ATLAS NOTE

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Muon momentum scale from Z bosons

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Abstract

This note describes a data-driven method for calculating muon momentum scale factors for positive and negative muons using events with Z bosons. Muon momentum scale factors are reported for the 2010 ATLAS data.

Contents

1 Introduction

Residual misalignments in the Inner Detector (ID) and the Muon Spectrometer (MS), as well as magnetic field effects, can result in noticeable muon momentum deviations from Monte-Carlo simulation. Charge-dependent momentum scales are particularly relevant for the W charge asymmetry and ratio measurements, where muon momentum scale is an important systematic. Good understanding of MS scale is also important in various searches involving high-p_T muons, where the ID measurement contributes little to the reconstruction of a combined muon.

We define the scale factors directly on the muon *transverse momentum*, as opposed to its inverse. For brevity, we will refer to this quantity in text as simply "momentum", even though we always mean "transverse momentum".

$$\begin{cases} \left.\frac{1}{p_T}\right|_{\mu_+,\mathrm{data}} = \frac{1}{k_+}\left.\frac{1}{p_T}\right|_{\mu_+,\mathrm{mc}} \\ \left.\frac{1}{p_T}\right|_{\mu_-,\mathrm{data}} = \frac{1}{k_-}\left.\frac{1}{p_T}\right|_{\mu_-,\mathrm{mc}} \end{cases} \quad (1)$$

It is possible to arrive at charge-separated momentum scale factors by comparing the momenta of muons from W^+ and W^- decays in data with those in Monte-Carlo simulation. While W bosons are substantially more abundant than Z bosons, their daughters decay with non-identical momentum distributions that depend on the charge of the boson - a feature of a proton-proton collider. Therefore, a method that relies on W bosons to derive muon momentum scale factors suffers from potential physics-related bias through Monte-Carlo modeling uncertainty (in particular, parton distribution functions). As an alternative, we develop a fully data-driven technique using Z events.

2 Procedure

By constraining the invariant mass of muons in Z candidate events to the expected Z mass, we can effectively determine the product of k_+ and k_- scale factors. Given a perfectly-aligned detector in Monte-Carlo simulation, the invariant mass scales as:

$$M_Z|_{\mathrm{data}} = M_Z|_{\mathrm{mc}} \cdot \sqrt{k_+ \cdot k_-} \quad (2)$$

Note that another equation is needed to constrain k_+ and k_- individually. It can be obtained by balancing the momenta of positive and negative muons from Z against each other.

In particular, we used Pythia $Z \to \mu\mu$ Monte-Carlo to verify that momentum spectra for tight positive and negative muons are statistically identical in the events passing standard Z selection.

$$\left.\frac{1}{p_T}\right|_{\mu_+,\mathrm{mc}} = \left.\frac{1}{p_T}\right|_{\mu_-,\mathrm{mc}} \quad (3)$$

By comparing the shapes of inverse p_T spectra of $Z \to \mu\mu$ muons in data, we can attribute any observed differences to detector and magnetic field effects.

The following procedure is used to obtain the ratio of k_+ to k_- scale factors, which we call the relative scale R_0:

- Build a smooth $1/p_{T_{\mu+}}$ kernel PDF template for positive muons (Figure 1).
- Overlay the $1/p_{T_{\mu-}}$ spectrum on the PDF. Due to misalignments, it may not match well and will potentially result in a bad χ^2.
- Instead, overlay $1/(R \cdot p_{T_{\mu-}})$ spectrum for many different values of "relative" scale R (Figure 2)
- Given a plot of χ^2 vs R, fit a parabola to find $R_0 \pm \delta R_0$ at which μ_+ and μ_- spectra are compatible.

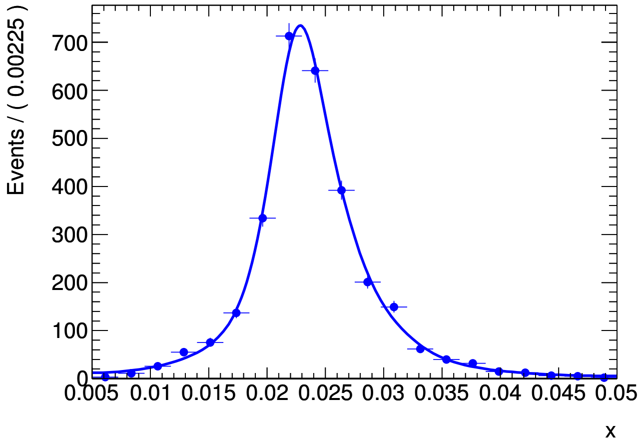

Figure 1: $1/p_T$ for μ_+ and its derived smooth PDF

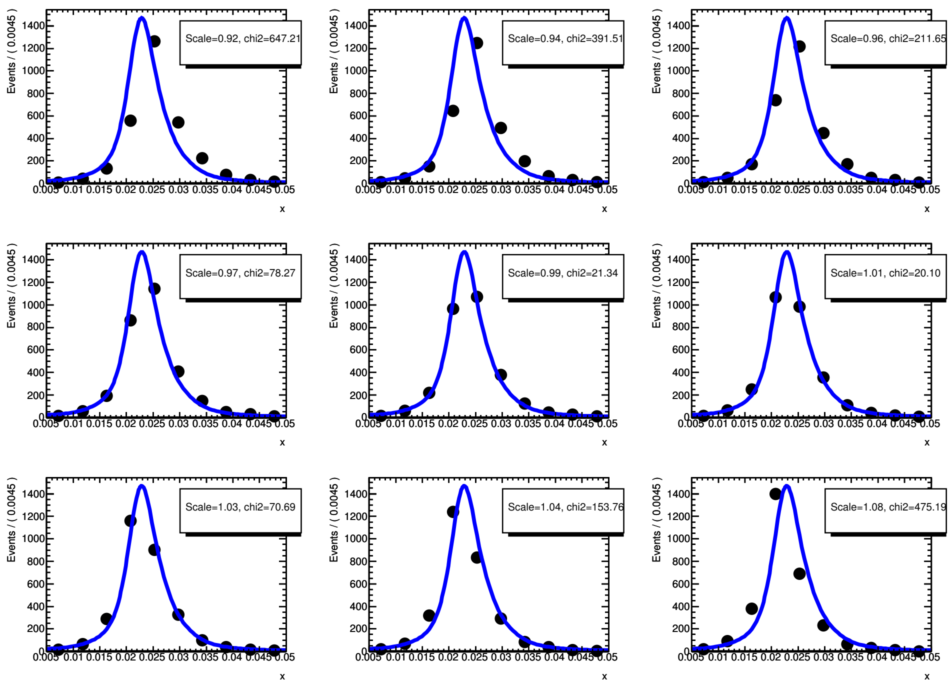

Figure 2: Scanning through several values of the relative scale for a fixed μ_+ $1/p_T$. Only nine values are shown, but 500 bins were used in the full scan.

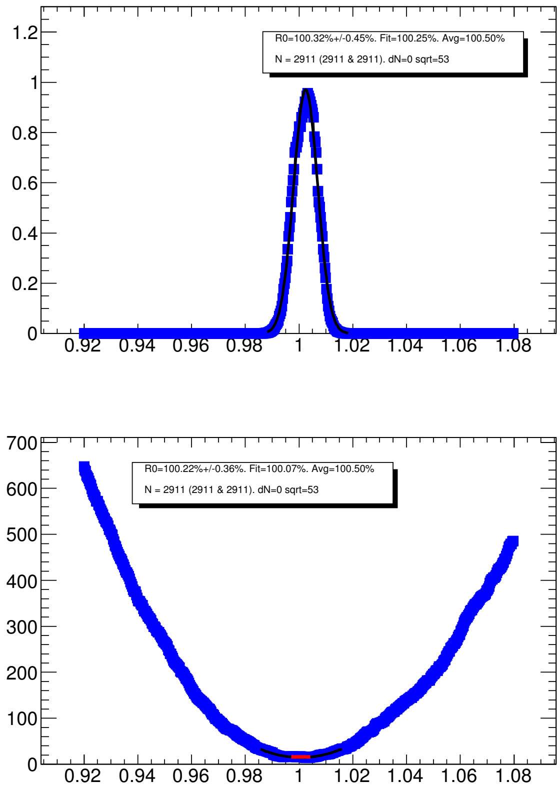

Figure 3: KS (above) and χ^2 (below) fit for the relative scale R_0 for Barrel-only combined muons

As an alternative, we also perform an unbinned Kolmogorov-Smirnov (KS) test that evaluates the compatibiliy of $1/(R \cdot p_T)$ spectra for positive and negative muons for a range of R values. The best value, R_0, is found when the KS probability is maximized. With large statistics, the KS and χ^2 methods yield identical results. The KS method is more robust with smaller statistics as far as the central value is concerned but doesn't provide a simple way to extract the error on the fitted parameter.

The χ^2-based method requires binning the spectrum in order to calculate the χ^2, which results in a slight loss of statistical power and introduces a systematic due to the choice of the bin size. More importantly, there is substantial freedom in the choice of the parameters used to create a kernel PDF. We investigated adaptive and non-adaptive kernels and several values of the "bandwidth" parameter (smoothing factor) and concluded that the precise location of the χ^2 minimum is somewhat sensitive to different settings but the curvature of the fitted parabola (and thus the estimate of the error on the fitted parameter) remains stable.

Given different relative strength of the two methods, we obtain the central value of R_0 from the KS method and the error δR_0 from the χ^2 method. Figure 3 illustrates both methods when applied to the same set of Z events.

2.1 Mathematical details

By definition of R_0, we have:

$$\frac{1}{p_T}\bigg|_{\mu_+,\text{data}} = \frac{1}{R_0} \cdot \frac{1}{p_T}\bigg|_{\mu_-,\text{data}} \tag{4}$$

Using the definition of k_+ and k_- scale factors, this becomes:

$$\frac{1}{k_+}\frac{1}{p_T}\Big|_{\mu_+,\mathrm{mc}} = \frac{1}{R_0}\cdot\frac{1}{k_-}\frac{1}{p_T}\Big|_{\mu_-,\mathrm{mc}} \tag{5}$$

Exploiting the fact that the spectra for $\mu+$ and $\mu-$ are identical in Monte-Carlo, we finally obtain:

$$\frac{1}{k_+} = \frac{1}{R_0}\cdot\frac{1}{k_-} \tag{6}$$

Together with the previously described constraint on Z mass, we have the following system of equations:

$$\begin{cases} k_+ = R_0 \cdot k_- \\ M_Z|_{\mathrm{data}} = M_Z|_{\mathrm{mc}} \cdot \sqrt{k_+ \cdot k_-} \end{cases} \tag{7}$$

This system can be solved for k_+ and k_-:

$$\begin{cases} k_+ = \frac{M_Z|_{\mathrm{data}} \cdot \sqrt{R_0}}{M_Z|_{\mathrm{mc}}} \\ k_- = \frac{M_Z|_{\mathrm{data}}}{M_Z|_{\mathrm{mc}} \cdot \sqrt{R_0}} \end{cases} \tag{8}$$

Standard formula for error propagation yields the uncertainty on absolute scale factors. Since Monte-Carlo simulation provides plenty of Z events to perform a robust mass constraint, the error on $M_Z|_{\mathrm{mc}}$ was neglected.

$$\begin{cases} \delta^2 k_+ = \delta^2(M_Z|_{\mathrm{data}}) \cdot \left(\frac{\sqrt{R_0}}{M_Z|_{\mathrm{mc}}}\right) + \delta^2(R_0) \cdot (\frac{M_Z|_{\mathrm{data}}}{M_Z|_{\mathrm{mc}}})^2 \cdot \frac{1}{4R_0} \\ \delta^2 k_- = \delta^2(M_Z|_{\mathrm{data}}) \cdot \left(\frac{1}{M_Z|_{\mathrm{mc}} \cdot \sqrt{R_0}}\right) + \delta^2(R_0) \cdot (\frac{M_Z|_{\mathrm{data}}}{M_Z|_{\mathrm{mc}}})^2 \cdot \frac{1}{4R_0^3} \end{cases} \tag{9}$$

3 Results for 2010 data

We report results for 2010 "fall reprocessing" reconstruction. Due to limited statistics in the 2010 dataset, the scale factors are calculated for only three η bins: $|\eta| < 1.05$ (Barrel), $-2.4 < \eta < -1.05$ (Endcap-C), and $1.05 < \eta < 2.4$ (Endcap-A).

3.1 Relative scale R_0

Table 1 shows the relative scale R_0 for Combined, MS, and ID muons.

3.2 Absolute scale factors k_+ and k_-

Table 2 shows the absolute scale factors k_+ and k_- for combined, MS, and ID muons.

4 Systematic effects

The following systematic effects are associated with the procedure that determines the relative scale R_0:

- **Choice of Z sample** (Table 4). Given a particular η range for which we want to compute the relative scale, we have two ways of choosing a subset of approprtiate Z events. We can require that both Z legs fall in the fiducial η region of the detector, or we can make no such requirement and include events where the leg with the opposite charge falls outside of the fiducial η region.

Region	R_0	δR_0
Combined muons		
Endcap-A	100.1%	0.5%
Barrel	100.3%	0.4%
Endcap-C	101.7%	0.5%
MS muons		
Endcap-A	101.1%	0.6%
Barrel	100.4%	0.4%
Endcap-C	98.0%	0.6%
ID muons		
Endcap-A	99.9%	0.4%
Barrel	100.2%	0.4%
Endcap-C	102.7%	0.6%

Table 1: Relative scales in 2010 data

Region	k_+	k_-
Combined muons		
Endcap-A	100.0 ± 0.2%	100.0 ± 0.2%
Barrel	100.3 ± 0.2%	100.0 ± 0.2%
Endcap-C	100.8 ± 0.3%	99.1 ± 0.3%
MS muons		
Endcap-A	100.6 ± 0.3%	99.6 ± 0.3%
Barrel	100.6 ± 0.2%	100.3 ± 0.2%
Endcap-C	99.2 ± 0.3%	101.3 ± 0.3%
ID muons		
Endcap-A	99.5 ± 0.2%	99.6 ± 0.2%
Barrel	100.1 ± 0.2%	99.9 ± 0.2%
Endcap-C	101.5 ± 0.3%	98.9 ± 0.3%

Table 2: Absolute scale factors in 2010 data. Only statistical uncertainty is shown.

- **Background contamination in the data** (Table 5). The window around nominal Z mass was reduced from 70..110 GeV to 80..100 GeV, which changed the composition of the Z sample used to determine R_0.
- χ^2 **versus KS method** (Table 6). As described earlier, we employ two alternative techniques to arrive at the relative scale R_0. The difference between these two methods is taken as a systematic uncertainty.

The following systematic effect is associated with the invariant mass constraint and happens after R_0 has already been determined:

- **Z fit function** (Table 7). Instead of using a Gaussian fit in the core, we also performed a full Z lineshape fit with a Crystal-Ball resolution function. Because the fit in the data is normalized to the corresponding fit in the Monte-Carlo, any systematic shifts introduced by either method are cancelled in the ratio.

	Combined			MS only			ID only		
	EA	B	EC	EA	B	EC	EA	B	EC
Statistical Uncertainties									
R_0 determination (Anton: update)	0.2	0.2	0.2	0.3	0.2	0.3	0.2	0.2	0.3
Z mass (Anton: update)	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0
Systematic Uncertainties									
Expanded sample	0.0	0.4	0.1	0.0	0.4	0.8	0.1	0.3	0.1
Mass window for Z selection	0.0	0.0	0.1	0.2	0.0	0.4	0.1	0.0	0.1
Shape comparison method	0.0	0.1	0.4	0.1	0.1	0.2	0.1	0.2	0.2
Z mass fit function	0.2	0.3	0.1	0.3	0.6	0.0	0.3	0.3	0.1
Summary									
Correlated (constant k_+/k_-)	0.2	0.3	0.1	0.3	0.6	0.0	0.3	0.3	0.1
Anti-correlated (constant k_+k_-) (update with stat)	0.0	0.4	0.4	0.2	0.4	0.9	0.2	0.4	0.2

Table 3: Summary of uncertainties on $k_\pm$ by region and muon type.

5 Applying the Results

We provide central values for the scales k_+ and k_- (Table 2), as well as correlated and anticorrelated uncertainties on these values (Table 3). These corrections are valid for muons in the approximate p_T range 20–50 GeV. To apply these corrections to Monte Carlo events, μ^+ and μ^- candidates should have their p_T multiplied by k_+ and k_- respectively for the appropriate muon type and η region. We factor the uncertainty in the scale measurements into correlated and anticorrelated components, where correlated means that the quantity k_+/k_- is kept constant (the scales for both charges change in the same direction), and anticorrelated means that k_+k_- is kept constant (the scales change in opposite directions). Note that the correlated and anticorrelated uncertainties are uncorrelated between η regions and between different types of muons. The effects of the correlated and anticorrelated terms should be evaluated separately and then added in quadrature to obtain an overall uncertainty due to momentum scale.

We also provide the deviations observed in the scales when we use smaller η bins. This gives a measure of how much structure in $k_\pm$ is hidden by our averages over large η regions. We caution that there can still be significant unresolved structure in ϕ and η and so these results should *not* be taken as limits on the momentum scale deviation on a per-muon basis.

A Systematic Uncertainty Details

Region	Δk_+	Δk_-
Combined muons		
Endcap-A	+0.0%	+0.0%
Barrel	−0.4%	+0.4%
Endcap-C	+0.0%	−0.1%
MS muons		
Endcap-A	+0.0%	+0.0%
Barrel	−0.4%	+0.4%
Endcap-C	+0.6%	−0.8%
ID muons		
Endcap-A	+0.1%	−0.1%
Barrel	−0.3%	+0.3%
Endcap-C	−0.1%	+0.1%

Table 4: Systematic change in k_+ and k_- when one of the Z legs is allowed to be outside of the fiducial region, expanding the sample size used in R_0 determination.

Region	Δk_+	Δk_-
Combined muons		
Endcap-A	+0.0%	+0.0%
Barrel	+0.0%	+0.0%
Endcap-C	−0.1%	+0.1%
MS muons		
Endcap-A	−0.2%	+0.2%
Barrel	+0.0%	+0.0%
Endcap-C	+0.4%	−0.4%
ID muons		
Endcap-A	−0.1%	+0.1%
Barrel	+0.0%	+0.0%
Endcap-C	−0.1%	+0.1%

Table 5: Systematic change in k_+ and k_- associated with a reduction of the mass window used in the R_0 fit procedure from 70..110 GeV to 80..100 GeV.

Region	Δk_+	Δk_-
Combined muons		
Endcap-A	+0.0%	+0.0%
Barrel	−0.1%	+0.1%
Endcap-C	−0.4%	+0.3%
MS muons		
Endcap-A	+0.1%	−0.1%
Barrel	−0.1%	+0.1%
Endcap-C	−0.2%	+0.2%
ID muons		
Endcap-A	−0.1%	+0.1%
Barrel	−0.1%	+0.2%
Endcap-C	−0.2%	+0.2%

Table 6: Systematic change in k_+ and k_- associated with the difference between χ^2 and KS methods of finding R_0

Region	Δk_+	Δk_-
Combined muons		
Endcap-A	+0.2%	+0.2%
Barrel	−0.3%	−0.3%
Endcap-C	+0.1%	+0.1%
MS muons		
Endcap-A	−0.3%	+0.3%
Barrel	−0.6%	−0.5%
Endcap-C	+0.0%	+0.0%
ID muons		
Endcap-A	+0.3%	+0.3%
Barrel	−0.3%	−0.2%
Endcap-C	−0.1%	−0.1%

Table 7: Systematic change in k_+ and k_- when the fit function is changed from simple Gaussian to full Z lineshape.