

Physics 142 Problem Set 1

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1 (1.4) Charges on a square

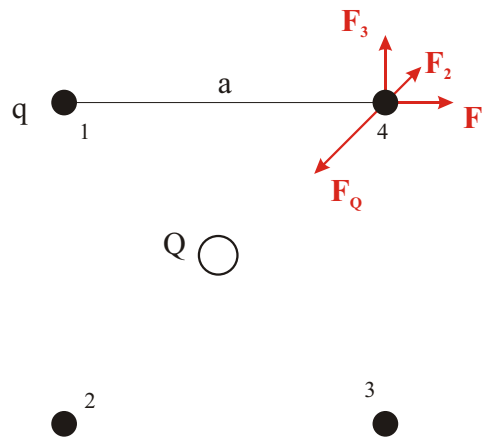


Figure 1: Charges.

To have an equilibrium we need the sum of the forces to be zero:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_Q = 0. \quad (1)$$

Now:

$$\begin{aligned} F_1 &= F_3 = \frac{q^2}{a^2} \\ F_2 &= \frac{q^2}{2a^2} \\ F_Q &= \frac{Qq}{\frac{a^2}{2}}. \end{aligned} \quad (2)$$

Projecting all forces in the \vec{F}_1 direction the equilibrium of the forces becomes:

$$\begin{aligned} 2\frac{q^2}{a^2} \cos \frac{\pi}{4} + \frac{q^2}{2a^2} &= \frac{Qq}{\frac{a^2}{2}} \\ \therefore Q &= \left(\frac{1}{4} + \frac{1}{\sqrt{2}} \right) q. \end{aligned} \quad (3)$$

The equilibrium will not be stable (which is true for any electrostatic system). To see that one can calculate the total force for small displacements from the equilibrium and observe that it points away from it.

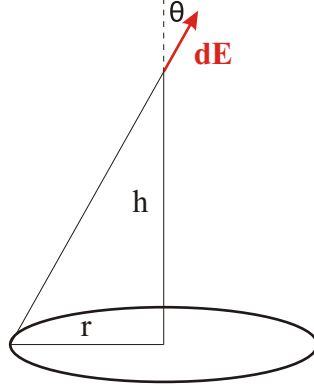


Figure 2: Ring charge.

2 (1.14) Ring charge

The electric field of a small piece dl of the wire $d\vec{E}$ is:

$$dE = \frac{dq}{h^2 + r^2} = \frac{\lambda r d\theta}{h^2 + r^2} = \frac{Q d\theta}{2\pi(h^2 + r^2)}, \quad (4)$$

where $\lambda = \frac{Q}{2\pi r}$ is the linear charge density of the wire.

When integrated over the whole circle the perpendicular components of the electric field of the opposite pieces of wire cancel out, so we need to take into account only the parallel to the axis component of $d\vec{E}$. Since $d\vec{E}$ (and also its parallel component) doesn't depend on which piece of wire you consider, integrating over the wire will just multiply by 2. Combining all together we have:

$$E = 2\pi dE \cos \theta = \frac{Qh}{(h^2 + r^2)^{\frac{3}{2}}}. \quad (5)$$

To find the h which gives the maximum value of E we take the derivative and equate it to zero:

$$\begin{aligned} \frac{dE}{dh} &= \frac{Q}{(h^2 + r^2)^{\frac{3}{2}}} (r^2 - 2h^2) = 0 \\ \therefore h &= \frac{r}{\sqrt{2}}. \end{aligned} \quad (6)$$

3 (1.16) Carving a sphere

To deal with the hole without committing a suicide, we consider the original sphere without any holes and imagine putting a negative charged sphere of the size of the hole where the hole is supposed to be. Electrostatically, i.e. in terms of charges, the two are the same, so we can proceed with it instead of our original problem.

At point A the big sphere has no effect, while the electric field of the second sphere is:

$$\frac{\rho \frac{4}{3}\pi \left(\frac{a}{2}\right)^3}{\left(\frac{a}{2}\right)^2} = \frac{2}{3}\pi\rho a, \quad (7)$$

pointing upward.

At point B both of the spheres contribute to the electric field and we have:

$$E_{big} - E_{small} = \frac{\rho \frac{4}{3}\pi a^3}{a^2} - \frac{\rho \frac{4}{3}\pi \left(\frac{a}{2}\right)^3}{\left(\frac{3a}{2}\right)^2} = \pi\rho a \left(\frac{4}{3} - \frac{2}{27}\right) = \frac{34}{27}\pi\rho a, \quad (8)$$

pointing downward.

4 (1.17) Flux through a cube

(a) By symmetry the flux through one face is equal to the total flux through the cube divided by the number of faces:

$$\text{flux through one face} = \frac{\text{total flux}}{6} = \frac{4\pi q}{6} = \frac{2}{3}\pi q. \quad (9)$$

(b) Solving the problem by directly calculating the flux seems hard, so we again try to use some symmetry arguments to simplify it. First note that the flux through adjacent to the charge edges is zero, since the electric field is parallel to the surface, and thus doesn't contribute to the flux integral. Now to calculate the flux through the other three faces (which is again the same through each face by symmetry), we imagine putting seven more cubes around the point charge and forming a large cube around it.

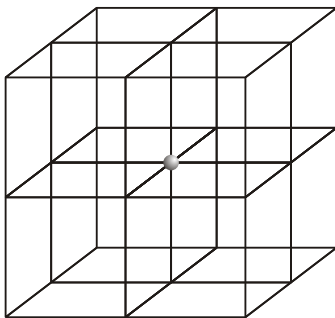


Figure 3: Constructing a large cube around the point charge.

Then the flux through a face is:

$$\text{flux through a face} = \frac{\text{total flux through the large cube}}{8 * 3} = \frac{1}{6}\pi q. \quad (10)$$

5 (1.19) A plane and a layer

The electric field of the infinite plane is:

$$E_{\text{plane}} = 2\pi\sigma, \quad (11)$$

outward.

Now for the layer of charge of density ρ , we have by Gauss's law, using same type of surface as in Fig. 1.23,

$$\begin{aligned} \text{if } r \geq \frac{d}{2}, \quad 2AE &= 4\pi\rho dA \\ \text{if } r < \frac{d}{2}, \quad 2AE &= 4\pi\rho 2rA, \end{aligned} \quad (12)$$

where r is taken to be zero at the center of the layer. Thus:

$$\begin{aligned} \text{if } r \geq \frac{d}{2}, \quad E &= 2\pi\rho d \\ \text{if } r < \frac{d}{2}, \quad E &= 4\pi\rho r. \end{aligned} \quad (13)$$

Now adding the electric fields and also shifting the coordinates so that zero of r is at the infinite plane, we get:

$$\begin{aligned} \text{if } r < 0, \quad \vec{E} &= (-2\pi\sigma - 2\pi\rho d)\hat{i} \\ \text{if } 0 \leq r \leq d, \quad \vec{E} &= \left(2\pi\sigma - 4\pi\rho\left(\frac{d}{2} - r\right)\right)\hat{i} \\ \text{if } r > d, \quad \vec{E} &= (2\pi\sigma + 2\pi\rho d)\hat{i} \end{aligned} \quad (14)$$

where \hat{i} is pointing to the right on the picture in book.

6 (1.26) Nice “coincidence” in a wire system

Since the system is symmetrical w.r.t. horizontal reflection to show that the electric field is zero at C, we need to show that the sum of the electrical fields of point A and B is horizontal. Thus we need

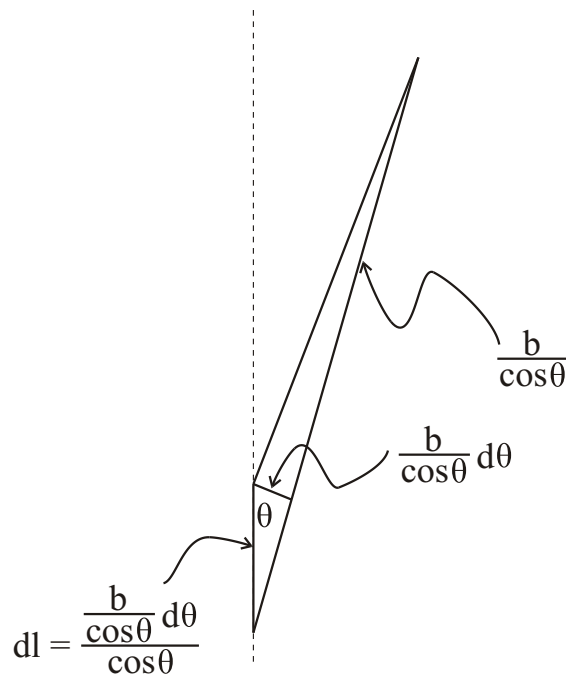
$$\begin{aligned} E_A \sin \theta &= E_B \sin \theta \\ \Leftrightarrow E_A &= E_B. \end{aligned} \tag{15}$$

The first one is easy to calculate:

$$E_A = \frac{\lambda b d\theta}{b^2} = \frac{\lambda d\theta}{b}. \tag{16}$$

We calculate E_B using the figure:

$$E_B = \frac{\lambda dl}{\left(\frac{b}{\cos \theta}\right)^2} = \frac{\lambda d\theta}{b}. \square \tag{17}$$



7 (1.30) Spherical capacitor

$$U = \frac{1}{8\pi} \int E^2 dV = \frac{1}{8\pi} \int_a^b \frac{Q^2}{r^4} 4\pi r^2 dr = \frac{1}{2} Q^2 \int_a^b \frac{dr}{r^2} = \frac{Q^2}{2} \left(\frac{1}{a} - \frac{1}{b} \right), \tag{18}$$

where we used that the electric field is non-zero only in between the two spherical shells (by Gauss’s law), and is equal to $\vec{E} = \frac{Q}{r^2} \hat{r}$.