

# ISR/FSR Uncertainties for Tevatron Studies

## Theory versus Practice

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- What *IS* the parton shower
- How good is it?
- Uncertainties



# Radioactive Decay Problem

$f(t) \equiv$  prob. 'something will happen' (a nucleus decay) at time  $t$   
*something happens at  $t$  only if it did not happen at  $t' < t$*

Equation for nothing  $\mathcal{N}(t)$  to happen *up to time  $t$*  is ( $\mathcal{N}(0) = 1$ ):

$$-\frac{d\mathcal{N}}{dt} = f(t)\mathcal{N}(t) = \mathcal{P}(t)$$

$$\mathcal{N}(t) = \exp\left\{-\int_0^t f(t') dt'\right\}$$

$$\mathcal{P}(t) = f(t) \exp\left\{-\int_0^t f(t') dt'\right\}$$

- Naive answer modified by exponential suppression
- $\mathcal{N}(t) \Rightarrow$  Sudakov form factor



# Toy Parton Shower

Consider a system that can emit a number of quanta (photons) with energy  $z_0 < x < x_{\max}(x)$ ,  $x_{\max}(1) = 1$

$$0 \leq Q(z) \leq 1, \quad \lim_{z \rightarrow 0} Q(z) = 1,$$

IF the prob. of one emission is  $a \frac{Q(x)}{x} dx$

THEN the *Sudakov form factor* is

$$\Delta(x_2, x_1) = \exp \left[ -a \int_{x_1}^{x_2} dz \frac{Q(z)}{z} \right],$$

Limit	Sudakov	# of Quanta
$a \ll 1$	$\Delta \sim 1 - a \frac{Q(x)}{x} dx$	few
$a \gg 1$	$\Delta \sim 0$	many

# Constructing an “Event” Generator

Event  $\equiv$  original system + emissions down to scale  $x_0$

Take  $Q(x) = 1$

To solve for the shower evolution:

- 1 Pick  $r = \exp\left(-a \int_x^{x_2} dx/x\right) = (x/x_2)^a$
- 2 Solve  $x = x_2 r^{(1/a)}$
- 3 Calculate remaining energy  $x_2$
- 4 Continue until  $x < x_0$

*This generates an energy-ordered shower with multiple photon emissions*



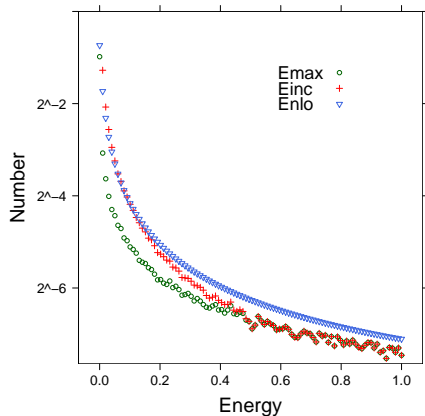
# Example Event Record

## Event listing (summary)

I	particle/jet	KS	KF	orig	E
1	e-	1	11	0	1.000
2	nu_e	1	12	0	0.000
3	(e-)	11	11	0	0.296
4	gamma	1	22	0	0.704
5	(e-)	11	11	3	0.285
6	gamma	1	22	3	0.011
7	(e-)	11	11	5	0.283
8	gamma	1	22	5	0.002
9	e-	1	11	7	0.282
10	gamma	1	22	7	0.001
	sum:	-1.00			1.000



# Spectra



Real (NLO) spectrum =  
$$\frac{d\sigma}{dx} = a \frac{R(x)}{x}$$

$R(x) \rightarrow Q(x)$  as  $x \rightarrow 0$

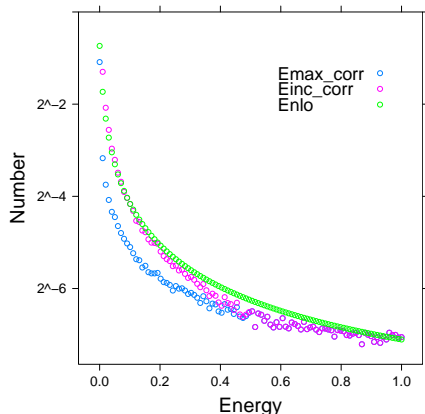
Here:  $R(x) = (1 + x/10)^2$

Enlo = Einc

Parton shower underestimates  
high  $x$



# Matrix Element Correction to Parton Shower



Assume the parton shower samples all of phase space and gives the hardest emission first

For the 1st emission, weight according to  $\frac{R(x)}{Q(x)}$

Here:  $(1 + x/10)^2 < 2$

Parton shower gets correct limit for large  $x$  and includes multiple photon emission

# How Good Is The Parton Shower?

Extensive LEP Experience and Tuning

- In general, a fair number of parameters are fit to data
- Most of these are related to the phenomenological model of hadronization
- For a given choice of Parton Shower, most sensitive parameters deal with the shower-hadronization transition and scale used in  $\alpha_s$

Range of  $Q_0, \Lambda_{LLA}$  gives approximate picture of our understanding of FSR in resonance production (numbers in GeV)

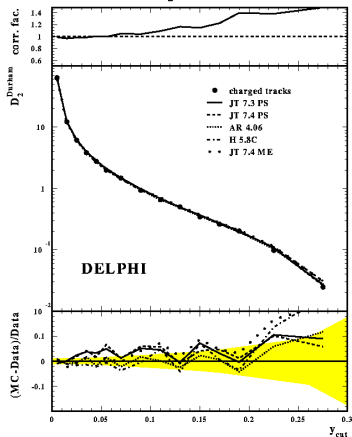
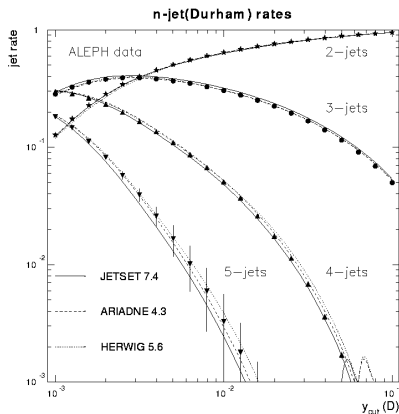
Parameter	Name	Default	Aleph	Delphi	L3	Opal
Shower $\Lambda_{LLA}$	PARJ(81)	0.290	0.320	0.297	0.306	0.250
Shower Cutoff $Q_0$	PARJ(82)	1.00	1.22	1.56	1.0	1.9



# Good description of complex topologies in $e^+e^-$

$R_2(y)$

$$D_2 = \frac{R_2(y) - R_2(\Delta y)}{\Delta y}$$



In general, Ariadne > Pythia > Herwig

# Quality of fit

Pythia 6.3

Distribution	bins	$\sum \chi^2$ of model	
		PY6.3 $p_{\perp}$ -ord.	PY6.1 mass-ord.
Sphericity	23	25	16
Aplanarity	16	23	168
1-Thrust	21	60	8
Thrust <sub>minor</sub>	18	26	139
jet res. $y_3(D)$	20	10	22
$x = 2p/E_{cm}$	46	207	151
$p_{\perp in}$	25	99	170
$p_{\perp out} < 0.7$ GeV	7	29	24
$p_{\perp out}$	(19)	(590)	(1560)
$x(B)$	19	20	68
sum	$N_{dof} =$ 190	497	765

- $\chi^2/dof > 1$
- “theory”  $T$  should have a systematic error  $qT$

$$\bullet \chi^2 = \frac{(O - T)^2}{(\sigma_O^2 + (qT)^2)}$$

q	0%	0.5%	1%
$\sum \chi^2$	523	364	234

for  $q = .01$ ,  $\chi^2/dof = 234/196 \sim 1 \Rightarrow$  generator good to 1%  
except  $p_{\perp out} > 0.7$  GeV (10%–20% error)

$p_{\perp out} \equiv$  one-particle inclusive  $p_{\perp}$  spectrum out of the event plane

- problem for all generators

## DGLAP

$$d\mathcal{P}_b = \frac{df_b(x, t)}{f_b(x, t)} = |dt| \sum_{a,c} \int \frac{dx'}{x'} \frac{f_a(x', t)}{f_b(x, t)} \frac{\alpha_{abc}}{2\pi} P_{a \rightarrow bc} \left( \frac{x}{x'} \right)$$

Summing up the cumulative effect of many small changes  $dt$ , the probability for no radiation exponentiates:

$$\begin{aligned} S_b(x, t_{\max}, t) &= \exp \left\{ - \int_t^{t_{\max}} dt' \sum_{a,c} \int \frac{dx'}{x'} \frac{f_a(x', t')}{f_b(x, t')} \frac{\alpha_{abc}(t')}{2\pi} P_{a \rightarrow bc} \left( \frac{x}{x'} \right) \right\} \\ &= \exp \left\{ - \int_t^{t_{\max}} dt' \sum_{a,c} \int dz \frac{\alpha_{abc}(t')}{2\pi} P_{a \rightarrow bc}(z) \frac{x' f_a(x', t')}{x f_b(x, t')} \right\} \end{aligned}$$

Probability that a parton  $b$  remains at  $x$  from  $t_{\max}$  to a  $t < t_{\max}$ .



- 1 Shower physics included, but only approximately
  - $LL + \alpha_s(p_T) + E - p \sim NLL$
- 2 Shower physics not included
  - genuine NLL effects:  $1 \rightarrow 3$  splittings
  - $2 \rightarrow 4$  “dipole” splittings (soft)
- 3 Beyond Shower Physics
  - Matrix Elements + Parton Shower



- 1 Estimate ME effects doing “matching” (hard emissions) or faking it
- 2 Vary logs with  $Q_{\max}$  and  $c \times p_T$  variations
  - changes range of Sudakov integration  $\rightarrow$  prob. of no radiation
- 3 Use Tevatron data:
  - High Mass  $M_{bb}/M_{jj}$
  - $H_T$  distribution?
  - many fits, several parameters

