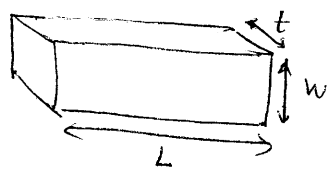
Physics 142 Homework 6 Solutions

Q1) In the lab frame +ve charge distribution $= \lambda_0$
and in the lab frame −ve charge distribution $= -\lambda_0$
Now in the rest frame of the test charge the separation between +ve charges decrease, but the separation between −ve charges decreases.
The factor the charge increases by $= \gamma_0$ as in the text.
So the +ve charge distribution $= \gamma_0 \lambda_0$
Now for the −ve charge the rest frame of the test charge & the −ve charges co-incide. Therefore the −ve charge distribution $= -\dfrac{\lambda_0}{\gamma_0}$.

Q2) The transverse field from 6.65 $= |\vec{E}_t| = \dfrac{JB}{nec}$



$\therefore$ the potential difference across $w = \dfrac{JBw}{nec}$

$JA = I = \dfrac{V}{R}$ where V is potential applied across L.

$= \dfrac{VA}{\rho L}$

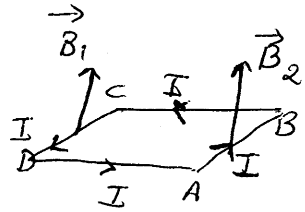
$V = 1\,V = \frac{1}{3}\times 10^{-2}$ Stat volts.
$w = 0.2\,cm$
$L = 0.5\,cm$
$B = 10^3\,G$
$\rho = 1.6\ \Omega\,cm = 1.6/9\times10^{11}$ sec/cm · cm $= \frac{1.6}{9}\times 10^{-11}$ sec.
$c = 3\times 10^{11}\,cm/s$
$q = 4.8\times 10^{-10}$ Stat C
$n = 2.5\times 10^{15}$ electrons/cm^3

$\Rightarrow J = \dfrac{V}{\rho L}$

$\therefore \phi = \dfrac{VBw}{nec\rho L}$

$\Rightarrow \phi = 7.7\times 10^{-3}\,V$

Q3) we know that the power $= \frac{dW}{dt} = \frac{\mathcal{E}^2}{R}$.



where R is the resistance.

Now we also have from 7.6 $\mathcal{E} = \frac{vw}{c}(B_1 - B_2)$

Now the force on AB $= \frac{IWB_2}{c}$

Force on CD $= -\frac{IWB_1}{c}$

$\Rightarrow$ Force on the loop $= -\frac{IW(B_1-B_2)}{c}$ $\Rightarrow$ Force need to keep the loop moving at $\vec{v}$ $= \frac{IW(B_1-B_2)}{c}$.

$\therefore$ power $= \vec{F}\cdot\vec{v}$

$= \frac{IWv(B_1-B_2)}{2} = I\mathcal{E} = \frac{\mathcal{E}^2}{R}$.

So power dissipated by the resistor is equal to that put into the system to keep the loop moving at constant velocity $\vec{v}$.

Q.4) From 6.41 we know the field along axis of a circular wire $= \frac{2\pi a^2 I}{c(a^2+z^2)^{3/2}}$

Now as $z = b$ $B = \frac{2\pi a^2 I}{c(a^2+b^2)^{3/2}} \approx \frac{2\pi a^2 I}{cb^3}$ as $a \ll b$.

$\therefore$ the flux passing through the other loop $= BA = \frac{2\pi^2 a^4 I}{cb^3}$

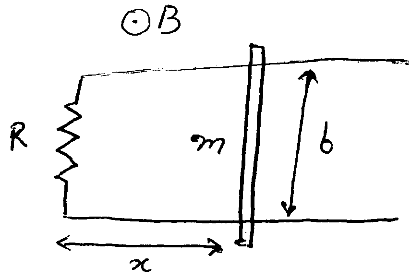
$\therefore$ the induced emf $= \mathcal{E} = -\frac{d}{dt}(BA) = \frac{2\pi^2 a^4}{cb^3}\frac{dI}{dt}$. $\left\{\begin{array}{l}\text{-ve sign as the} \\ \vec{A} \text{ is antiparallel to} \\ \vec{B}\end{array}\right\}$

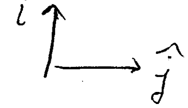
$\therefore M_{12} = \frac{2\pi^2 a^4}{cb^3}$

p5

7.14)

a)





Flux through the loop $= BA = Bbx$.

$\therefore$ induced emf $= -\dfrac{d}{dt}(BA) = -Bbv$ where $v = \dfrac{dx}{dt}$.

Now the current through the loop $= \dfrac{\mathcal{E}}{R} = -\dfrac{Bbv}{R}$.

The current flows counter clockwise. So the force on the bar

$$= \frac{I}{c} Bb = \frac{B^2b^2v}{Rc} \quad \text{pointing in the } -\hat{j} \text{ direction}$$

$$\therefore \quad m\frac{dv}{dt} = -\frac{B^2b^2v}{Rc}$$

$$\Rightarrow \frac{dv}{v} = -\frac{B^2b^2}{Rcm}dt \Rightarrow \frac{v}{v_0} = e^{-B^2b^2t/Rcm} \quad \Rightarrow v = v_0 e^{-B^2b^2t/Rcm}$$

$\therefore$ the rod stops when $t \to \infty$

b) Now $v = \dfrac{dx}{dt} = v_0 e^{-B^2b^2t/Rcm}$

$$\Rightarrow x - x_0 = \left.\frac{v_0 e^{-B^2b^2t/Rcm}}{(-B^2b^2/Rcm)}\right|_0^t \Rightarrow x = x_0 + \frac{v_0Rcm}{B^2b^2}\left(1 - e^{-B^2b^2t/Rcm}\right)$$

$\therefore$ the distance after it stop $= \dfrac{v_0Rcm}{B^2b^2}$

c) Clearly energy is not conserved, but the amount of energy dissipated through ~~energy~~ the ~~energy~~ resistor = the initial kinetic energy of the rod.

Q6) We know $\Phi = -\vec{B}\cdot\vec{A}N$ as $\vec{B}$ is constant spatially & $\vec{B} \parallel \vec{A} \Rightarrow \Phi = BA$

$$\Rightarrow \mathcal{E} = -\frac{A}{c}\frac{dB}{dt}N.$$

Now $I = \frac{\mathcal{E}}{RC} = -\frac{A}{CR}\frac{dB}{dt}N \Rightarrow \frac{dQ}{dt} = -\frac{A}{CR}\frac{dBN}{dt} \Rightarrow dQ = -\frac{AdBN}{RC}$

$$\Rightarrow Q = -\frac{A}{RC}NB\Big|_{B_0}^{0} = \frac{AB_0N}{RC}$$

The reason rapidity of change does not affect the total charge flow is because the current is proportional to the change in field, which is not the case for circuit with both L & C.

Q7) We know $\mathcal{E} = -\frac{A}{c}\frac{dB}{dt}$

$\therefore$ the electric field that the wire feels $= \frac{\mathcal{E}}{2\pi a} = E$ $\left\{\begin{array}{l}\text{as length of loop of} \\ \text{radius } a = 2\pi a\end{array}\right\}$

$$\therefore E = -\frac{\pi a^2}{2\pi a c}\frac{dB}{dt} = -\frac{a}{2c}\frac{dB}{dt}.$$

The force the loop feels $= qE = -\frac{qa}{2c}\frac{dB}{dt}$

$\therefore$ the torque $= |\vec{r}\times\vec{F}| = aqE = -\frac{qa^2}{2c}\frac{dB}{dt} = \frac{dL}{dt}.$

$$\Rightarrow dL = \frac{-qa^2}{2c}dB \qquad \Rightarrow L = \frac{qa^2}{2c}B_0$$

Now as $I = ma^2 \Rightarrow m\omega a^2 = \frac{qa^2B_0}{2c} \Rightarrow \omega = \frac{qB_0}{2mc}$