60th Compton Lectures
The Origin of Mass in Particle Physics

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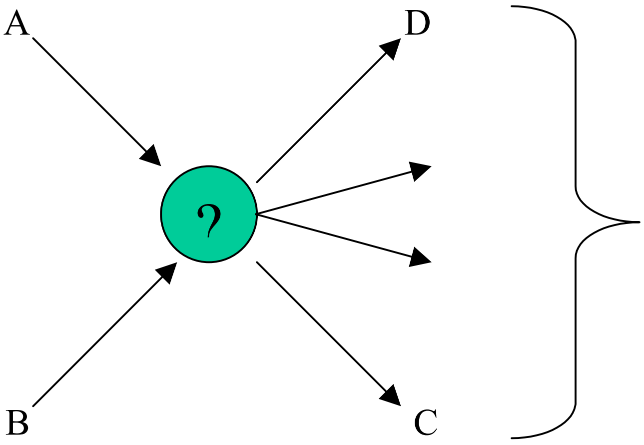
Lecture III: The World of the Small and the Fast

1. Introducing the Bubble of Ignorance

As the understanding of the microscopic world evolved in the early 20th century, it became increasingly clear that, in order to get a response from nature, asking right questions and trying to understand them in specific experimental and theoretical framework was necessary to make progress. In a simplified picture, a particle physics experiments can be summarized with the following diagram



Where A and B are two incoming particles that are prepared with specific experimental condition of particle energy and type etc. They interact at the bubble (with question mark) of ignorance and resultant output particles emerge. A detector can measure the energy, momentum and position etc., of the outputs. At high enough energies, more than two particles can emerge from the reaction since mass and energy are related. We cannot look at what happens in the interactions in the bubble directly because that would deform the interaction under study. What we can do is calculate probabilities of different interactions using the laws of quantum mechanics. Once the full set of probability is calculated for different output possibilities, we can perform experiments to establish if the theoretical model predicting the probabilities actually conforms to nature.

Since, now we are working in the regime of small particles and high energy i.e., world of Quantum Mechanics and Special Relativity we need a theory that accounts for both of them in one framework.

2. Combining Quantum Mechanics and Special Relativity

P.A.M Dirac introduced a general formulation of Quantum Mechanics of which Schoredinger's (wave equation) and Heisenberg's (matrix mechanics) were special cases. These formulations of quantum mechanics describe a single particle interacting with an external potential of some kind. The behavior of the particle is encapsulated in the evolution of the wave function associated with the particle. The formulation describes particles moving at velocity much smaller compared to the velocity of light. In 1927 Dirac combined Quantum Mechanics and Special Relativity to produce the equation of relativistic quantum mechanics. One of the triumphs of this equation was that the electron spin was naturally incorporated in this equation.

Electron Spin: A brief interlude

When an electron rotates around the nucleus, we can associate with this rotation a property called angular momentum. It is the product of the momentum (mass times velocity) and the distances of the electron from the axis of rotation. Since electrons have charge, when it rotates it produces a current and hence acts like a tiny magnet. The strength with which a magnet feels force in magnetic field is proportional to the magnetic moment, which in the case of the electron orbiting around the nucleus is equal to the current times the area covered by electron orbit. It can be shown that the magnetic moment (μ) is proportional to the angular momentum (L)

$$\mu = g\mu_B L$$

Where g is called as the gyromagnetic ratio and μ_B is a natural constant called Bohr magneton. The value of g is one in this classical case.

Electrons also have 'intrinsic angular momentum' called spin. The electron spin, which is a quantum mechanical property, does not have an exact classical counterpart. In fact, the gyromagnetic ratio in the case of electron is close to two, suggesting that electron spin is half. Like other quantum mechanical properties, spin is quantized and comes in integrals of $\frac{1}{2}(h/2\pi)$ (Planck's constant h has the unit of angular momentum). Particles that have half-integral (1/2, 3/2, etc.) spins are called fermions and particles that have integral (0,1,2 etc.) spin are called bosons. One of the strangest feature quantum spins is shown by of half-integral spin particles. If an object like earth rotates 360 degrees it returns to where it started. But and object like an electron, which has half-integral spin, if it rotates 360 degree it gets in a quantum state which is measurably different from the initial state. In order for it to get back to initial quantum state it has to rotate by 720 degrees. One way to picture this is to say that quantum particle 'sees' the universe differently. What we see if we turn 360 degrees twice are identical copies of the universe, but the quantum particle can somehow discern a difference between the two copies.

End of Interlude

Dirac's equation applied to an electron in an electromagnetic field showed that electrons should have spin ½. Thus the gyromagnetic ratio, g, in the case of electron spin was 2 instead of 1 as in the classical case. This value g is only approximately correct, which also points to the limitation of Dirac's equations.

Also, for the above example of electron in an electro magnetic field, Dirac's equation predicted two solutions, one was a positive in energy and the other one was negative in energy. Dirac recognized the problem of negative energy solution and proposed a bold solution that leads him to predict the existence of anti-particles. His proposal was that negative energy states are filled by electrons and hence unobservable. Also, since electrons obey Pauli exclusion principle each state is occupied by only one electron, the catastrophe that all electrons quickly decaying to lower energy state are avoided. If an electron moves from negative energy state to positive energy state, it will create a hole, which is a positively charged electron. Carl Anderson discovered the positron in 1931 by observing the cosmic ray data, thus confirming the existence of anti-particles.

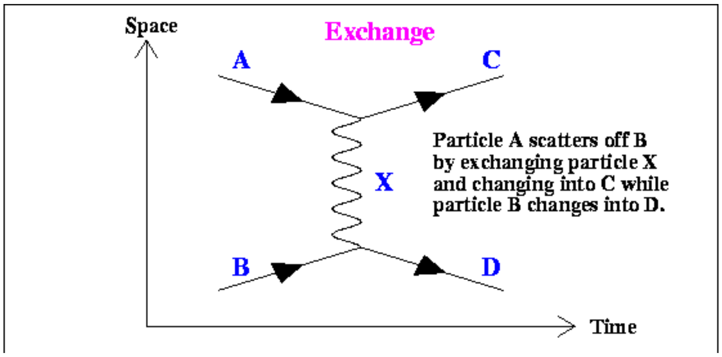
In spite of its successes, this formulation of Quantum Mechanics and Special Relativity together had problems. The whole argument of sea of negative energy state did not fit well with any physical picture since no negative energy states in vacuum were observed. Another problem was that the same theory could have been applied to charged pions, which are spin zero particles and to not have to obey Pauli's exclusion principle. How then do the charged pions not quickly transition to the sea?

A complete relativistic quantum theory has to account for particle creation and annihilation. The formalism of wave equation and wave function is based on fixed number of particles of definite type and it cannot account for particle creation and annihilation.

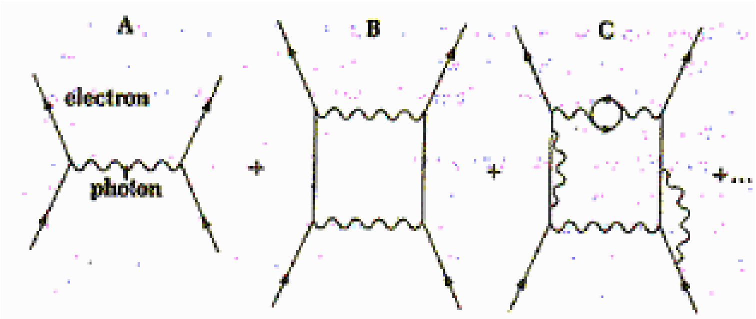
3. Quantum field theory of electrons and photons

We have defined field in previous lecture as a set of number at each space-time point. An example is the electromagnetic field which is the "medium" in which electromagnetic waves propagates. Quantum field theory describes the quantum mechanics of fields, such as the electromagnetic field and the electron field. In this setup, particles and waves, both are different faces of the same type of object: the quantum field.

The quantum description of electron and photon field and their interaction is called Quantum electrodynamics. For the fundamental work on this theory Feynman, Schwinger and Tomonaga were awarded the Nobel Prize in 1965. QED works be considering all possible reaction that might take place in the bubble of ignorance discussed earlier. It calculates probability of each possibility and adds them to get overall probability. Feynman developed a highly pictorial technique for making sure that all possibilities were included. An example is an electron electron elastic scattering, which can be represented by the diagram below. The electrons A and B interact by exchanging a 'virtual' photon before they fly away. The point where the three lines (incoming electron, outgoing electron and the photon line) meet is called the vertex. Virtual photon is a term used when photon for a brief moment, allowed by the Heisenberg's uncertainty principle, violates energy conservation. All the exchange particles in Feynman diagrams are virtual.



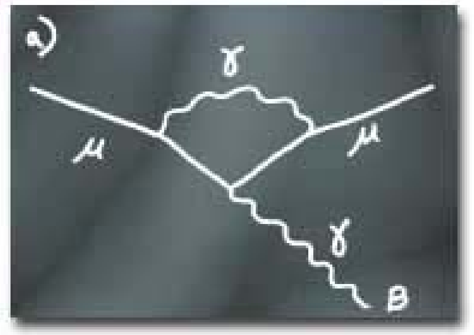
It can be imagined that, in the bubble of ignorance, more than one photon could be exchanged and more complex diagrams can be created. The following diagrams are some example.



Each diagram represents a possibility to consider when calculating the overall probability for the interaction in the bubble of ignorance. Feynman devised mathematical rules to calculate probability for each diagram. As the diagrams increase in complexity, they decrease in probability. This is due to the fact that the highest contributions come from the simplest diagrams, but as the experimental precision improves higher order diagrams need to be included in order to explain experimental results. One of the spectacular successes of quantum electrodynamics is the prediction of the dipole moment of the electron, which is measured to a part in a billion.

4. Dipole moment of electrons

One of the successes of Dirac's equation was that it showed that the magnetic moment of an electron, using appropriate units, was exactly one. But, experiments also showed deviations from unity. An early triumph of quantum electrodynamics was Julian Schwinger's calculation of first order radiative correction to the magnetic moment of the electron i.e., including the following diagram in the calculation.



The radiation and reabsorption of a single virtual photon will contribute to the magnetic moment. The magnetic moment of the muon, the heavier sibling of the electron, is also measured to a high precision. These high precision measurements require more and more diagrams to be included in bubble of ignorance. Inclusion of similar diagrams from other known interactions (such are the strong and weak forces) is needed to explain precision measurements. If we run out of known processes to explain experimental result, it would be a hint for the existence of a new physics beyond what we know in the Standard Model of particle physics.