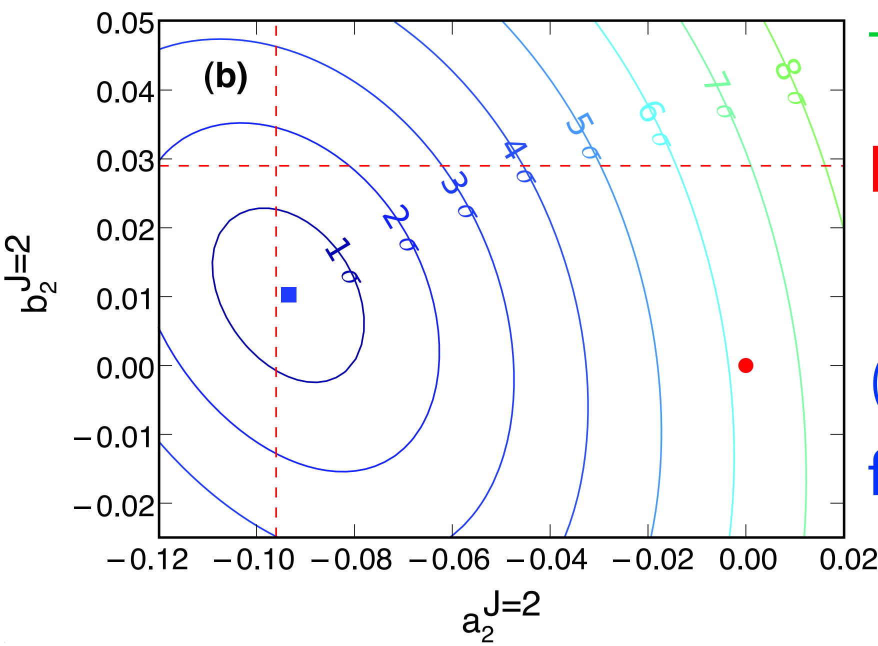
$J_\chi = 1,\ 2$ LIKELIHOODS



Two-parameter fit, $J_\chi = 1$:

Prediction for $\kappa_c = 0$, $m_c = 1.5$ GeV is $(a_2, b_2) = (-0.065, 0.029)$

$(a_2, b_2) = (0, 0)$ ($\bullet$) is 11.1σ from maximum-likelihood solution ($\Box$)

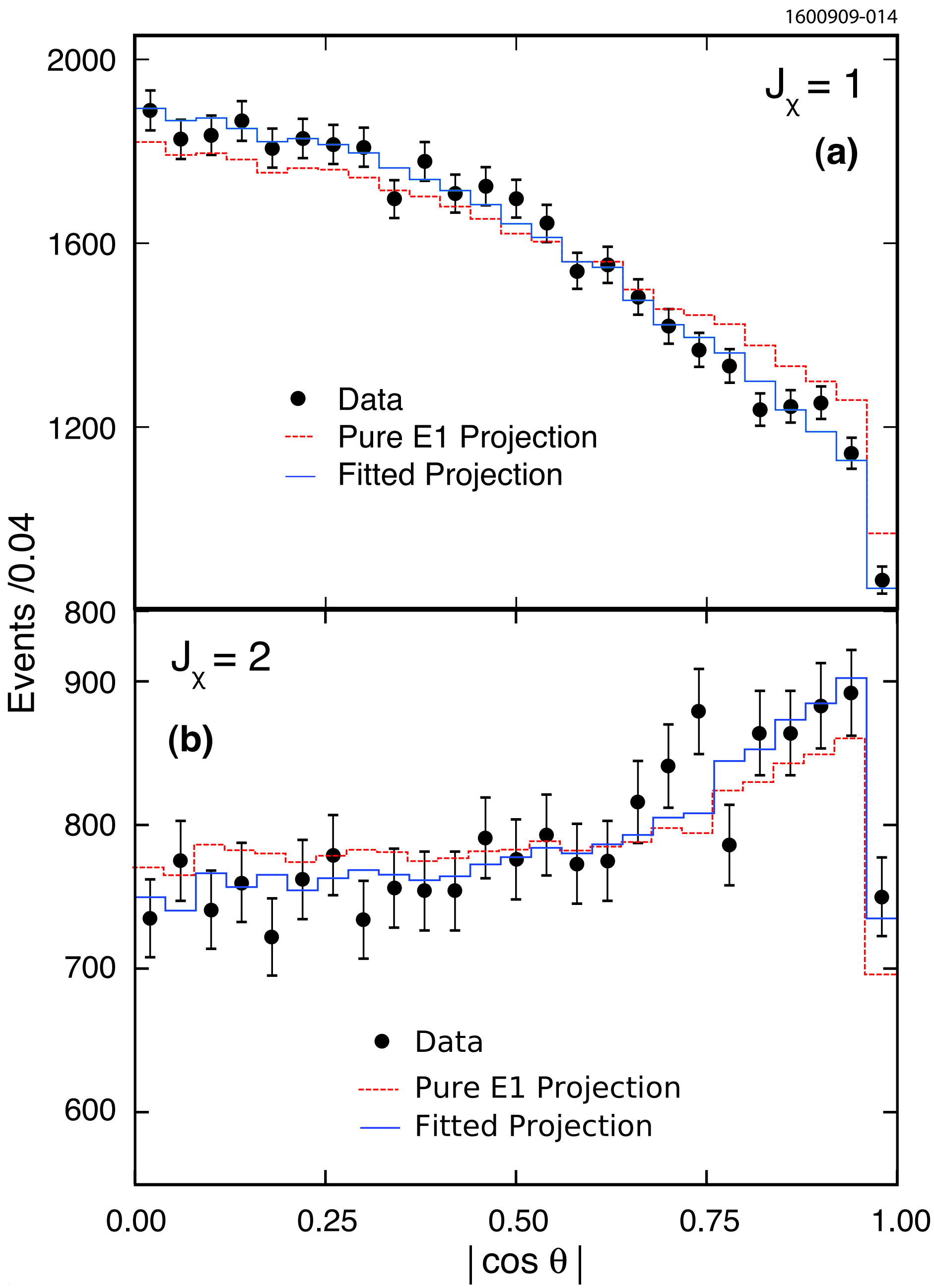


Two-parameter fit, $J_\chi = 2$:

Prediction for $\kappa_c = 0$, $m_c = 1.5$ GeV is $(a_2, b_2) = (-0.096, 0.029)$

$(a_2, b_2) = (0, 0)$ ($\bullet$) is 6.2σ from maximum-likelihood solution ($\Box$)

$J_\chi = 1,\ 2\ \cos\theta$ PROJECTIONS



Two-parameter fit, $J_\chi = 1$:

$\chi^2/N_{\mathrm{d.o.f.}} = 16.2/22 = 0.74$

for fitted projection;

$80.29/24 = 3.35$ for pure E1

Two-parameter fit, $J_\chi = 2$:

$\chi^2/N_{\mathrm{d.o.f.}} = 20.3/22 = 0.92$

for fitted projection;

$35.5/24 = 1.48$ for pure E1

SYSTEMATIC ERRORS

Toy Monte Carlo check of fitting procedure: Ensembles of 200 toy MC trials, each with 40K (20K) signal events for $J_\chi = 1\ (2)$. No systematic bias or uncertainty

Amount of phase space MC needed for efficiency integrals: Ensembles of 31 (37) $J_\chi = 1\ (2)$ simulations; no evidence of a systematic uncertainty when using $> 10^5$ events. Actual number used: 4.5M for each $J_\chi = 1, 2$

Impurity systematic uncertainties: Ensembles of 31 (37) $J_\chi = 1\ (2)$ simulations with and without impurity background. For $J_\chi = 1$, find significant impurity bias relatively constant over trials; correct for bias and assign systematic uncertainty of the full bias. For $J_\chi = 2$, impurity bias fluctuates; assign systematic uncertainty of size of fluctuations

Final state radiation: No statistically significant evidence for systematic uncertainty

Choice of kinematic fits (e.g., with/without bremsstrahlung recovery): No statistically significant evidence for systematic uncertainty

Variation of selection criteria: Max. shower energy $\rightarrow (18, 50)$ MeV, max. $\chi^2_R \rightarrow (16, 30)$, χ_c mass window $\rightarrow \pm(10, 20)$ MeV, max. barrel $|\cos\theta| \rightarrow (0.77, 0.80)$: effects included in summary tables

SYSTEMATICS: $J_\chi = 1$

Source of systematic error	$a_2^{J_\chi=1}$ Uncertainty $\times 10^{-2}$	$a_2^{J_\chi=1}$ Bias $\times 10^{-2}$	$b_2^{J_\chi=1}$ Uncertainty $\times 10^{-2}$	$b_2^{J_\chi=1}$ Bias $\times 10^{-2}$
Generic MC impurities	0.15	0.15	0.05	0.05
Selection criteria	0.19	-	0.22	-
Total systematic uncert.	0.24	0.15	0.23	0.05
Statistical uncertainty	0.63	-	0.73	-

Results for bias-corrected multipole amplitudes:

$$a_2^{J=1} = (-6.26 \pm 0.63 \pm 0.24) \times 10^{-2} ,$$
$$b_2^{J=1} = (2.76 \pm 0.73 \pm 0.23) \times 10^{-2} .$$

Recall theoretical predictions (for $m_c = 1.5$ GeV):

$$a_2^{J=1} = -6.5(1 + \kappa_c) \times 10^{-2} ,$$
$$b_2^{J=1} = 2.9(1 + \kappa_c) \times 10^{-2} .$$

SYSTEMATICS: $J_\chi = 2$

Two-, three-parameter fits:

	Two-param.		Three-param.		
Systematic uncertainty	$a_2^{J=2}$ $\times 10^{-2}$	$b_2^{J=2}$ $\times 10^{-2}$	$a_2^{J=2}$ $\times 10^{-2}$	$b_2^{J=2}$ $\times 10^{-2}$	$b_3^{J=2}$ $\times 10^{-2}$
Generic MC impurities	0.04	0.07	0.04	0.07	0.03
Selection criteria	0.33	0.33	0.33	0.34	0.20
Total systematic uncert.	0.3	0.3	0.3	0.3	0.2
Statistical uncertainty	1.6	1.3	1.6	1.4	1.2

Two-parameter fit (a_2, b_2) with $a_3 = b_3 \equiv 0$:

$$a_2^{J=2} = (-9.3 \pm 1.6 \pm 0.3) \times 10^{-2} ,$$
$$b_2^{J=2} = (1.0 \pm 1.3 \pm 0.3) \times 10^{-2} .$$

Three-parameter fit (a_2, b_2, b_3) with $a_3 \equiv 0$:

$$a_2^{J=2} = (-9.3 \pm 1.6 \pm 0.3) \times 10^{-2} ,$$
$$b_2^{J=2} = (0.7 \pm 1.4 \pm 0.3) \times 10^{-2} ,$$
$$b_3^{J=2} = (-0.8 \pm 1.2 \pm 0.2) \times 10^{-2} .$$

Theory ($m_c = 1.5$ GeV): $(a_2, b_2, a_3) = (-9.6, 2.9, 0)(1 + \kappa_c) \times 10^{-2}$

$J_\chi = 2$, CONTINUED

Fits with fixed b_2/a_2 and $a_3 = 0$; four free parameters:

	Fixed b_2/a_2		Four free parameters			
Systematic uncertainty	$a_2^{J=2}$ $\times 10^{-2}$	$b_3^{J=2}$ $\times 10^{-2}$	$a_2^{J=2}$ $\times 10^{-2}$	$b_2^{J=2}$ $\times 10^{-2}$	$a_3^{J=2}$ $\times 10^{-2}$	$b_3^{J=2}$ $\times 10^{-2}$
Generic MC impurities	0.04	0.04	0.06	0.08	0.08	0.03
Selection criteria	0.34	0.23	0.24	0.39	0.28	0.20
Total systematic uncert.	0.3	0.2	0.3	0.4	0.3	0.2
Statistical uncertainty	1.6	1.1	1.9	1.5	1.4	1.2

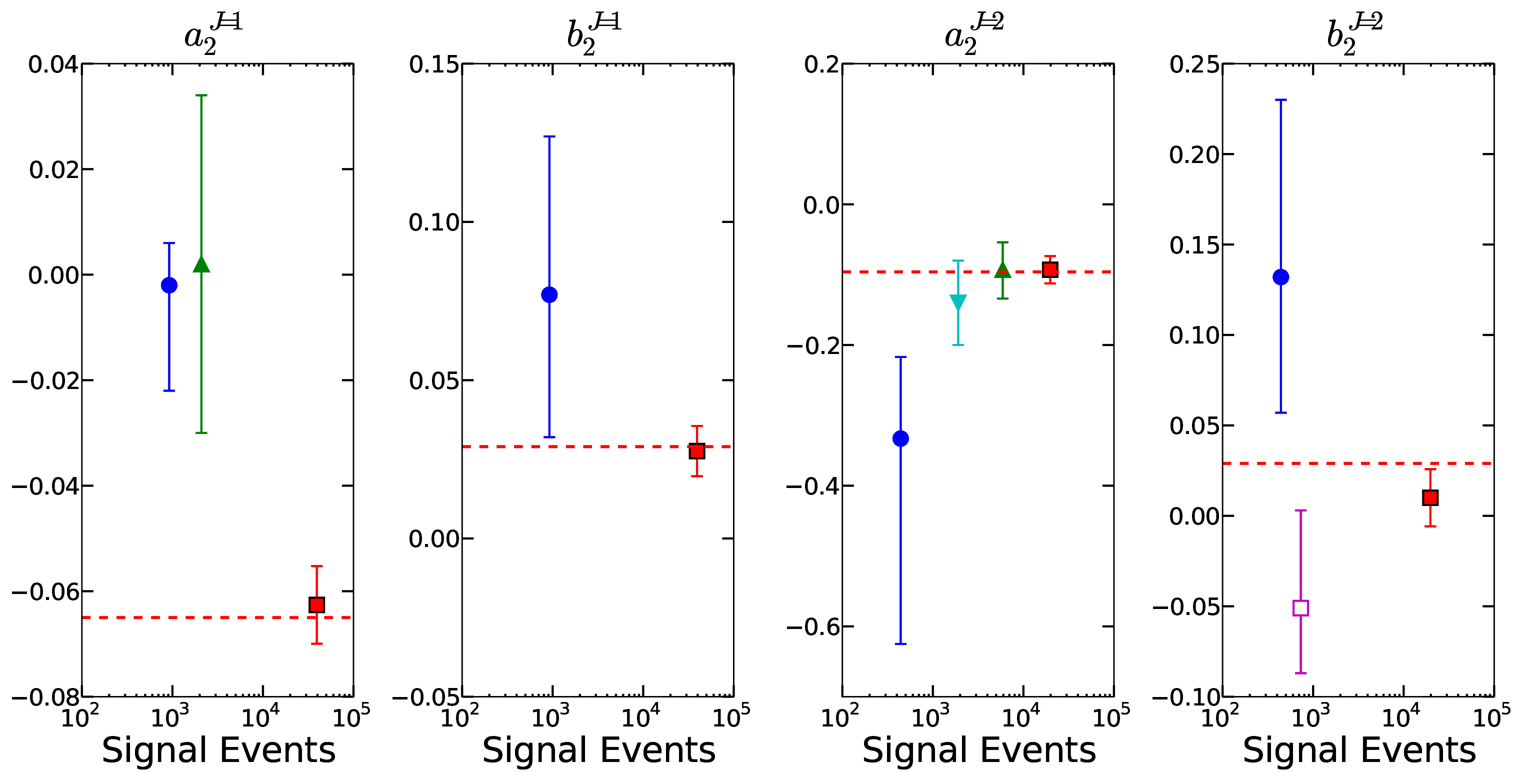
Fit with fixed b_2/a_2 and $a_3 = 0$:

$$
\begin{aligned}
a_2^{J=2} &= (-9.2 \pm 1.6 \pm 0.3) \times 10^{-2} \ , \\
b_2^{J=2} &\equiv -a_2^{J=2}/3.367 = (2.7 \pm 0.5 \pm 0.1) \times 10^{-2} \ , \\
b_3^{J=2} &= (-0.1 \pm 1.1 \pm 0.2) \times 10^{-2} \ .
\end{aligned}
$$

Fit with four free parameters:

$$
\begin{aligned}
a_2^{J=2} &= (-7.9 \pm 1.9 \pm 0.3) \times 10^{-2} \ , \\
b_2^{J=2} &= (0.2 \pm 1.5 \pm 0.4) \times 10^{-2} \ , \\
a_3^{J=2} &= (1.7 \pm 1.4 \pm 0.3) \times 10^{-2} \ , \\
b_3^{J=2} &= (-0.8 \pm 1.2 \pm 0.2) \times 10^{-2} \ .
\end{aligned}
$$

COMPARISON WITH PREVIOUS RESULTS



CLEO-c: triangles (two-parameter fit for $J_\chi = 2$); Crystal Ball: circles; E760: $+$; E835: $\times$; BES-II: square; theory ($m_c = 1.5$ GeV. $\kappa_c = 0$): dashed line

First statistically significant evidence for non-zero $a_2^{J=1}$; in accord with prediction

Multipoles $b_2^{J=1}$ and $a_2^{J=2}$ also significantly non-zero and in accord with prediction

Multipole $b_2^{J=2}$ not significant but error reduced w.r.t. previous measurements

No statistically significant evidence for E3 transitions

RATIOS INDEPENDENT OF m_c AND κ_c

Charmed quark mass could easily vary by ± 0.3 GeV (20%); anomalous moment was proposed when $\Gamma(J/\psi \to \gamma\eta_c)$ was thought to be lower than at present

$$\left(\frac{a_2^{J=1}}{a_2^{J=2}}\right)_{\rm CLEO} = 0.67^{+0.19}_{-0.13} \quad \stackrel{?}{=} \quad \left(\frac{a_2^{J=1}}{a_2^{J=2}}\right)_{\rm th} = 0.676 \pm 0.071 \ ,$$

$$\left(\frac{a_2^{J=1}}{b_2^{J=1}}\right)_{\rm CLEO} = -2.27^{+0.57}_{-0.99} \quad \stackrel{?}{=} \quad \left(\frac{a_2^{J=1}}{b_2^{J=1}}\right)_{\rm th} = -2.27 \pm 0.16 \ ,$$

$$\left(\frac{b_2^{J=2}}{b_1^{J=2}}\right)_{\rm CLEO} = 0.37^{+0.53}_{-0.47} \quad \stackrel{?}{=} \quad \left(\frac{b_2^{J=2}}{b_2^{J=1}}\right)_{\rm th} = 1.000 \pm 0.015 \ ,$$

$$\left(\frac{b_2^{J=2}}{a_2^{J=2}}\right)_{\rm CLEO} = -0.11^{+0.14}_{-0.15} \quad \stackrel{?}{=} \quad \left(\frac{b_2^{J=2}}{a_2^{J=2}}\right)_{\rm th} = -0.297 \pm 0.025 \ .$$

Ratios now appear to be compatible with predictions

CONCLUSIONS

CLEO measures significant non-zero magnetic quadrupole amplitudes for the transitions $\chi_{c1} \to \gamma J/\psi$, $\chi_{c2} \to \gamma J/\psi$, and $\psi' \to \gamma' \chi_{c1}$.

Outstanding disagreement on ratios between theory and experiment is now resolved

Strengths of these amplitudes agree with predictions to $\mathcal{O}(E_{\gamma^{(\prime)}}/m_c)$ with $m_c = 1.5$ GeV and $\kappa_c = 0$

No significant M2 contribution observed for $\psi' \to \gamma' \chi_{c2}$ but this has fewer signal events and a low photon energy

$\cos\theta$ distributions in $\chi_{c(1,2)} \to \gamma J/\psi$ display M2 contribution

Five-angle fits exclude pure E1 by more than $(11, 6)\sigma$ for $J_\chi = (1, 2)$

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