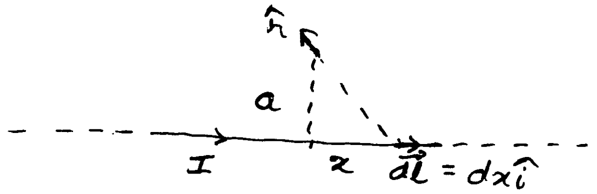
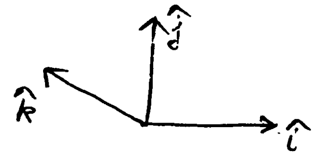
Physics 142 Homework 5 Solutions

Q1)





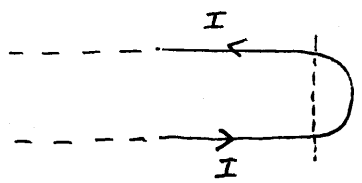
Biot Savart Law $\Rightarrow d\vec{B} = \dfrac{I}{c}\dfrac{\vec{dl}\times\hat{r}}{r^2} = \dfrac{I}{c}\dfrac{dx\sin\theta}{r^2}\hat{k}$

As $r = \sqrt{x^2+a^2}$ & $\sin\theta = \dfrac{a}{\sqrt{x^2+a^2}}$ $\Rightarrow d\vec{B} = \dfrac{I}{c}\dfrac{a\,dx}{(x^2+a^2)^{3/2}}\hat{k}$

$\therefore \vec{B} = \dfrac{I}{c}\hat{k}\displaystyle\int_{-\infty}^{\infty}\dfrac{a\,dx}{(x^2+a^2)^{3/2}} = \dfrac{I}{c}\hat{k}\int_{-\pi/2}^{\pi/2}\dfrac{a^2\sec^2\theta\,d\theta}{a^3\sec^3\theta}$ where $x = a\tan\theta$

$= \dfrac{2I}{ca}\hat{k}$

Q2)

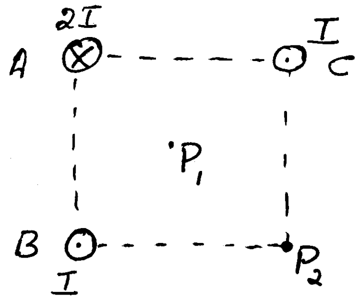


Since we have two semi-infinite wires we have only half the magnetic field due to one them. So the total contribution due to the semi-infinite arms $= \dfrac{2I}{ca}\hat{k}$

Now as we have half a loop the field is $= \dfrac{\pi I}{ca}\hat{k}$

$\therefore$ total field $= \dfrac{I}{ca}(2+\pi)\hat{k}$

(3)

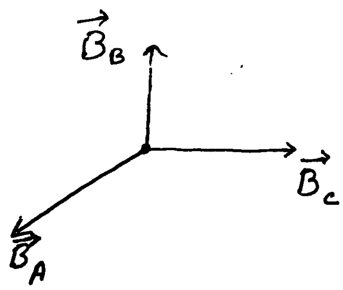


~~Field~~ At P_1 the field due B has the same magnitude as C but as they are on opposite sides of P_1 they point in opposite directions. So the field at P_1 is purely due to A

$$= -\frac{4I\sqrt{2}}{cd}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{-4I}{cd}\left(\hat{i}+\hat{j}\right)$$

as the field at P_1 points at an angle 45° to the -ve x-axis.

Now at P_2 we have



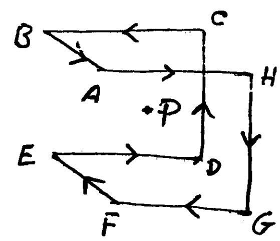
the field due to B+C points at 45° + the ~~mag~~ magnitude is exactly the same as

$$A = \frac{2I}{c}\cdot\frac{\sqrt{2}}{d} = \frac{2\sqrt{2}\,I}{cd}.$$

However at P_2 the field due to A points in the exact opposite direction to resultant field due to B+C.

Q4) The field on a bisector of wire segment of length d is exactly the same as in Q1 except the limits of the integral go from $(-\infty, \infty)$ to $(-d/2, d/2)$. So at distance a from the wire

$$\Rightarrow \vec{B} = \frac{2I}{ca} \sin\theta_0 \hat{k} \quad \text{where } \sin\theta_0 = \frac{a}{\sqrt{a^2 + d^2/4}}$$



$$\Rightarrow \vec{B} = \frac{2I}{c\sqrt{a^2 + d^2/4}} \hat{k}.$$

Now if $a = d/\sqrt{2}$ as the point P is at the center of the cube.

$$\Rightarrow \vec{B} = \frac{4I}{\sqrt{3}cd} \hat{k}.$$ So the magnitude of the field at P due to any of the segments that make up the loop $= \frac{4I}{\sqrt{3}cd}$.

Now the components due to the different segments AH, BC, FG, ED all cancel each other. So the only components that do not cancel completely. The verticle components of AB + EF cancel. while there is a similar cancellation between CD + GH components perpendicular to the page. So there a total of 4 horizontal components that do not cancel $\therefore$ resultant field at $P = \frac{4I}{\sqrt{3}cd}(4\cos 45°)\hat{j}$

$$= 8\sqrt{\frac{2}{3}} \frac{I}{cd} \hat{j}$$

b) By similar arguements as in a) the only components that survive are again the horizontal ones which again give the same field at P.

b) Now from the loop we have

field due to HG $= 2\sqrt{\frac{2}{3}}\,\frac{I}{cd}(-\hat{i}+\hat{j})$

field due to CD $= 2\sqrt{\frac{2}{3}}\,\frac{I}{cd}(\hat{i}+\hat{j})$

field due to CH $= 2\sqrt{\frac{2}{3}}\,\frac{I}{cd}(\hat{j}-\hat{k})$

field due to GD $= 2\sqrt{\frac{2}{3}}\,\frac{I}{cd}(\hat{j}+\hat{k})$

$\therefore$ total field $= 8\sqrt{\frac{2}{3}}\,\frac{I}{cd}\,\hat{j}$

(Q5) Using ampere's law we have $\oint \vec{B}\cdot d\vec{l} = \frac{4\pi}{c} I_{enc}$

The total current flowing through the wire $= I_0$

and the area of the wire $= \pi R^2 - \pi \frac{R^2}{4} = \frac{3}{4}\pi R^2$

$\therefore$ the current density $= \frac{4 I_0}{3\pi R^2} = J$

Now we know that by the principle of superposition we can make hollowed out wire from two wires of radius R & $R/2$ with the current flowing in the opposite direction in the $R/2$ wire.

$\therefore$ Magnitude of the field at P is $= B = \dfrac{\frac{2\cancel{4\pi}}{c} J \frac{\pi R^2}{4}}{2\cancel{\pi} R} = \dfrac{\frac{2}{c}\left(\frac{\cancel{4} I_0}{3\cancel{\pi R^2}}\right)\frac{\cancel{\pi R^2}}{\cancel{4}}}{R}$

$= \dfrac{2 I_0}{3cR}$

The direction of the ~~magnetic~~ magnetic field is counter clockwise

Q6) We know $d\vec{F} = \frac{I}{c}\, d\vec{l} \times \vec{B}$ & $d\vec{\Gamma} = \vec{r} \times d\vec{F}$

$$\Rightarrow d\vec{\Gamma} = \frac{I}{c}\, \vec{r} \times (d\vec{l} \times \vec{B})$$

$$= \frac{I}{c}\{(\vec{r}\cdot\vec{B})\, d\vec{l} - (\vec{r}\cdot d\vec{l})\,\vec{B}\}$$

As we are in the x-y plane $d\vec{l} = dx\left(\hat{i} + \frac{\partial y}{\partial x}\hat{j}\right)$ where $y(x)$ is parametrization of the equation of curve of the loop.

& $\vec{r} = x\hat{i} + y(x)\hat{j}$

As $\vec{B}$ is the y-z plane $\vec{B} = B_y\hat{j} + B_z\hat{k}$

$$\therefore d\vec{\Gamma} = \frac{I}{c}\left\{ y B_y\, dx\left(\hat{i} + \frac{\partial y}{\partial x}\hat{j}\right) - \left(x\, dx + y\frac{\partial y}{\partial x}dx\right)(B_y\hat{j} + B_z\hat{k})\right\}$$

$$= \frac{I}{c}\left\{ y B_y\, dx\,\hat{i} - B_y\, x\, dx\,\hat{j} - B_z\left(x + y\frac{\partial y}{\partial x}\right)dx\,\hat{k}\right\}$$

$$\Rightarrow \vec{\Gamma} = \frac{I}{c}\left\{ B_y \int_c y\, dx\,\hat{i} - B_y\int_c x\, dx\,\hat{j} - B_z\left(\int_c x\, dx + \int_c y\, dy\right)\hat{k}\right\}$$

$$= \frac{I}{c} B_y\, a\,\hat{i}$$ {as on the closed curve C each of the integrals $= 0$ & $\int_c y\, dx = A$}

$$\Rightarrow \vec{\Gamma} = \left(\frac{Ia}{c}\right) B_y\,\hat{i}$$

Now $\vec{m} = \frac{-Ia}{c}\hat{k}$

$$\therefore \vec{m}\times\vec{B} = -\frac{Ia}{c}(\hat{k}\times\hat{j}) = \frac{Ia}{c}\hat{i}$$

$$\therefore \vec{\Gamma} = \vec{m}\times\vec{B}$$

Q1) $\vec{F}_1 = (x+y)\hat{i} + (-x+y)\hat{j} - 2z\hat{k}$

a)

$$\Rightarrow \vec{\nabla}\cdot\vec{F}_1 = 1+1-2 = 0$$

$$\Rightarrow \vec{\nabla}\times\vec{F}_1 = \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & -x+y & -2z \end{pmatrix} = -2\hat{k}$$

b) $\vec{G}_1 = 2y\hat{i} + (2x+3z)\hat{j} + 3y\hat{k}$

$$\Rightarrow \vec{\nabla}\cdot\vec{G}_1 = 0$$

$$\vec{\nabla}\times\vec{G}_1 = \mathrm{Det}\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & 2x+3z & 3y \end{pmatrix} = 0\hat{i} - \hat{j}0 + \hat{k}0 = 0$$

Now $\vec{G}_1 = \vec{\nabla}\phi_1 \quad \Rightarrow \frac{\partial\phi_1}{\partial x} = 2y \quad \frac{\partial\phi_1}{\partial y} = 2x+3z \quad \frac{\partial\phi}{\partial z} = 3y$

$$\Rightarrow \phi_1 = 2xy + 3yz$$

c) $\vec{H} = (x^2 - z^2)\hat{i} + 2\hat{j} + 2xz\hat{k}$

$$\Rightarrow \vec{\nabla}\cdot\vec{H} = 2x + 2x = 4x$$

$$\vec{\nabla}\times\vec{H} = \mathrm{Det}\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2-z^2 & 2 & 2xz \end{pmatrix} = 0\hat{i} - \hat{j}4z + \hat{k}0 = -4z\hat{j}$$

Q8) The ~~part~~ electron will move in a circular path as $\vec{F} = -\frac{e}{c}\vec{v}\times\vec{B}$ and so $\vec{F} \perp \vec{v}$ at any point in its motion in the field.

$\therefore$ as $\vec{v}$ points initial in the +ve x-direction & $\vec{B}$ is out of the page so $\vec{v}\times\vec{B}$ points downwards $\therefore$ $\vec{F}$ points upwards. So the displacement of the electron after it has passed through the region of magnetic field $= 2R$ where R is the radius of its circular path. Now as $\vec{F}$ is a centripetal force $\frac{mv^2}{R} = \frac{e}{c}vB$

$$\Rightarrow R = \frac{mvc}{eB}$$