Technical Mumbo-Jumbo

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1 Introduction

If you read the literature about K^0 decay experiments you will find lots of physicist jargon that has not been used in the "*Anti-Matter in the Universe*" presentation. Below I will make an attempt to relate the concepts in this talk with some of the common technical mumbo-jumbo that we physicists like to use. The discussion below assumes that you have a good understanding of the "*Anti-Matter in the Universe*" presentation and that you have some familiarity with quantum mechanics.

2 What is CP Violation?

The key point in this talk is that we have experimentally demonstrated the "partial decay loophole," in which a particle (K^0) and its anti-particle ($\bar{K}^0$) have a slightly different decay rate into a particular channel ($\pi\pi$). In physics jargon this is called "CP violation." To explain this I must first tell you what C and P refer to.

C is a quantum mechanical operator that flips particles into anti-particles. P applies a parity transformation, $x, y, z \rightarrow -x, -y, -z$. Next recall that K^0 and $\bar{K}^0$ mix into each-other five billion times per second. In the language of quantum mechanics, the physical states are super-positions of K^0 and $\bar{K}^0$:

$$
\begin{aligned}
K_{even} &= K^0 + \bar{K}^0 \qquad (1) \\
K_{odd} &= K^0 - \bar{K}^0 \qquad (2)
\end{aligned}
$$

The *even* and *odd* sub-scripts refer to the CP eigen-states. Let's take that last step a little more slowly. The parity is the same for both K^0 and $\bar{K}^0$ so the parity operator has no effect on K_{even} and K_{odd}. Now we can apply the CP operator to K_{even},

$$CP[K_{even}] = CP[K^0 + \bar{K}^0] = [\bar{K}^0 + K^0] = +[K_{even}] \qquad (3)$$

so we say that it is an even state of CP. Now apply C to the K_{odd}, and you get a very important minus sign:

$$CP[K_{odd}] = CP[K^0 - \bar{K}^0] = [\bar{K}^0 - K^0] = -[K_{odd}] \qquad (4)$$

The two-pion final state has even CP,

$$CP[\pi\pi] = +[\pi\pi] \qquad (5)$$

so that we expect only the CP-even kaon to decay into two pions, if CP is a good symmetry. To see how this relates to the particle vs. anti-particle concept, we must use the language of decay "*amplitudes*" in quantum mechanics. Physical observables are always the square of decay amplitudes, but interesting things can happen when you add amplitudes before squaring them. Define

$$\mathcal{A} = Ampl(K^0 \to \pi\pi) \qquad \bar{\mathcal{A}} = Ampl(\bar{K}^0 \to \pi\pi) \ . \tag{6}$$

If CP is a good symmetry then $\mathcal{A} = \bar{\mathcal{A}}$; i.e., the particle and anti-particle decay amplitudes are the same. For the mixed states in Eqs 1,2 we must add/subtract amplitudes before squaring. Assuming CP symmetry is good, the amplitude for K_{even} is $\mathcal{A} + \bar{\mathcal{A}} = 2\mathcal{A}$ while the amplitude for K_{odd} is $\mathcal{A} - \bar{\mathcal{A}} = 0$! This shows again that K_{odd} cannot decay into two pions unless CP symmetry is violated.

So how does K_{odd} decay ? It turns out that a 3π state has odd CP, and indeed about 1/3 of the K_{odd} decay are into three pions. The remaining 2/3 of the K_{odd} decay into either $\pi e\nu$ or $\pi\mu\nu$. Furthermore, the decay $K_{odd} \to 3\pi$ is *much* slower than the decay $K_{even} \to 2\pi$. Thus we can create a pure K_{odd} beam by simply waiting until all the K_{even} have decayed.

3 Discovery of CP Violation

In 1964, the odd kaon was found to decay into two pions; $K_{odd} \to 2\pi$. This was a remarkable discovery! From the discussion above it seemed that the K^0 had to have a different decay amplitude than $\bar{K}^0$; i.e., $\mathcal{A} \neq \bar{\mathcal{A}}$. But nature was just teasing us. After many more experiments it was realized that the even and odd kaon states above are not quite what nature created. Instead the true kaon states are:

$$K_S = 1.00228K^0 + 0.99772\bar{K}^0 \tag{7}$$

$$K_L = 1.00228K^0 - 0.99772\bar{K}^0 \tag{8}$$

which are slightly asymmetric mixtures of K^0 and $\bar{K}^0$. The K_S is called K-short because it decays rapidly into two pions with a relatively "short" life-time. The K_L is called K-long because it takes a relatively "long" time before it decays into three pions. The mixing asymmetry in Eq. 7,8 is indeed CP violating. It shows that $\bar{K}^0 \to K^0$ is slightly preferred over $K^0 \to \bar{K}^0$, which demonstrates a slight difference between a matter particle and its anti-matter partner. However, the mixing asymmetry does *not* involve decays, and we really believe that CP violation should also manifest itself in decays. Despite the lack of CP violation in the decay, this discovery in 1964 was profound and eventually earned the discovery team a Nobel prize in 1981. What the original discovery actually found was $K_L \to \pi\pi$, and from Eq. 8 we see that it now has a non-zero decay amplitude of $1.00228\mathcal{A} - 0.99772\bar{\mathcal{A}} \simeq 0.00456\mathcal{A}$. Before moving on there is some more physics jargon: the mixing asymmetry in Eqs 7,8 is known as "*indirect*" CP violation. An asymmetry in the decay, yet to be discovered, is know as "*direct*" CP violation.

Following the discovery in 1964, a holy quest began to find an asymmetry in the decay; i.e., to show that $\mathcal{A} \neq \bar{\mathcal{A}}$. However, the mixing asymmetry poses a serious experimental difficulty. To see this, suppose that the K^0 and $\bar{K}^0$ decay amplitudes differ by 1 part in $10,000$. The problem is that instead of finding $\mathcal{A} \neq \bar{\mathcal{A}}$ we would determine the mixing asymmetry to be

$$K'_S = 1.00218K^0 + 0.99782\bar{K}^0 \quad (9)$$

$$K'_L = 1.00218K^0 - 0.99782\bar{K}^0 \quad (10)$$

i.e., the asymmetry factors 1.00228 and 0.99772 in Eqs 7,8 would have been measured as 1.00218 and 0.99782 and we could never tell that the kaon decay amplitudes $\mathcal{A}$ and $\bar{\mathcal{A}}$ were different. From many other kaon experiments we know that the difference in decay amplitudes must be much smaller than the 0.228% mixing asymmetry, but how can we ever un-tangle the *direct* CP violation from the much more dominant *indirect* CP violation?

4 Direct CP Violation

The key to finding *direct* CP violation is to compare the decays for two different modes, $K_L \to \pi^+\pi^-$ and $K_L \to \pi^0\pi^0$. We essentially measure the 0.228% mixing asymmetry for the charged ($\pi^+\pi^-$) and neutral ($\pi^0\pi^0$) decay mode; if the two mixing asymmetries are different, then this must be due to a difference in the decay amplitudes.

This leads us to the final piece of jargon known as $Re(\epsilon'/\epsilon)$, pronounced *real part of epsilon-prime over epsilon*. The quantity $\epsilon = 0.00228$ is just the size of the asymmetry in Eqs 7,8. The quantity ϵ' is what we really want to measure because it is:

$$\epsilon' \simeq \frac{\mathcal{A} - \bar{\mathcal{A}}}{\mathcal{A} + \bar{\mathcal{A}}} \tag{11}$$

i.e., the fractional difference in the decay amplitudes[1]. As shown in Appendix A, we can experimentally measure the ratio $Re(\epsilon'/\epsilon)$, where "*Re*" refers to the real part of the complex amplitude. A non-zero value of $Re(\epsilon'/\epsilon)$ is un-ambiguous evidence of direct CP violation in a decay. This situation is very fortunate because it allows us to determine ϵ' about 1000 times more precisely than the experimental precision. To see this,

[1] For clarity I have used a slightly modified definition of ϵ' here.

suppose that $\epsilon' \sim 2 \times 10^{-6}$. Then $Re(\epsilon'/\epsilon) = 2 \times 10^{-6}/0.0023 \simeq 0.001$, so we need to do an experiment with part-per-thousand accuracy to have part-per-million sensitivity on ϵ'. This is not cheating; it's just picking a clever experiment!

To give an example of determining something much more accurately than your experimental precision, suppose you want to measure the length *difference* of two football fields to make sure that neither stadium has cheated on the distance. You get a ball of string and a 12" ruler to work with. If you try to measure each football field with the 12" ruler, you will generate a very larger error. However, if you are clever, you might cut yourself a piece of string that has exactly the length of one football field, and then compare the length of this string with the other football field. You won't know if the football fields are exactly 100 yards long, but you will be able to accurately measure any difference to much better than an inch! The key is to have a common reference of length. For the kaons, the nearly even mixture of K^0 and $\bar{K}^0$ in the K_L provides the common reference. If the mixture were exactly even, i.e, if $K_L = K^0 - \bar{K}^0$, then seeing just one $K_{odd} \rightarrow \pi\pi$ decay would be sufficient to determine that $\mathcal{A} \neq \bar{\mathcal{A}}$. The mixing asymmetry adds a complication, but this can be overcome by comparing the charged and neutral pion decay modes.

In terms of $Re(\epsilon'/\epsilon)$, the average of the four experiments (two from Fermilab and two from CERN) is about $(17 \pm 2) \times 10^{-4}$. This rather obscure way of presenting the result is the asymmetry in the decay amplitudes relative to the mixing asymmetry. In this talk I simply converted the amplitude-asymmetry to the decay rate asymmetry between

K^0 and $\bar{K}^0$. If you like math, the derivation is given in Appendix B. The result in terms of the decay rates (R) is

$$\frac{R(K^0 \to \pi^+\pi^-) - R(\bar{K}^0 \to \pi^+\pi^-)}{R(K^0 \to \text{all})} = (7.4 \pm 0.7) \times 10^{-6} \qquad (12)$$

which is less than 1 part per hundred thousand.

5 Time Reversal and the CPT Theorem

The last thing that I would like to discuss is an *operator* called time reversal (T). No it does NOT mean going backward in time! The application of time-reversal is more like playing a movie backward; if what you see in the reversed video is allowed by the laws of physics then we say that T is a good symmetry. Note that breaking glass, or water rushing out of a hose, does *not* violate T; the reverse situations are indeed allowed by the laws of physics, but the probability is too small to ever see a broken glass re-assemble itself, or water fly back into a hose.

The reason for bringing up T is so I can discuss the CPT theorem. This is a general and rigorous proof that the laws of physics must be invariant under the combined operations of C, P and T. So if you play a movie backward (T), look at it in a mirror (P) and change everything into anti-matter (C), the laws of physics must be the same. There is NO such proof for any subset, which means that combinations like CP, CT, PT, P and C can all be violated without contradicting quantum field theory. So what's the difference between CP symmetry and CPT symmetry?

CPT symmetry implies that particles and anti-particles have exactly the same mass and exactly the same total decay rate. It is a very strong statement, and clever experiments have shown this with sensitivities of parts-per billion and better! The record is held by the neutral kaon; the kaon and anti-kaon mass difference is less than 1 part in 10^{18} ! CP symmetry says that **partial** decay rates are the same if a particle has more than one channel to decay into. To see how this works with kaon decays, let R and $\bar{R}$ be the total decay rates for K^0 and $\bar{K}^0$, respectively. The total decay rate is the sum of partial decay rates,

$$R = R_{\pi^+\pi^-} + R_{\pi^0\pi^0} + \sum_i r_i \quad (13)$$

$$\bar{R} = \bar{R}_{\pi^+\pi^-} + \bar{R}_{\pi^0\pi^0} + \sum_i \bar{r}_i \quad (14)$$

where $R_{\pi^+\pi^-}$ and $R_{\pi^0\pi^0}$ are the decay rates into charged and neutral pions, and r_i are the decay rates into rare channels such as $\pi e\nu$, $\pi\mu\nu$, $3\pi^0$ and $\pi^+\pi^-\pi^0$.

CPT symmetry demands that $R = \bar{R}$. CP symmetry implies equality for each partial decay rate, i.e., $R_{2\pi} = \bar{R}_{2\pi}$ and $r_i = \bar{r}_i$ for each i. Assuming that CPT symmetry is good, a CP violating asymmetry in one decay mode must be compensated by CP violation in another decay mode. To see this re-write the asymmetry in Eq. 12 as

$$Asym(\pi^+\pi^-) = (R_{\pi^+\pi^-} - \bar{R}_{\pi^+\pi^-})/R \quad (15)$$

$$Asym(\pi^0\pi^0) = (R_{\pi^0\pi^0} - \bar{R}_{\pi^0\pi^0})/R \quad (16)$$

Neglecting possible CP violating contributions from the rare channels r_i, the total decay asymmetry is

$$Asym(tot) = Asym(\pi^+\pi^-) + Asym(\pi^0\pi^0) \quad (17)$$

But CPT symmetry says that the *total* decay asymmetry must be zero,

$$Asym(tot) = (R - \bar{R})/R = 0 \tag{18}$$

which shows that the $\pi^+\pi^-$ and $\pi^0\pi^0$ partial decay asymmetries must be equal-and-opposite because of CPT symmetry.

If CPT symmetry is good, which all experiments to date have verified, and CP symmetry is violated, then doesn't that suggest T violation? Absolutely yes! T-violation refers to microscopic irreversibility. Recall that broken glass and water out of a hose are *not* T-violating because the reverse situations are forbidden by probability rather than by any fundamental laws of physics. T-violation is easiest to explain in the kaon asymmetry since $\bar{K}^0 \to K^0$ is preferred over $K^0 \to \bar{K}^0$ by a meager 0.23%.

APPENDIX

A Derivation of $Re(\epsilon'/\epsilon)$

The key point is that we measure experimentally the double ratio ($\mathcal{DR}$) of decay rates,

$$\mathcal{DR} \equiv \frac{R(K_L \to \pi^+\pi^-)/R(K_S \to \pi^+\pi^-)}{R(K_L \to \pi^0\pi^0)/R(K_S \to \pi^0\pi^0)} \simeq 1 + 6Re(\epsilon'/\epsilon) \qquad (19)$$

The derivation follows using the fact that

$$\epsilon'_{+-} = \epsilon' \qquad (20)$$

$$\epsilon'_{00} = -2\epsilon' \qquad (21)$$

where ϵ'_{+-} is the fractional K^0-$\bar{K}^0$ amplitude differences for $\pi^+\pi^-$ decays and ϵ'_{00} is the same quantity for $\pi^0\pi^0$ decays.

(need to finish this section).

B Derivation of the Decay Asymmetry

This appendix converts the measured quantity $Re(\epsilon'/\epsilon)$ into a decay rate asymmetry between $K^0 \to \pi\pi$ and $\bar{K}^0 \to \pi\pi$.

The asymmetry that we want to determine is

$$a \equiv \frac{R(K^0 \to \pi^+\pi^-) - R(\bar{K}^0 \to \pi^+\pi^-)}{R(K^0 \to \pi^+\pi^-)} \qquad (22)$$

$$= \frac{|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2}{|\mathcal{A}|^2} = 1 - |\bar{\mathcal{A}}/\mathcal{A}|^2 \qquad (23)$$

where $\mathcal{A}$ ($\bar{\mathcal{A}}$) is the decay amplitude for the K^0 ($\bar{K}^0$) to decay into a $\pi^+\pi^-$ pair. Now recall that ϵ' is the amplitude-asymmetry defined as

$$\epsilon' = (\mathcal{A} - \bar{\mathcal{A}})/(\mathcal{A} + \bar{\mathcal{A}}) \tag{24}$$

which can be inverted to solve for $\bar{\mathcal{A}}/\mathcal{A} = 1 - 2\epsilon'$. In Eq. 23 we need the square of $\bar{\mathcal{A}}/\mathcal{A}$. We must be careful since these are complex amplitudes. Noting that $\epsilon'^\star$ is the complex conjugate of ϵ',

$$|\bar{\mathcal{A}}/\mathcal{A}|^2 = (1 - 2\epsilon') \cdot (1 - 2\epsilon'^\star) = 1 - 4Re(\epsilon') + |\epsilon'|^2 \tag{25}$$

$$\simeq 1 - 4Re(\epsilon') \tag{26}$$

where I have dropped the very small term $|\epsilon'|^2$. Now just plug Eq. 26 into Eq. 23 and we get the decay asymmetry,

$$a = 1 - [1 - 4Re(\epsilon')] = 4Re(\epsilon') \tag{27}$$

Next we have to convert $Re(\epsilon')$ in terms of the experimentally determined quantity $Re(\epsilon'/\epsilon)$.

$$Re(\epsilon') = Re(\epsilon \cdot \frac{\epsilon'}{\epsilon}) = [Re(\epsilon) \times Re(\epsilon'/\epsilon) - Im(\epsilon) \times Im(\epsilon'/\epsilon)] \tag{28}$$

where "Re" and "Im" refer to the real and imaginary parts of the complex numbers. To reduce this further we note that ϵ and ϵ' are complex quantities that make a 45^0 angle (also called phase) in the complex plane. It is fortunate the ϵ and ϵ' are parallel[2] because this means that $Im(\epsilon'/\epsilon) \simeq 0$ and we can ignore the imaginary terms in Eq. 28. We now have

$$Re(\epsilon') \simeq Re(\epsilon) \times Re(\epsilon'/\epsilon) = 0.00228 \times 0.00212/\sqrt{2} \tag{29}$$

$$= 3.42 \times 10^{-6} \tag{30}$$

[2]a proof of this is beyond the scope of this presentation.

The $\sqrt{2}$ in the denominator comes from the complex 45^0 phase, which leads to a term of $\cos(45^0) = 1/\sqrt{2}$. I don't have a good explanation for this phase so you will just have to take it in good faith.

The last step is to note that the denominator in Eq. 22 is the partial decay rate into $\pi^+\pi^-$. Instead we would like the total decay rate to be in the denominator. Since the K^0 decays into $\pi^+\pi^-$ 68.6% of the time (and 31.4% into $\pi^0\pi^0$), the total decay rate is $1/0.686 = 1.458$ times larger than the partial decay rate into $\pi^+\pi^-$. Thus define a new asymmetry

$$Asym(\pi^+\pi^-) \equiv \frac{R(K^0 \to \pi^+\pi^-) - R(\bar{K}^0 \to \pi^+\pi^-)}{R(K^0 \to \text{all})} \tag{31}$$

$$= 0.686 \times a = 2.744 Re(\epsilon') = 7.4 \times 10^{-6} \ . \tag{32}$$