PHYSICS 142 HOMEWORK 4 SOLUTIONS

Q1) The force on a parallel plate capacitor $= \frac{1}{2} Q^2 \frac{d}{dx}\frac{1}{C}$ where

$C = \frac{A}{4\pi x}$ $\Rightarrow$ $F = \frac{1}{2} Q^2 \frac{4\pi}{A}$. Now we also know that the capacitor was charged ~~to~~ by a battery of voltage V $\Rightarrow$ $Q = C_0 V$ where C_0 is the initial capacitance where the separation between plates $= d$.

$\Rightarrow Q = \frac{AV}{4\pi d}$ $\Rightarrow$ $F = \frac{2\pi}{A}\left(\frac{AV}{4\pi d}\right)^2 = \frac{AV^2}{8\pi d^2} \simeq 1.7 \times 10^2$ dynes

$\therefore F =$

Now the work done externally $= \vec{F}\cdot\vec{d} = \frac{AV^2}{8\pi d} = 1.7 \times 10^3$ ergs

Finally the electric field inside the plates $= E = 4\pi\sigma = \frac{4\pi Q}{A} = \frac{V}{d}$

$\therefore$ energy stored $= \frac{1}{(8\pi)} E^2 A d = \frac{1}{8\pi}\frac{V^2}{d^2} A d = \frac{1}{8\pi}\frac{V^2 A}{d}$

So the electric field stores the same amount of energy as the external work done if the capacitor was to collapse on itself.

Q2) We have from Ohms law $V = IR$

Now if ρ = resistivity of material, l is length of wire & ~~A = dee~~

A = area of cross-section of the wire $\Rightarrow R = \frac{\rho l}{A}$

If there are n electrons per cubic centimeter & the drift velocity $= v_d$.

$\Rightarrow J = n e v_d$ is the current density.

$\therefore$ current $I = JA = n e v_d A$

$\therefore V = n e v_d A \left(\frac{\rho l}{A}\right) = n e v_d \rho l \Rightarrow v_d = \cancel{\frac{V}{ne\rho l}} = \frac{V}{n e \rho l}$

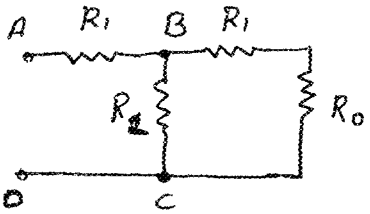
$V = 6\,V = 2 \times 10^{-2}$ Stat volt

$n = 8 \times 10^{22}$ electron/cm^3 $\quad e = 4.8 \times 10^{-10}$ Stat Coul

$\rho = 1.7 \times 10^{-6}\ \Omega$ cm

$l = 10^5$ cm $\quad \Rightarrow v_d = 2.5 \times 10^{-15}$ cm/s

$\therefore$ time it takes to go a distance of 1 km $= \frac{10^5}{2.5 \times 10^{-15}} = 4 \times 10^{20}$ s

Q3) 

The equivalent resistance between BC $= \frac{(R_1 + R_0) R_1}{2R_1 + R_0}$

$\therefore$ the total resistance $= R_1 + \frac{(R_1 + R_0) R_1}{(2R_1 + R_0)} = R_0$

$\cancel{R_1(R_1+R_0) R_1 + R_1^2 + R_0 R_1 = 2R_1 R_0 + R_0^2}$ $\Rightarrow 2R_1^2 + \cancel{2R_1R_0} + R_1^2 = \cancel{2R_1R_0} + R_0^2$

$\cancel{\Rightarrow 2R_1^2 + R}$

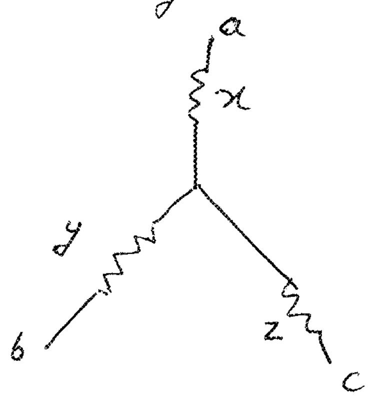
$\Rightarrow R_0^2 = 3R_1^2 \Rightarrow R_1 = (\sqrt{3})^{-1} R_0 = R_0/\sqrt{3}$

P4) We have 3 terminals a, b, c & resistors x, y, z.

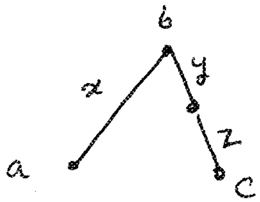
Case analysis:

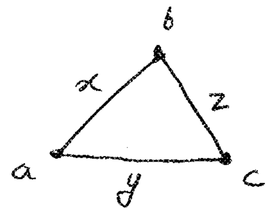
1) If one terminal has three wire connected to it there are no remaining wires to create a potential drop across the other two terminals.

2) If all have only one wire connected to ~~the~~ each terminal then the only possibility is that they all meet at a point.

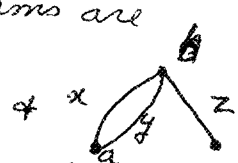


3) The only remaining case is when there are two wires connected to atleast one terminal. So possible diagrams are



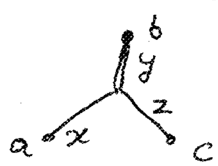


&



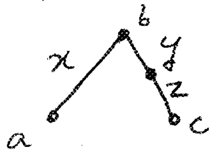
where the first & third diagram is possible with a permutation of a, b, c.

Now if we consider the first possible case.



$$\Rightarrow R_{ab} = x + y = 30$$
$$R_{ac} = x + z = 60$$
$$\Rightarrow z - y = 30$$
$$R_{bc} = y + z = 70$$
$$\Rightarrow 2z = 100$$
$$\Rightarrow z = 50$$
$$\Rightarrow y = 20$$
$$\Rightarrow x = 10$$

2nd possible diagram.

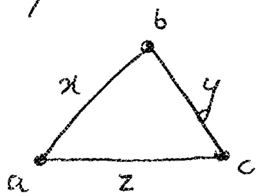


$$R_{ab} = x = 30$$
$$R_{bc} = \cancel{x}\, y+z = 60$$
$$R_{ac} = x+y+z = 70$$

} Impossible as $y+z = 40$ using the first & 3rd equation.

as R_{ac} is the maxim drop the other permutations also does not work.

3rd possible diagram.



$$R_{ab} = \frac{x(y+z)}{x+y+z} = 30$$
$$R_{bc} = \frac{y(x+z)}{x+y+z} = 60$$
$$R_{ac} = \frac{z(x+y)}{x+y+z} = 70$$

$$\Rightarrow \frac{(y-x)z}{(x+y+z)} = 30 \quad \Rightarrow \frac{y-x}{y+x} = \frac{3}{7}$$

$$\Rightarrow 7y - 7x = 3x + 3y$$
$$\Rightarrow 4y = 10x \Rightarrow 2y = 5x$$

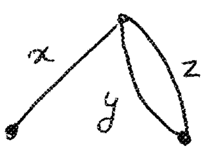
Also we have $\frac{(z-y)x}{x+y+z} = 10 \Rightarrow \frac{z-y}{z+y} = \frac{1}{3} \Rightarrow 3z - 3y = z+y$

$$\Rightarrow 2z = 4y \Rightarrow z = 2y = 5x$$

& $\frac{x(\frac{5}{2}x + 5x)}{x + \frac{5}{2}x + 5x} = 30 \Rightarrow \frac{15x}{17} = \cancel{30}\,2 \Rightarrow x = 34$

$\therefore y = 85$ & $z = 170$

Forth Possible case.

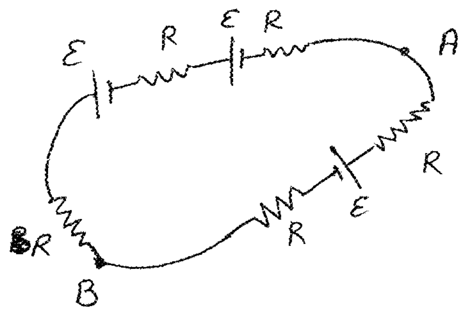


Is not possible as the sum of of any of the potential is not equal to the third one.

So the only possible configurations are the ones listed in the question

There is no way of differentiating between the two diagrams from an external measurement.

Q5) The circuit is



Let i the the current flowing through loop counterclockwise

$\therefore\ i = \frac{3\varepsilon}{5R}$ $\quad\therefore$ potential drop across $AB = 2\varepsilon - 3iR = 2\varepsilon - \frac{9\varepsilon}{5} = \frac{\varepsilon}{5}$

To find the equivalent resistance between A & B we can neglect the batteries and treat the circuit purely as made up of resistors

$$\therefore\ R_{eq} = \frac{1}{\frac{1}{3R} + \frac{1}{2R}} = \frac{6}{5}R$$

$\therefore$ short circuit current $= \varepsilon/5 \cdot \frac{5}{6R} = \frac{\varepsilon}{6R}$

Another way to do this is to calculate the short circuit current first.

So short circuit current is the sum of the current entering & leaving A or B

Current enerty $A = \frac{\varepsilon}{2R}$

Current leaving $A = \frac{2\varepsilon}{3R}$

$\therefore$ net current through $A = \frac{\varepsilon}{2R} - \frac{2\varepsilon}{3R} = -\frac{1\varepsilon}{6R}$

$$\therefore\ R_{eq} = \varepsilon_{eq}/I_{sc} = \frac{6R}{5}$$

So the thevinin equivalent circuit have $\varepsilon_0 = \varepsilon/5$ & $R_0 = 6R/5$

Q6) The charge initially $= Q = CV_0$

$\therefore$ energy stored $= \frac{1}{2}CV_0^2 = \frac{1}{2}\frac{Q^2}{C}$ $\therefore$ energy stored at time $t = \frac{1}{2}CV_0^2 e^{-2t/RC}$

Now we know the current ~~dissipated~~ $I = \frac{V_0}{R} e^{-t/RC}$

$\therefore$ energy dissipate at time $I = \frac{V_0^2}{R} e^{-2t/RC}$

$$\therefore \text{ total energy lost } = \int_0^\infty \frac{V_0^2 e^{-2t/RC}}{R}\,dt = \frac{V_0^2}{R} \frac{e^{-2t/RC}}{(-2/RC)}\Big|_0^\infty = \frac{1}{2}\frac{V_0^2}{\cancel{R}}\cancel{R}C$$
$$= \frac{1}{2}CV_0^2.$$

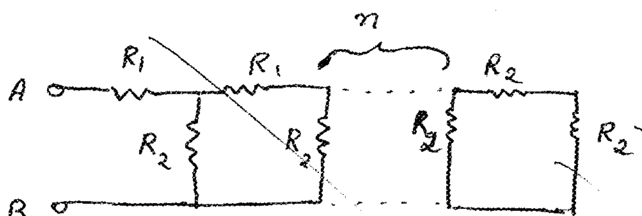
To show energy lost in time T we have

$$E = \int_0^T \frac{V_0^2 e^{-2t/RC}}{R} = \frac{1}{2}V_0^2 C\left(1 - e^{-2T/RC}\right)$$

and using our initial value of energy & energy stored in the capacitor at time T we have energy lost by capacitor

$= \frac{1}{2}CV_0^2 - \frac{1}{2}CV_0^2 e^{-2T/RC}$ the same as that dissipated through the resistor.

Q7)

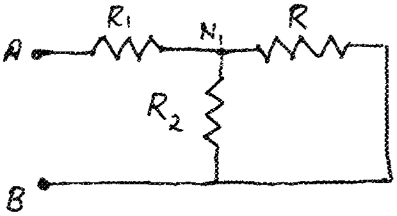


Let i_n be the current entering the nth R_1 resistor. & ~~J_n be the current entering the~~, Therefore the current entering the nth R_2 resistor $= i_n - i_{n+1}$

P7) Let R be the equivalent resistance of the entire infinite chain of resistors.

∴ Now consider stripping of the initial R_1 & R_2 of and treating the rest of the resistors as R. As the chain is infinite removing any finite number of R_1's & R_2's does not change the equivalent resistance of the remaining chain.

So if we treat ~~it~~ the chain as $1R_1$ & $1R_2$ & $1R$ we have the circuit.

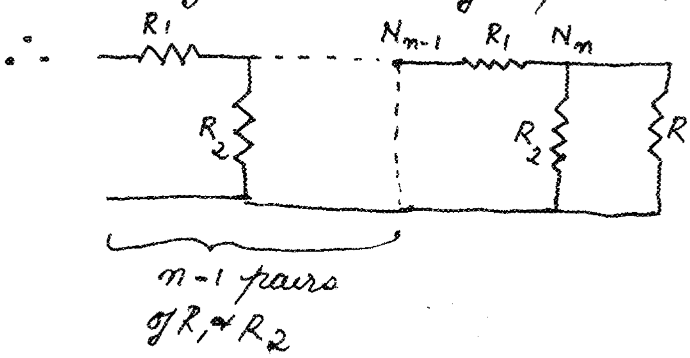


has potential at $N_1 = V_0 - i_1 R_1$ where i_1 is the current through 1st R_1. But as entire circuit has $R_{eq} = R$ $\quad i_1 = \frac{V_0}{R}$

$$\therefore V_1 = V_0 - V_0\left(\frac{R_1}{R}\right) = V_0\left(\frac{R-R_1}{R}\right).$$

Now lets consider n such pairs of R_1 & R_2 treated independently & the remaining chain being replaced by R.

∴



Now using Thevenins theorem we can replace the initial $n-1$ resistors attenuation by the voltage source V_{n-1}

$$\therefore \text{ current through } n\text{th } R_1 = \frac{V_{n-1}}{R_1} = i_n$$

$$\therefore \text{ potential at } N_n = V_{n-1} - i_n R_1 = V_{n-1}\left(1 - \frac{R_1}{R}\right)$$

$$\Rightarrow \quad V_n = V_{n-1}\left(\frac{R-R_1}{R}\right)$$

$$\therefore \text{ we find that } V_n = V_0\left(\frac{R-R_1}{R}\right)^n$$

Now we have also that $R = R_1 + \frac{RR_2}{R+R_2}$ using the first ~~ccct~~ circuit on the previous page $\Rightarrow R^2 \cancel{+RR_2} = RR_1 + R_1R_2 + \cancel{RR_2}$

$$\Rightarrow R^2 - RR_1 - R_1R_2 = 0 \quad \Rightarrow R = \frac{R_1 \pm \sqrt{R_1^2 + 4R_1R_2}}{2}$$

$$R > 0 \quad \Rightarrow R = \frac{R_1 + \sqrt{R_1^2 + 4R_1R_2}}{2}$$

$$\Rightarrow V_n = V_0 \left(\frac{\sqrt{R_1^2 + 4R_1R_2} - R_1}{\sqrt{R_1^2 + 4R_1R_2} + R_1} \right)^n.$$

∴ The potential decrease as an geometric series.

Now if we want the sequence of potentials to go down by half

$$\left(\frac{\sqrt{R_1^2 + 4R_1R_2} - R_1}{\sqrt{R_1^2 + 4R_1R_2} + R_1} \right) = \frac{1}{2} \quad \Rightarrow \sqrt{R_1^2 + 4R_1R_2} = 3R_1$$

$$\Rightarrow R_1^2 + 4R_1R_2 = 9R_1^2 \Rightarrow 4R_1R_2 = 8R_1^2 \Rightarrow R_1 = 0 \text{ or } R_2 = 2R_1$$

Clearly $R_1 = 0$ is not a solution so $R_2 = 2R_1 \Rightarrow R = 2R_1$

From our solution we can create such an attenuator to say the n^{th} power by replacing all the $n+1$ to infinite remain resistors by R.