Kinematic variables for Dark Matter at the LHC and Beyond

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University of Chicago HEP Seminar – March 9, 2014
Talk Outline

- Weakly interacting particles at the LHC
  - Why? How?

- Kinematic handles for studying them
  - MET
  - singularity variables

- Recursive Jigsaw Reconstruction
  - ex. di-leptons (i.e. super-razor variables)
  - ex. di-leptonic tops/stops (something sexier)
Why are they interesting?

- Dark Matter
  - It exists - but what is it? Would like to know if we’re producing these particles at the LHC
- Electroweak bosons
  - Decays of W and Z often produce neutrinos
- New symmetries
  - Discrete symmetries (ex. R-parity) make lightest new ‘charged’ particles stable
How do we study them?

- Can infer their presence through *missing transverse energy*
- Hermetic design of LHC experiments allows us to infer ‘what’s missing’

- Full azimuthal coverage, up to $|\eta|$ of ~5
- Stopping power of ~12-20 interaction lengths
- ECAL+HCAL components with segmentation comparable to lateral shower sizes

\[ \vec{E}_T^{miss} = - \sum_i \vec{E}_T \]
Missing transverse energy

Figures from SUSY10 conference talk:

Missing transverse energy is a powerful observable for inferring the presence of weakly interacting particles.

But, it only tells us about their transverse momenta – often we can better resolve quantities of interest by using additional information.
Missing transverse energy only tells us about the momentum of weakly interacting particles in an event…
Missing transverse energy

…not about the identity or mass of weakly interacting particles
Missing transverse energy

…not about the identity or mass of weakly interacting particles
Missing transverse energy

We can learn more by using other information in an event to contextualize the missing transverse energy.
Resolving the invisible

$m_T(\ell \nu)$ has kinematic edge at $m_W \sim 80$ GeV

Can use visible particles in events to contextualize missing transverse energy and better resolve mass scales
Missing transverse energy

We can learn more by using other information in an event to contextualize the missing transverse energy ⇒ multiple weakly interacting particles?
Open vs. closed final states

**CLOSED** \( H \rightarrow Z(\ell\ell)Z(\ell\ell) \)
- Can calculate all masses, momenta, angles
- Can use masses for discovery, can use information to measure spin, CP, etc.

**OPEN** \( H \rightarrow W(\ell\nu)W(\ell\nu) \)
- Under-constrained system with multiple weakly interacting particles – can’t calculate all the kinematic information

What useful information can we calculate?
What can we measure?
Multiple weakly interacting particles?

- Dark Matter
- Higgs quadratic divergences
- ....

Canonical open / $\mathbb{Z}_2$ topology

visible

Can be single or multiple decays steps

invisible

Can be one or more particles

visible

Can be one or more particles

visible

Can be one or more particles

invisible

Can be one or more particles

Theory
- SUSY
- Little Higgs
- UED

$\mathbb{Z}_2$
- R-parity
- T-parity
- KK-parity

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Example: slepton pair-production

Experimental signature: di-leptons final states with missing transverse momentum
Example: slepton pair-production

Main background:
Example: slepton pair-production

What quantities, if we could calculate them, could help us distinguish between signal and background events?

\[ \sqrt{s} = 2 \gamma^{\text{decay}} m_{\tilde{\ell}} \]

\[ M_\Delta \equiv \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}^0}^2}{m_{\tilde{\chi}^0}} \]
Example: slepton pair-production

What information are we missing?

We don’t observe the weakly interacting particles in the event. We can’t measure their momentum or masses.
Example: slepton pair-production

What do we know?

We can reconstruct the 4-vectors of the two leptons and the transverse momentum in the event.
Example: slepton pair-production

Can we calculate anything useful?
With a number of simplifying assumptions...

\[ E_{T}^{miss} = \sum \vec{p}_{T} \tilde{\chi}^{0} \quad m_{\tilde{\chi}^{0}} = 0 \]

…we are still 4 d.o.f. short of reconstructing any masses of interest
Singularity Variables

- State-of-the-art for LHC Run I was to use singularity variables as observables in searches.

- Derive observables that bound a mass or mass-splitting of interest by
  - Assuming knowledge of event decay topology
  - Extremizing over under-constrained kinematic degrees of freedom associated with weakly interacting particles.
Singularity Variable Example: $M_{T^2}$

Generalization of transverse mass to two weakly interacting particle events

$$M_{T^2}^2(m_\chi) = \min_{\vec{p}_T^{\chi_1} + \vec{p}_T^{\chi_2} = \vec{E}_{T}^{miss}} \max \left[ m_T^2(\vec{p}_T^{\ell_1}, \vec{p}_T^{\chi_1}, m_\chi), m_T^2(\vec{p}_T^{\ell_2}, \vec{p}_T^{\chi_2}, m_\chi) \right]$$

with:

$$m_T^2(\vec{p}_T^{\ell_i}, \vec{p}_T^{\chi_i}, m_\chi) = m_\chi^2 + 2 \left( E_T^{\ell_i} E_T^{\chi_i} - \vec{p}_T^{\ell_i} \cdot \vec{p}_T^{\chi_i} \right)$$

From:

Singularity Variable Example: $M_{T2}^2$

Generalization of transverse mass to two weakly interacting particle events

LSP ‘test mass’

\[
M_{T2}^2(m_\chi) = \min_{\not{p}_T^{x1} + \not{p}_T^{x2} = E_{miss}^T} \max \left[ m_T^2(\not{p}_T^{\ell1}, \not{p}_T^{\chi1}, m_\chi), m_T^2(\not{p}_T^{\ell2}, \not{p}_T^{\chi2}, m_\chi) \right]
\]

with:

\[
m_T^2(\not{p}_T^{\ell i}, \not{p}_T^{\chi i}, m_\chi) = m_\chi^2 + 2 \left( E_T^{\ell i} E_T^{\chi i} - \not{p}_T^{\ell i} \cdot \not{p}_T^{\chi i} \right)
\]

From:

Singularity Variable Example: $M_{T2}$

Generalization of transverse mass to two weakly interacting particle events

LSP ‘test mass’

$$M_{T2}^2(m_\chi) = \min \left\{ \max \left[ m_T^2(\vec{p}_T^{\ell_1}, \vec{p}_T^{\chi_1}, m_\chi), m_T^2(\vec{p}_T^{\ell_2}, \vec{p}_T^{\chi_2}, m_\chi) \right] \right\}$$

with: $m_T^2(\vec{p}_T^{\ell_i}, \vec{p}_T^{\chi_i}, m_\chi) = m_\chi^2 + 2 \left( E_T^{\ell_i} E_T^{\chi_i} - \vec{p}_T^{\ell_i} \cdot \vec{p}_T^{\chi_i} \right)$

Subject to constraints

Extremization of unknown degrees of freedom

From:
Singularity Variable Example: $M_{T2}$

Generalization of transverse mass to two weakly interacting particle events

LSP ‘test mass’

\[
M_{T2}^2(m_\chi) = \min_{\vec{p}_T^{x_1} + \vec{p}_T^{x_2} = \vec{E}_T^{miss}} \max \left[ m_T^2(\vec{p}_T^{\ell_1}, \vec{p}_T^{\chi_1}, m_\chi), m_T^2(\vec{p}_T^{\ell_2}, \vec{p}_T^{\chi_2}, m_\chi) \right]
\]

Extremization of unknown degrees of freedom

Subject to constraints

\[
\text{with: } m_T^2(\vec{p}_T^{\ell_i}, \vec{p}_T^{\chi_i}, m_\chi) = m_\chi^2 + 2 \left( E_T^{\ell_i} E_T^{\chi_i} - \vec{p}_T^{\ell_i} \cdot \vec{p}_T^{\chi_i} \right)
\]

Constructed to have a kinematic endpoint (with the right test mass) at:

\[
M_{T2}^{\text{max}}(m_\chi) = m_{\tilde{\ell}}, \quad M_{T2}^{\text{max}}(0) = M_\Delta \equiv \frac{m_\chi^2 - m_{\tilde{\ell}}^2}{m_{\tilde{\ell}}}
\]

From:

$M_{T2}$ in practice

From:
ATLAS-CONF-2013-049

Backgrounds with leptonic W decays fall steeply once $M_{T2}$ exceeds the W mass

Searches based on singularity variables have sensitivity to new physics signatures with mass splittings larger than the analogous SM ones
Recursive Jigsaw Reconstruction

New approach to reconstructing final states with weakly interacting particles: *Recursive Jigsaw Reconstruction*

- The strategy is to transform observable momenta iteratively *reference-frame to reference-frame*, traveling through each of the reference frames relevant to the topology.
- At each step, *extremize only the relevant d.o.f. related to that transformation*.
- Repeat procedure recursively, using only the momenta encountered in each reference frame.

- Rather than obtaining one observable, get a *complete basis* of useful observables for each event.
Recursive rest-frame reconstruction

For two lepton case, these are the ‘super-razor variables’:


\[ \ell_1^{\text{lab}}, \ell_2^{\text{lab}} \quad \text{Begin with reconstructed lepton 4-vectors in lab frame} \]
Recursive rest-frame reconstruction

For two lepton case, these are the ‘super-razor variables’:


\( \ell_{1}^{\text{lab}}, \ell_{2}^{\text{lab}} \) Begin with reconstructed lepton 4-vectors in lab frame

\[
\frac{\partial (E_{\ell_{1}}^{\text{lab}} z + E_{\ell_{2}}^{\text{lab}} z)}{\partial \beta_{z}} = 0 \rightarrow \beta_{z}
\]

Remove dependence on unknown longitudinal boost by moving from ‘lab’ to ‘lab z’ frames
Recursive rest-frame reconstruction

For two lepton case, these are the ‘super-razor variables’:


\[(\tilde{\chi}_1 + \tilde{\chi}_2)^2 = (\ell_1 + \ell_2)^2\]

Determine boost from ‘lab z’ to ‘CM (\tilde{\ell}\tilde{\ell})’ frame by specifying Lorentz-invariant choice for invisible system mass.
Recursive rest-frame reconstruction

For two lepton case, these are the ‘super-razor variables’:


\[
\frac{\partial (E_{\ell_1} + E_{\ell_2})}{\partial \beta_{\ell\ell \to \ell_i}} = 0 \rightarrow \beta_{\ell\ell \to \ell_i}
\]

Determine asymmetric boost from CM to slepton rest frames by minimizing lepton energies in those frames.
Recursive rest-frame reconstruction

For two lepton case, these are the ‘super-razor variables’:


\[ \ell_1^{\text{lab}}, \ell_2^{\text{lab}} \]
Begin with reconstructed lepton 4-vectors in lab frame

\[ \frac{\partial (E_{\ell_1}^{\text{lab}} z + E_{\ell_2}^{\text{lab}} z)}{\partial \beta_z} = 0 \rightarrow \beta_z \]
Remove dependence on unknown longitudinal boost by moving from ‘lab’ to ‘lab \(z\)’ frames

\[ (\tilde{\chi}_1 + \tilde{\chi}_2)^2 = (\ell_1 + \ell_2)^2 \]
Determine boost from ‘\(\text{lab} \ z\)’ to ‘CM (\(\tilde{\ell}\tilde{\ell}\))’ frame by specifying Lorentz-invariant choice for invisible system mass

\[ \frac{\partial (E_{\ell_1}^{\tilde{\ell}} + E_{\ell_2}^{\tilde{\ell}})}{\partial \beta_{\tilde{\ell}\tilde{\ell}\rightarrow\tilde{\ell}_i}} = 0 \rightarrow \tilde{\beta}_{\tilde{\ell}\tilde{\ell}\rightarrow\tilde{\ell}_i} \]
Determine asymmetric boost from CM to slepton rest frames by minimizing lepton energies in those frames
Recursive rest-frame reconstruction

1st transformation: extract variable sensitive to invariant mass of total event: $\sqrt{\hat{S}_R}$

Resulting variable is \textasciitilde invariant under $p_T$ of di-slepton system

Lab frame  \hspace{1cm} di-slepton CM frame

$\ell\ell \rightarrow \ell\ell$

$\beta_{lab} \; z \rightarrow \tilde{\ell}\tilde{\ell}$

$\sqrt{s} = 8$ TeV

MadGraph+PGS

$pp \rightarrow \bar{\ell}\ell; \; \bar{\ell} \rightarrow l\tilde{\chi}_1^0; \; m_l = 150$ GeV

$p_T^{CM}$ [GeV]

[Graph showing the distribution of $p_T^{CM}$ for different masses]

$M_{\Delta}$

[Graph showing the distribution of $M_{\Delta}$ for different masses]
Resonant Higgs production

\[ H \rightarrow WW^* \rightarrow 2\ell2\nu\]

Using information from the two leptons, and the missing transverse momentum, the observable \( \sqrt{\hat{s}} \) is directly sensitive to the Higgs mass.

From:
Recursive rest-frame reconstruction

2\textsuperscript{nd} transformation(s): extract variable sensitive to invariant mass of squark: $M_\Delta^R$

Resulting variable has kinematic endpoint at:

$$M_\Delta \equiv \frac{m^2_\tilde{l} - m^2_{\tilde{\chi}^0}}{m_\tilde{l}}$$

MadGraph+PGS

$\sqrt{s}=8\text{ TeV}$

$\nu^\pm l \rightarrow \nu^\pm l^\pm \nu$
Variable comparison

Three different singularity variables, all attempting to measure the same thing

\[ M^R_\Delta \geq M_{T2}(0) \geq M_{CT\perp} \]

More details about variable comparisons in PRD 89, 055020 (arXiv:1310.4827) and backup slides
But what else can we calculate?

With recursive scheme can extract the two mass scales $\sqrt{\hat{S}_R}$ and $M^R_\Delta$ almost completely independently.

\[ \Delta M = 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \quad 1.2 \quad 1.4 \quad 1.6 \quad 1.8 \quad 2 \]

\[ \Delta M / M_\Lambda = 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \]

MadGraph+PGS
\[ \sqrt{s} = 8 \text{ TeV} \]
\[ pp \to \tilde{t}\tilde{t}; \tilde{t} \to t\tilde{\chi}_1^0 \]

\[ m_{\tilde{t}} = 150 \text{ GeV} \]
\[ m_{\tilde{\chi}_1^0} = 50 \text{ GeV} \]
Angles, angles, angles…

Recursive scheme fully specifies approximate event decay chain, also yielding angular observables.

Two transformations mean at least two independent angles of interest (essentially the decay angle of the state whose rest-frame you are in).
Towards a kinematic basis

but \[ \sqrt{\hat{S}}_R \sim 2 \gamma^{\text{decay}} M_\Delta \]

while \[ \sqrt{\hat{S}} = 2 \gamma^{\text{decay}} m_{\tilde{\ell}} \]

Underestimating the real mass means over-estimating the boost magnitude:

\[ \beta_{\text{lab} \to CM} = \frac{\vec{p}_T^{CM}}{\sqrt{|\vec{p}_T^{CM}|^2 + \hat{S}}} \]

From PRD 89, 055020 (2014)
Towards a kinematic basis

but \( \sqrt{\hat{S}}_R \sim 2\gamma^{\text{decay}} M_\Delta \)

while \( \sqrt{\hat{S}} = 2\gamma^{\text{decay}} m_\tilde{\ell} \)

Underestimating the real mass means over-estimating the boost magnitude:

From PRD 89, 055020 (2014)
Angular Variables

Incorrect boost magnitude induces correlation

Angle between lab → CM frame boost and di-leptons in CM frame is sensitive to

\[ \frac{m_\chi}{m_{\tilde{\ell}}} \] rather than \[ M_\Delta \]

~Uncorrelated with other super-razor variables
Angular Variables

In the approximate di-slepton rest frame, reconstructed decay angle sensitive to particle spin and production.
Angular Variables

In the approximate slepton rest frames, reconstructed slepton decay angle sensitive to particle spin correlations.
Angular Variables

Also allows us to better resolve the kinematic endpoint of interest

MadGraph+PGS
$\sqrt{s}=8$ TeV
$pp \to \bar{\ell}\ell; \bar{\ell} \to l\tilde{\chi}_1^0$

$m_{\tilde{\ell}} = 150$ GeV
$m_{\chi_1} = 50$ GeV

$M_\Delta$ [GeV] vs. $|\cos \theta_{R+1}|$
Super-razor variable basis

Can re-imagine a di-lepton analysis in new basis of variables

Can improve sensitivity while removing MET cuts!

\[ \Delta \phi_R^\beta \] Sensitive to ratio of invisible and visible masses

\[ \cos \theta_R \] Spin and production

\[ M^R_{\Delta} \] Mass-squared difference resonant/non-resonant prod.

\[ \sqrt{\hat{S}} \] Sensitive to mass of CM

Good for resonant production of heavy parents

\[ \sigma N \] CMS selection

MadGraph+PGS

\[ \sqrt{s} = 8 \text{ TeV} \int L dt = 20 \text{ fb}^{-1} \]
Generalizing further

Recursive Jigsaw approach can be generalized to arbitrarily complex final states with weakly interacting particles.
Example: the di-leptonic top basis

In more complicated decay topologies there can be many masses/mass-splittings, spin-sensitive angles and other observables of interest that can be used to distinguish between the SM and SUSY signals
Singularity Variables for Tops

\[ m_{bl}^{\text{max}} = \frac{\sqrt{(m_t^2 - m_W^2)(m_W^2 - m_\nu^2)}}{m_W} \]

\[ M_{CT}^{\max}(\ell_1, \ell_2) = \frac{m_W^2 - m_\nu^2}{m_W} \]

\[ M_{CT}^{\max}(b_1 \ell_1, b_2 \ell_2) = \frac{m_t^2 - m_W^2}{m_t} + \frac{m_t}{m_W} \left( \frac{m_W^2 - m_\nu^2}{m_W} \right) \]

\[ M_{CT}^{\max}(b_1, b_2) = \frac{m_t^2 - m_W^2}{m_t} \]

If there are multiple masses or mass splittings of interest in the event, appearing at different points in the decay chains, then singularity variables correspond to different extremizations of missing d.o.f.

What are the correlations between the different singularity variables estimating different mass splittings? Can be large/complicated when using $M_{CT}$ or $M_{T2}$ in this case.

What if the two tops/stops are decaying differently through different particles?
Recursive Jigsaw reconstruction

Move through each reference frame of interest in the event, specifying only d.o.f. relevant to each transformation:

- Lab frame
- Di-top frame
- Top frame
- W frame
- Two X

2b2l system treated as one visible object

Each bl system treated as visible object

Top frame

W frame

Individual leptons resolved independently
Recursive Jigsaw reconstruction

$M_W / M_{top}$

$M_{top}$ [GeV]

The scales can be extracted independently
In fact the scales can be extracted independently for each top – the reconstruction chains are \textit{decoupled}. 

\[ M_{\text{top}} \text{ [GeV]} \]
The di-leptonic top basis

\[ M_{t\bar{t}}, \ \vec{p}_{t\bar{t}}, \cos \theta_{TT} \]
\[ \Delta \phi_{T1,T2} \]

2 X

\[ E^\text{top-frame}_b, \cos \theta_T \]
\[ \Delta \phi_{T,W} \]

\[ E^\text{W-frame}_\ell, \cos \theta_W \]
The di-leptonic top basis

largely independent information about five different masses
Previous state-of-the-art

Mass-sensitive singularity variables are not necessarily independent
The di-leptonic top basis

largely independent information about decay angles

Here, the decay angle of the top/anti-top system
The di-leptonic top basis

largely independent information about decay angles

Here, the decay angle of one of the top quarks
The di-leptonic top basis vs. signals

Different variables in the basis are useful for different signals

First, we consider resonant ttbar production through a gravitton
Distributions of top/W/neutrino mass-splitting-sensitive observables are nearly identical since graviton signal and non-resonant background both contain on-shell tops.
The di-leptonic top basis vs. gravitons

Different variables in the basis are useful for different signals

Instead, observables related to the production of the two tops are sensitive to the intermediate resonance

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The di-leptonic top basis vs. stops

SUSY stops decaying through charginos

V.S.
The di-leptonic top basis vs. stops

Mass-splitting-sensitive observables can be used to distinguish presence of signals.
The di-leptonic top basis vs. stops

$m_{\tilde{t}} = 700$ GeV

$m_{\tilde{\chi}^\pm} = 500$ GeV

$m_{\tilde{\chi}^0} = 300$ GeV

Here, the azimuthal angle between the the top and W decay planes $\Delta \phi_{T1,W1}$ and the angle between the two top decay planes $\Delta \phi_{T1,T2}$
Summary

- The strategy of **Recursive Jigsaw Reconstruction** is to not only develop ‘good’ mass estimator variables, but to decompose each event into a *basis of kinematic variables*
  - Through the recursive procedure, each variable is (as much as possible) *independent of the others*
  - The interpretation of variables is straightforward; they each correspond to an *actual, well-defined, quantity in the event*
  - Can be generalized to arbitrarily complex final states with *many weakly interacting particles*
- Code package to be released imminently (*RestFrames* - see back-up)
- First papers nearing completion
  - (*Recursive Jigsaw Reconstruction*,
  - *The Di-leptonic ttbar basis* - with Paul Jackson)
Recursive Jigsaw Reconstruction is a systematic recipe for deriving a kinematic basis for any open final state

- Mixed decay cases can now be treated in a sensible way

\[ \tilde{t} \rightarrow t\tilde{\chi}^0 \text{ vs. } \tilde{t} \rightarrow b\tilde{\chi}^\pm \]

\[ \tilde{\chi}^\pm \rightarrow W^\pm \tilde{\chi}^0 \text{ vs. } \]
\[ \tilde{\chi}^\pm \rightarrow \ell^\pm \tilde{\nu} \text{ vs. } \]
\[ \tilde{\chi}^\pm \rightarrow \nu \ell^\pm \]

-Lots of potential applications – some I’m currently thinking about:
  - Di-lepton searches and differential measurements
  - stop / top-quark partner and resonant $t\bar{t}$bar searches
  - other $\mathbb{Z}_2$ signatures (SUSY, exotica)
  - $H \rightarrow WW^* \rightarrow \ell\nu\ell\nu$ spin measurements
  - …
BACKUP SLIDES
**RestFrames** software library

**RestFrames**: Soon-to-be-public code that can be used to calculate kinematic variables associated with any decay chain and can implement “Recursive Jigsaw” rules

**Example**: Di-leptonic ttbar decays - how to use the code to initialize a decay tree, implement a jigsaw rule for determining the neutrino four-momenta, and analyze events

[www.RestFrames.com](http://www.RestFrames.com)
RestFrames software library

initialize all your reference frames of interest

connect them according to the decay tree you want to impose on the event
// draw your decay tree
FramePlot* DecayTree_plot = new FramePlot("tree","Decay Tree");
DecayTree_plot->AddFrameTree(LAB);
DecayTree_plot->DrawFramePlot();

...and you’ll see this
RestFrames example: di-leptonic top

Set the 4-vector input and MET for an event

Get observables directly from decay tree

// example analysis methods
double TTmass = TT.GetMass();  // mass
double costhetaT1 = T1.GetCosDecayAngle();  // decay angle
double P_T2_inTT = T2.GetMomentum(TT);  // momentum in some frame
TLorentzVector V_W1_in_W2 = W1.GetFourVector(W2);  // 4-vector in some frame
double dphi_T1_W1 = T1.GetDeltaPhiDecayPlanes(W1);  // angle between decay planes
RestFrames example: di-leptonic top

DecayTree_plot->AddFrameTree(TT);
DecayTree_plot->DrawFramePlot();

DecayTree_plot->AddFrameTree(T1);
DecayTree_plot->DrawFramePlot();
RestFrames example: di-leptonic top

// Invisible Frame Group
InvisibleGroup INV("INV","Invisible Frame Jigsaws");
INV.AddFrame(Neu1);
INV.AddFrame(Neu2);
RestFrames example: di-leptonic top

CombinatoricGroup BTAGS("BTAGS","B-tagged jet Jigsaws");
BTAGS.AddFrame(Bjet1);
BTAGS.AddFrame(Bjet2);

BTAGS.SetNElementsForFrame(Bjet1,1,true);
BTAGS.SetNElementsForFrame(Bjet2,1,true);

number of elements that _have_ to go to this frame from the group each event

exclusive (true) or inclusive (false) counting
```
InvisibleMassJigsaw MinMassJigsaw("MINMASS_JIGSAW",
    "Invisible system mass Jigsaw");
INV.AddJigsaw(MinMassJigsaw);

InvisibleRapidityJigsaw RapidityJigsaw("RAPIDITY_JIGSAW",
    "Invisible system rapidity Jigsaw");
INV.AddJigsaw(RapidityJigsaw);
RapidityJigsaw.AddVisibleFrame((LAB.GetListVisibleFrames()));
// Same as this
RapidityJigsaw.AddVisibleFrame(T1.GetListVisibleFrames());
RapidityJigsaw.AddVisibleFrame(T2.GetListVisibleFrames());
// Same as this
RapidityJigsaw.AddVisibleFrame(Bjet1);
RapidityJigsaw.AddVisibleFrame(Bjet2);
RapidityJigsaw.AddVisibleFrame(Lep1);
RapidityJigsaw.AddVisibleFrame(Lep2);
```
RestFrames example: di-leptonic top

```cpp
ContraBoostInvariantJigsaw TTJigsaw("TT_JIGSAW","Contraboost invariant Jigsaw");
INV.AddJigsaw(TTJigsaw);
TTJigsaw.AddVisibleFrame((T1.GetListVisibleFrames()), 0);
TTJigsaw.AddInvisibleFrame((T1.GetListInvisibleFrames()), 0);
TTJigsaw.AddInvisibleFrame((T2.GetListInvisibleFrames()), 1);
TTJigsaw.AddVisibleFrame((T2.GetListVisibleFrames()), 1);

// draw your decay tree with jigsaws
FramePlot* JigsawTree_plot = new FramePlot("tree","Decay");
JigsawTree_plot->AddFrameTree(TT);
JigsawTree_plot->AddJigsaw(TTJigsaw);
JigsawTree_plot->DrawFramePlot();
```
RestFrames example: di-leptonic top

```c
MinimizeMassesCombinatoricJigsaw BLJigsaw("BL_JIGSAW",
    "Minimize m_{b\#it{l}}'s Jigsaw");
BTAGS.AddJigsaw(BLJigsaw);
BLJigsaw.AddFrame(Bjet1,0);
BLJigsaw.AddFrame(Lep1,0);
BLJigsaw.AddFrame(Bjet2,1);
BLJigsaw.AddFrame(Lep2,1);
```

event-by-event will choose b-tag jet combinatoric assignment that minimizes the masses of these two collections
\textbf{RestFrames} example: di-leptonic top

MinimizeMassesCombinatoricJigsaw BLJigsaw("BL_JIGSAW",
"Minimize m_{b#it{l}}'s Jigsaw");

BTAGS.AddJigsaw(BLJigsaw);
BLJigsaw.AddFrame(Bjet1,0);
BLJigsaw.AddFrame(Lep1,0);
BLJigsaw.AddFrame(Bjet2,1);
BLJigsaw.AddFrame(Lep2,1);

JigsawTree_plot->AddJigsaw(BLJigsaw);
JigsawTree_plot->DrawFramePlot();
Invisible Frame Jigsaws

\[ \nu + \nu \]

\[ a \quad b \]

Invisible system mass Jigsaw

\[ \nu + \nu \]

\[ a \quad b \]

Invisible system rapidity Jigsaw

\[ \nu + \nu \]

\[ a \quad b \]

Contraboost invariant Jigsaw

\[ \nu \]

\[ a \]

\[ \nu \]

\[ b \]

//draw invisible group tree
FramePlot* INVgroup_plot = new FramePlot("INVtree","Invisible Group");
INVgroup_plot->AddGroupTree(INV);
INVgroup_plot->DrawFramePlot();
RestFrames example: di-leptonic top

B-tagged jet Jigsaws

```cpp
//draw b-tag group tree
FramePlot* BTAGgroup_plot = new FramePlot("BTAGtree","B-tagged Jet Group");
BTAGgroup_plot->AddGroupTree(BTAGS);
BTAGgroup_plot->DrawFramePlot();
```

Minimize m_{b}'s Jigsaw
// check that tree topology is consistent
cout << "Is consistent decay topology? : " << LAB.InitializeTree() << endl;

cout << "Is the tree ok for analysis? : " << LAB.InitializeAnalysis() << endl;
RestFrames example: di-leptonic top

```cpp
TLorentzVector lepA, lepB, b1, b2, MET;
lepA.SetPxPyPzE(1., 0., 0., 1.);
lepB.SetPxPyPzE(0., 1., 0., 1.);
b1.SetPxPyPzE(1., 0., 0., 1.);
b2.SetPxPyPzE(0., 1., 0., 1.);
MET.SetPxPyPzE(-.5, -.5, 0., 1.);

LAB.ClearEvent();

Lep1.SetLabFrameFourVector(lepA);
Lep2.SetLabFrameFourVector(lepB);
BTAGS.AddLabFrameFourVector(b1);
BTAGS.AddLabFrameFourVector(b2);
INV.SetLabFrameFourVector(MET);
// same as
INV.SetLabFrameThreeVector(MET.Vect());

cout << "Analyzed event ok? : " << LAB.AnalyzeEvent() << endl;
```
// example analysis methods
double TTmass = TT.GetMass(); // mass
double costhetaT1 = T1.GetCosDecayAngle(); // decay angle
double P_T2_inTT = T2.GetMomentum(TT); // momentum in some frame
TLorentzVector V_W1_in_W2 = W1.GetFourVector(W2); // 4-vector in some frame
double dphi_T1_W1 = T1.GetDeltaPhiDecayPlanes(W1); // angle between decay planes

tracking of combinatoric elements

GroupElementID b1ID = BTAGS.AddLabFrameFourVector(b1);
GroupElementID b2ID = BTAGS.AddLabFrameFourVector(b2);

// object tracking in combinatoric groups
cout << "Did b1 end up as Bjet1? : ";
cout << BTAGS.GetFrame(b1ID)->IsSame(Bjet1) << endl;
RestFrames setup

- To make shared library ‘lib/libRestFrames.so’
  
  <$ make

- To set environmental variables 
  (ex. for running ROOT macros):

  <$ source setup.sh
To run ‘macros/TestDiLeptonicTop.C’ macro:

<$ root

root [0] .x macros/TestDiLeptonicTop.C
Singularity variables

Kinematic Singularities. A singularity is a point where the local tangent space cannot be defined as a plane, or has a different dimension than the tangent spaces at non-singular points.

From:
Singularity variables

The guiding principle we employ for creating useful hadron-collider event variables, is that: *we should place the best possible bounds on any Lorentz invariants of interest, such as parent masses or the center-of-mass energy $\hat{s}^{1/2}$, in any cases where it is not possible to determine the actual values of those Lorentz invariants due to incomplete event information.*

From:
Example: $M_{CT}$

Assuming ~mass-less leptons

$$M_{CT}^2 = 2 \left( p_T^{\ell_1} p_T^{\ell_2} + \vec{p}_T^{\ell_1} \cdot \vec{p}_T^{\ell_2} \right)$$

Constructed to have a kinematic endpoint at:

$$M_{CT}^{\text{max}} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2}{m_{\tilde{\chi}_1^0}} = M_\Delta$$

From:

Singularity variables (like $M_{CT}$) can be sensitive to quantities that can vary dramatically event-by-event.

Kinematic endpoint ‘moves’ with nonzero CM system $p_T$.
The mass challenge

The invariant mass is invariant under coherent Lorentz transformations of two particles

$$m_{inv}^2(p_1, p_2) = m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2)$$

The Euclidean mass (or contra-variant mass) is invariant under anti-symmetric Lorentz transformations of two particles

$$m_{eucl}^2(p_1, p_2) = m_1^2 + m_2^2 + 2(E_1 E_2 + \vec{p}_1 \cdot \vec{p}_2)$$

Even the simplest case requires variables with both properties!
Correcting for CM $p_T$

- Want to boost from lab-frame to CM-frame
- We know the transverse momentum of the CM-frame:

$$\vec{p}_T^{CM} = \vec{p}_T^{\ell_1} + \vec{p}_T^{\ell_2} + \vec{E}_T^{\text{miss}}$$

- But we don’t know the energy, or mass, of the CM-frame:

$$\vec{\beta}_{lab \rightarrow CM} = \frac{\vec{p}_T^{CM}}{\sqrt{|\vec{p}_T^{CM}|^2 + \hat{S}}}$$
\( p_T \) corrections for \( M_{CT} \)

Attempts have been made to mitigate this problem:

(i) ‘Guess’ the lab \( \rightarrow \) CM frame boost:

\[
M_{CT(\text{corr})} = \begin{cases} 
M_{CT} & \text{after boosting by } \beta = p_b / E_{\text{cm}} \quad \text{if } A_{x(lo)} \geq 0 \text{ or } A'_{x(lo)} \geq 0 \\
M_{CT} & \text{after boosting by } \beta = p_b / \hat{E} \\
M_{Cy} & \text{if } A'_{x(hi)} < 0 \\
M_{Cy} & \text{if } A'_{x(hi)} \geq 0
\end{cases}
\]

\( x \) – parallel to boost

\( y \) – perp. to boost

\[ A_x = p_x[q_1]E_y[q_2] + p_x[q_2]E_y[q_1] \]

\[ M_{Cy}^2 = (E_y[q_1] + E_y[q_2])^2 - (p_y[q_1] - p_y[q_2])^2 \]


(ii) Only look at event along axis perpendicular to boost:

\( M_{CT \perp} \)

`peak position’ of signal and backgrounds due to other cuts ($p_T$, MET) and only weakly sensitive to sparticle masses

From:
CMS-SUS-PAS-13-006
Recursive rest-frame reconstruction

\( M^R_\Delta \) is a singularity variable – in fact it is essentially identical to \( M_{CT} \) but evaluated in a different reference frame. Boost procedure ensures that new variable is invariant under the previous transformations
Resonant Higgs production

\[ H \rightarrow WW^* \rightarrow 2\ell 2\nu \]

The \( \Delta \phi \) between the leptons is evaluated in the \( R \)-frame, removing dependence on the \( p_T \) of the Higgs and correlation with \( \sqrt{s_R} \).

CMS uses 2D fit of variables to measure Higgs mass in this channel.

From:
Resonant Higgs production

\[ H \rightarrow WW^* \rightarrow 2\ell 2\nu \]

The shape of the \( \sqrt{s_R} \) distribution, for the Higgs signal and backgrounds, is used to extract both the Higgs mass and signal strength – even while information is lost with the two escaping neutrinos.

From:
What other info can we extract?

Ex. $M_{T2}$ extremization assigns values to missing degrees of freedom – if one takes these assignments literally, can we calculate other useful variables?

From:

Mass and Spin Measurement with $M(T2)$ and MAOS Momentum - Cho, Won Sang et al.

When we assign unconstrained d.o.f. by extremizing one quantity, what are the general properties of other variables we calculate? What are the correlations among them?
Example: the di-leptonic top basis

A rich basis of useful Recursive Jigsaw observables can be calculated, each with largely independent information

Christopher Rogan - University of Chicago HEP - March 9, 2015
The di-leptonic top basis vs. stops

Observables sensitive to intermediate resonances cannot distinguish between non-resonant signals and background
$t\bar{t} \rightarrow (b\ell\nu)(b\ell\nu)$ vs. $\tilde{t}\tilde{t} \rightarrow (b\tilde{\chi}^{\pm})(b\tilde{\chi}^{\pm}) \rightarrow (b\ell^{\pm}\nu\tilde{\chi}^{0})(b\ell^{\pm}\nu\tilde{\chi}^{0})$
The di-leptonic top basis vs. stops

Decay angles are also sensitive to differences between stop signals and ttbar background
The di-leptonic top basis vs. stops

\begin{align*}
\tilde{m}_t &= 700 \text{ GeV} \\
\tilde{m}_{\chi^\pm} &= 500 \text{ GeV} \\
\tilde{m}_{\chi^0} &= 300 \text{ GeV}
\end{align*}

Decay angles are also sensitive to differences between stop signals and ttbar background.

Christopher Rogan - University of Chicago HEP - March 9, 2015
The di-leptonic top basis vs. stops

\[ m_{\tilde{t}} = 700 \text{ GeV} \]
\[ m_{\tilde{\chi}^\pm} = 500 \text{ GeV} \]
\[ m_{\tilde{\chi}^0} = 300 \text{ GeV} \]

Decay angles are also sensitive to differences between stop signals and ttbar background.
The di-leptonic top basis vs. stops

Here, the azimuthal angle between the top and W decay planes $\Delta \phi_{T1,W1}$, for each of the two decay chains

$m_{\tilde{t}} = 700$ GeV

$650$

$m_{\tilde{\chi}^\pm} = 500$ GeV

$340$

$m_{\tilde{\chi}^0} = 300$ GeV

Christopher Rogan - University of Chicago HEP - March 9, 2015
The di-leptonic top basis vs. stops

Here, the azimuthal angle between the top and W decay planes $\Delta \phi_{T1,W1}$ and the angle between the two top decay planes $\Delta \phi_{T1,T2}$

$m_{\tilde{t}} = 700 \text{ GeV}$

$m_{\tilde{\chi}^\pm} = 500 \text{ GeV}$

$m_{\tilde{\chi}^0} = 300 \text{ GeV}$
Razor kinematic variables

- Assign every reconstructed object to one of two mega-jets.
- Analyze the event as a ‘canonical’ open final state:
  - two variables: $M_R$ (mass scale), $R$ (scale-less event imbalance).
- An inclusive approach to searching for a large class of new physics possibilities with open final states.

Razor variables

CMS+ATLAS analyses

arXiv:1006.2727v1 [hep-ph]
PRD 85, 012004 (2012)
EPJC 73, 2362 (2013)
PRL 111, 081802 (2013)
CMS-PAS-SUS-13-004
Razor kinematic variables

- Assign every reconstructed object to one of two mega-jets.
- Analyze the event as a ‘canonical’ open final state:
  - two variables: $M_R$ (mass scale), $R$ (scale-less event imbalance)

\[
M_R \sim \sqrt{s} \quad R = \frac{M_T^R}{M_R} \sim \frac{M_\Delta}{\sqrt{s}}
\]

Two distinct mass scales in event
Two pieces of complementary information
**Baseline Selection**

- Exactly two opposite sign leptons with $p_T > 20$ GeV/c and $|\eta| < 2.5$
- If same flavor, $m(\ell \ell) > 15$ GeV/c²
- $\Delta R$ between leptons and any jet (see below) > 0.4
- Veto event if b-tagged jet with $p_T > 25$ GeV/c and $|\eta| < 2.5$

**Kinematic Selection**

- ‘CMS selection’
  
  \[ |m(\ell \ell) - m_Z| > 15 \text{ \, GeV} \]
  \[ E_{T}^{\text{miss}} > 60 \text{ \, GeV} \]

- ‘ATLAS selection’
  
  \[ |m(\ell \ell) - m_Z| > 10 \text{ \, GeV} \]
  \[ E_{T}^{\text{miss,rel}} = \begin{cases} 
  E_{T}^{\text{miss}} & \text{if } \Delta\phi_{\ell,j} \geq \pi/2 \\
  E_{T}^{\text{miss}} \times \sin \Delta\phi_{\ell,j} & \text{if } \Delta\phi_{\ell,j} < \pi/2 
  \end{cases} \]
  \[ > 40 \text{ \, GeV} \]
1D Shape Analysis

Analysis Categories

- Consider final 9 different final states according to lepton flavor and jet multiplicity – simultaneous binned fit includes both high S/B and low S/B categories

\[(ee, \mu\mu, e\mu) \times (0, 1, \geq 2 \text{ jets}) \quad \text{with} \quad p_T^{\text{jet}} > 30 \text{ GeV/c}, \ |\eta^{\text{jet}}| < 3\]

Fit to kinematic distributions (in this case, \(M_{\Delta R}, M_{T2}\) or \(M_{CT\text{perp}}\) in 10 GeV bins), over all categories for \(WW, \ tt\) and \(Z/\gamma^* + \text{jets}\) yields

From PRD 89, 055020

Other jet multiplicity and lepton flavor categories
Systematic uncertainties

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])

- 2% lepton ID (correlated btw bkg, uncorrelated between lepton categories)
- 10% jet counting (per jet) (uncorrelated between all processes)
- 10% x-section uncertainty for backgrounds (uncorrelated) + theoretical x-section uncertainty for signal (small)
- ‘shape’ uncertainty derived by propagating effect of 10% jet energy scale shift up/down to MET and recalculating shapes templates of kinematic variables
- Uncertainties are introduced into toy pseudo-experiments through marginalization (pdfs fixed in likelihood evaluation but systematically varied in shape and normalization in toy pseudo-experiment generation)
Compared to Reality

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])

\[ pp \rightarrow \tilde{\ell}_L \tilde{\ell}_L; \quad \tilde{\ell}_L \rightarrow \tilde{\chi}_1^0 \ell \]
Expected Limit Comparison

\[ \sqrt{s} = 8 \text{ TeV} \int L \, dt = 20 \text{ fb}^{-1} \]

MadGraph+PGS

\[ \tilde{t}_L \tilde{t}_L \rightarrow \tilde{\chi}_0^0 \tilde{\chi}_0^0 \]

M_{T2} + ATLAS selection

\[ m_{\tilde{t}_L} = m_{\tilde{\chi}_0^0} \]

\[ \chi \sim \tilde{\chi}_0 \]

\[ \sqrt{s} = 8 \text{ TeV} \int L \, dt = 20 \text{ fb}^{-1} \]

MadGraph+PGS

\[ \tilde{t}_L \tilde{t}_L \rightarrow \tilde{\chi}_0^0 \tilde{\chi}_0^0 \]

M_{A^0} + ATLAS selection

\[ m_{\tilde{t}_L} = m_{\tilde{\chi}_0^0} \]

\[ \chi \sim \tilde{\chi}_0 \]

\[ \sqrt{s} = 8 \text{ TeV} \int L \, dt = 20 \text{ fb}^{-1} \]

MadGraph+PGS

\[ \tilde{t}_L \tilde{t}_L \rightarrow \tilde{\chi}_0^0 \tilde{\chi}_0^0 \]

M_{CT1} + CMS selection

\[ m_{\tilde{t}_L} = m_{\tilde{\chi}_0^0} \]

\[ \chi \sim \tilde{\chi}_0 \]

\[ \sqrt{s} = 8 \text{ TeV} \int L \, dt = 20 \text{ fb}^{-1} \]

MadGraph+PGS

\[ \tilde{t}_L \tilde{t}_L \rightarrow \tilde{\chi}_0^0 \tilde{\chi}_0^0 \]

M_{A^0} + CMS selection

\[ m_{\tilde{t}_L} = m_{\tilde{\chi}_0^0} \]

\[ \chi \sim \tilde{\chi}_0 \]

From PRD 89, 055020 (arXiv:1310.4827)
Charginos

From PRD 89, 055020 (arXiv:1310.4827)
Super-Razor Basis Selection

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])