The Glashow-Iliopoulos-Maiani (GIM-) mechanism-suppressed processes $K^+ \to \pi^+ \nu \bar{\nu}$ (Refs. 2-5) and $K^0_L \to \pi^0 e^+ e^-$ (Ref. 6) have been much discussed recently as tests of the standard model (SM). In each case the current experimental limit$^{7,8}$ lies more than 2 orders of magnitude above the SM prediction, affording a large window for new physics. If the predicted levels can be reached, these decays put interesting constraints on the Kobayashi-Maskawa (KM) matrix parameters and on the top-quark mass. The latter process is particularly interesting from the point of view of CP since the predicted direct CP violation is of the same order of magnitude as the indirect (state-mixing) contribution. By contrast, relatively little attention has been paid to the closely related and no less interesting process $K^0_L \to \pi^0 \nu \bar{\nu}$ (Ref. 10). As I will discuss below, this decay is expected to have a branching ratio of $\sim 10^{-11}$. Since there is no published upper limit on this decay, it offers a potentially enormous range in which to search for new effects. As in the case of $K_L \to \pi^0 e^+ e^-$, $K^0_L \to \pi^0 \nu \bar{\nu}$ is CP violating in leading order. However, unlike the former process, there is no potentially large, $2\gamma$-mediated CP-conserving contribution.$^{11}$ In fact the potential long-distance contributions in general are suppressed by CP violation and/or the GIM mechanism to extremely small levels.

In the excellent approximation that $K^+ \to \pi^+ \nu \bar{\nu}$ and $K^0_L \to \pi^0 \nu \bar{\nu}$ are short-distance dominated,$^{12}$ their amplitudes are related by isospin: $A(K^0 \to \pi^0 \nu \bar{\nu}) = (1/\sqrt{2}) A(K^+ \to \pi^+ \nu \bar{\nu})$. It then follows that the amplitudes for decays of the CP eigenstates $K^0_1$ and $K^0_2$ into $\pi^0 \nu \bar{\nu}$ are equal to the real and imaginary parts, respectively, of the amplitude for $K^+ \to \pi^+ \nu \bar{\nu}$ (Ref. 10). Ignoring higher-order CP-violating effects,

$$A(K^0 \to \pi^0 \nu \bar{\nu}) = \epsilon A(K^0 \to \pi^0 \nu \bar{\nu}) + A(K^0 \to \pi^0 \nu \bar{\nu})$$

In principle this leads to interference effects, but as will be shown, the first term is so much smaller than the second that these can be ignored. Note that modulo very small QCD corrections and assuming massless leptons,$^2$$^3$ for each neutrino flavor, where $V_{ij}$ are the KM matrix elements, $x_j = (m_j^2/m_W^2)$, and $D(x)$ is a kinematic function which is $\sim 0.004$ for $m_z$, and of order 1 for reasonable values of $m_t$. Substituting for the constants and the $s_3$ branching ratio, assuming small mixing angles, and ignoring QCD corrections,$^{13}$

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = \frac{2\alpha^2}{16\pi^2 \sin^4 \theta_W}$$

$$\times \left| \sum_{j = c, t} V_{ij}^* V_{jd} D(x_j) \right|^2 / V_{us}^2$$

for each neutrino flavor, where $s_2$, $s_3$, and $\delta$ are the usual KM parameters. Currently favored values of the KM parameters and $m_t$ give $0.5 - 8.0 \times 10^{-10}$ for the branching ratio summed over three neutrino flavors.$^5$ The branching ratio for the indirect CP-violating contribution is then

$$B(K_L^0 \to \pi^0 \nu \bar{\nu})_\epsilon = \left| c_4 \frac{\tau_{K_L}}{\tau_{K^+}} \times [D(x_e) + s_2(s_2 + s_3 c_3) D(x_e)] \right|^2$$

while that of the direct is

$$B(K_L^0 \to \pi^0 \nu \bar{\nu})_{\text{direct}} = \frac{\tau_{K_L}}{\tau_{K^+}} \times [s_2 s_3 s_8 D(x_e)]^2$$

In the context of the standard model with three generations, bounds have been derived$^5$ on $s_2$, $s_3$, and to some extent on $\delta$ and $m_t$, from measurements of or limits on $\tau_5$, $B(b \to c e \nu)$, $\bar{B} - B$ mixing, $\Gamma(b \to u e \nu)/\Gamma(b \to c e \nu)$, exclusive $B$ decay branching ratios, $\epsilon$, $\epsilon'$, etc. Neither experiment nor theory is sufficiently advanced to allow specific predictions, but sets of parameters which are con-
sistent with all known input can be obtained. For the purpose of estimating the size of the various contributions to $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$, I choose one such set: $s_2 = 0.08$, $s_3 = 0.03$, $s_6 = 165^\circ$, $m_1 = 100$ GeV, $m_2 = 1.5$ GeV. Then, ignoring QCD corrections, for three generations, this gives

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 2.5 \times 10^{-10}$$

and remembering that $|\epsilon| = 0.00275$,

$$B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})_e = 5.18 \times 10^{-6} \times 4.18 \times 3$$

$$\times 0.70 \times 10^{-6}(10.99 \times 10^{-3})^2$$

$$\approx 5.5 \times 10^{-15}$$

$$B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})_{\text{direct}} = 4.18 \times 3$$

$$\times 0.70 \times 10^{-6}(1.07 \times 10^{-3})^2$$

$$\approx 10^{-11}.$$  

Note that although the real part of the $K^+$ amplitude is 10 times as large as the imaginary for these values of the parameters, this is far outweighed by the small value of $\epsilon$. The ratio of direct to indirect $CP$ violation is roughly $1800$. This is to be contrasted with the case of $K_L^0 \rightarrow \pi^0 e^+ e^-$ where this ratio is $\sim 1:1$ and $K^0 \rightarrow 2\pi$ where it is $\sim 1:300$. In addition, as in the case of $K_L^0 \rightarrow \pi^0 e^+ e^-$, but not of $K^0 \rightarrow 2\pi$, the relevant hadronic matrix element need not be calculated ab initio, but as we have seen, may be obtained from the known rate of $K_{s3}$ decay. Thus from the point of view of the standard model, one has the potential for an extremely clean determination of the $CP$-violating product $s_2 s_3 s_6$, if one can measure $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ at the requisite level. As will be discussed below, such an experiment is very difficult. However, as will also be discussed, the effective present bound on this process is quite weak, so that a huge window for new physics (nonstandard $CP$-violating currents, $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ + new scalar, etc.) exists.

Although there has been no dedicated search for $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ our ignorance of the branching ratio is not quite complete. Following a suggestion by Hoffman, it is possible to extract such a limit from the work of Cronin et al., an early measurement of $K_L^0 \rightarrow 2\pi^0$. In that experiment, the signal for $K_L^0 \rightarrow 2\pi^0$ was a single photon whose c.m. energy was greater than that possible from $K_L^0 \rightarrow 3\pi^0$ (maximum 165 MeV) and less than that of $K_L^0 \rightarrow 2\nu$ (249 MeV). The photon spectrum from $K_L^0 \rightarrow 2\pi^0$ is a flat box extending from 19 to 229 MeV. The interval between 180 and 225 MeV was selected as the signal region. The overall acceptance for photons from the normalizing reaction, $K_L^0 \rightarrow 3\pi^0$, was 0.269 times that for those photons from $K_L^0 \rightarrow 2\pi^0$ that fall into this interval. As shown in Fig. 1(a), photons from $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ also span the interval from 180 to 225 MeV (Ref. 20) although on the average they are somewhat softer than those from $K_L^0 \rightarrow 2\pi^0$. As a result, the experimental efficiency in the 180–225-MeV range for photons from the former decay is about 2% less than that for photons from the latter decay. Furthermore, this interval contains a larger fraction of the photons produced in the latter process than in the former (0.214 vs 0.135). There were 156 events observed in the interval 180–225 MeV.

To be conservative, I do not subtract the contribution of $K_L^0 \rightarrow 2\pi^0$ or other backgrounds and use 181 events as an upper limit for $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ events. There were 4031 $K_L^0 \rightarrow 3\pi^0$ events in the normalizing sample. A limit on the branching ratio for $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ can then be extracted as follows:

$$B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu}) < \frac{B(K_L^0 \rightarrow 3\pi^0)(N_{\pi^0 \nu \bar{\nu}}/N_{3\pi^0})}{\epsilon_3/\epsilon_2}(\epsilon_2/\epsilon_{\pi^0 \nu \bar{\nu})^3},$$

where the $\epsilon$'s are the acceptances for single photons from the various processes and the factor 3 is the ratio of the number of photons emanating from $K_L^0 \rightarrow 3\pi^0$ to the number emanating from $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$. As discussed above $\epsilon_{\pi^0 \nu \bar{\nu}} = 0.269$ and $\epsilon_3/\epsilon_2 = 1.02 \times 0.214/0.135 = 1.62$. This then yields

$$B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu}) < 0.13 \text{ (90\% c.l.)}.$$  

A tighter bound can be obtained by subtracting the backgrounds calculated in Ref. 19 as well as the expected contribution from $K_L^0 \rightarrow \pi^0 \pi^0$. This leaves 85\pm12 events

![FIG. 1. Distributions of daughters of $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$. (a) Distribution of gamma energy in the $K_L^0$ center of mass. Vertical lines indicate region used to obtain the upper limit discussed in the text. (b) Distribution of $p_T$ of $\pi^0$ $p_T$. Vertical line indicates maximum $p_T$ of $\pi^0$ from $K_L^0 \rightarrow \pi^0 \pi^0$.](image-url)
in the signal region. The corresponding 90\%-C.L. upper limit is then

$$B(K_L^0 \rightarrow \pi^0 \nu \overline{\nu}) < 7.6 \times 10^{-3}.$$  

There is a window of almost 9 orders of magnitude between this number and the standard-model prediction. The detection of this decay at the $\sim 10^{-12}$ level is surely a formidable experimental challenge. Searches at the $10^{-10}$–$10^{-11}$ level for the kinematically similar (but topologically much more tractable) process $K_L^0 \rightarrow \pi^0 \nu \overline{\nu}$ have been proposed, and the $K_L^0$ flux required for these would probably suffice to reach a sensitivity about an order of magnitude worse for $K_L^0 \rightarrow \pi^0 \nu \overline{\nu}$. The signature would be an unaccompanied $\pi^0$ emerging out of a $K_L^0$ beam. It might be necessary for triggering (and background rejection) purposes to confine the acceptance to $\pi^0 p_T > 209$ MeV/c, the maximum $p_T$ allowed for a $\bar{K}_L^0 \rightarrow 2\pi^0$ decay. According to Fig. 1(b), this cut accepts $\sim 0.9$% of the signal (more like 7% when a reasonable allowance for 3% of the signal (more like 7% when a reasonable allowance for 3% of the signal is accounted for). The only significant decays which can produce $\pi^0$s with $p_T > 209$ MeV/c are $K_L^0 \rightarrow \pi^0 \nu \overline{\nu}$, $\pi^0 \mu^+ \mu^-$, and $\pi^0 \nu \overline{\nu}$. The first is expected to be of order $10^{-11}$ and is easily eliminated. The second, $K_L^0 \rightarrow \pi^0 \nu \overline{\nu}$, is expected at $\sim 10^{-6}$ and must therefore be vetoed to at least $10^{-6}$ ($\sim 10^{-3}$ photon) which seems possible. If extra gammas can indeed be vetoed at the $10^{-3}$ level, and the beam is well defined, the dominant background is likely to be that due to $\pi^0$ production by beam neutrons off residual gas atoms in the vacuum decay tank. These will have a $p_T$ spectrum which spans the signal region. This background will necessitate extremely good vacuum, and probably the use of neutron-sensitive calorimetry down to small angles with respect to the beam.

To define the kinematics in the presence of background it will be necessary to determine the direction of at least one of the gammas from $K_L^0 \rightarrow \pi^0 \nu \overline{\nu}$. If this is done through conversion to $e^+ e^-$, it will cost a factor $\sim 10/10$ in the conversion rate, and will use only those events in which the $\pi^0$ undergoes Dalitz decay. It would certainly be preferable to eliminate the need for either expedient, perhaps through the advent of directional photon detectors. Although the $p_T$ cut mentioned above will limit the acceptance for this decay to less than $\sim 7\%$, the fact that only two gammas need be detected out of a relatively large geometric acceptance, leading to the $10^{-9}$–$10^{-10}$ estimate made above.

A measurement of $K_L^0 \rightarrow \pi^0 \nu \overline{\nu}$ at the $10^{-12}$ level seems clearly beyond the present state of the art, but with the development of improved detectors and kaon sources, such a measurement might become possible on a time scale competitive with that for measurements of CP violation in the B system. In any case a measurement at even the $10^{-8}$ level would be of great interest.

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13. QCD corrections here primarily have the effect of reducing the charm-quark contribution by 30% or 40%. For a discussion of the numerical factors, see Ref. 12.
15. Note that the disparity in size of the two terms justifies our ignoring the interference. Variation of the parameters within limits given by present data maintain the ratio of direct/indirect branching ratio as $\sim 10$.
17. C. M. Hoffman (private communication).
20. See, e.g., N. G. Deshpande and G. Eilam, Phys. Rev. Lett. 53, 2289 (1984), for the $\pi^+$ spectrum from $K_L^0 \rightarrow \pi^+ \nu \overline{\nu}$. The $\pi^0$ spectrum from $K_L^0 \rightarrow \pi^0 \nu \overline{\nu}$ will be similar except for small effects due to the slightly different masses involved.
24. The current experimental state of the art for this type of measurement is probably more like $10^{-8}$ according to B. Weinstein (private communication).