

Problem 1

Work through Equ. 7.31 to Equ. 7.33.

Explain the scale of R .

$e^+e^- \rightarrow Z^0 \rightarrow \text{anything}$

$$\sigma = \underbrace{\frac{4\pi \alpha^2 (2J+1)}{(2s_e+1)^2} \cdot \frac{\Gamma^2/4}{[(E-E_0)^2 + \Gamma^2/4]}}_{\text{Breit-Wigner cross-section}} \cdot \frac{\Gamma_e}{\Gamma} \quad \text{Br}(Z^0 \rightarrow e^+e^-)$$

$E = E_0 = M_Z$ $\alpha = \frac{2k}{E_0}$ $s_e = 1/2$ $J = 1$ $k = 1$
↑ at the resonance:

$$\sigma_{\max} = \frac{3\pi \cdot 4}{E^2} \cdot \left(\frac{\Gamma_e}{\Gamma}\right) \quad E = E_0 = M_Z$$

The cross-section for $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ is:

$$\sigma(\text{point}) = \frac{4\pi \alpha^2}{3E^2}$$

$$R = \frac{\sigma_{\max}}{\sigma(\text{point})} = \frac{9}{\alpha^2} \cdot \left(\frac{\Gamma_e}{\Gamma}\right) \quad R \approx 5.7 \cdot 10^3$$

$$\Gamma_e/\Gamma \sim 0.034 \quad \alpha = \frac{1}{137}$$

The value of R is about $\sim 10^4$ is due to two reasons

a) You compare a resonant and non-resonant cross-sections

b) Z^0 decays inclusively by $\gamma \rightarrow \mu^+\mu^-$. This adds up another 10^2 to the value of R

Problem 2.

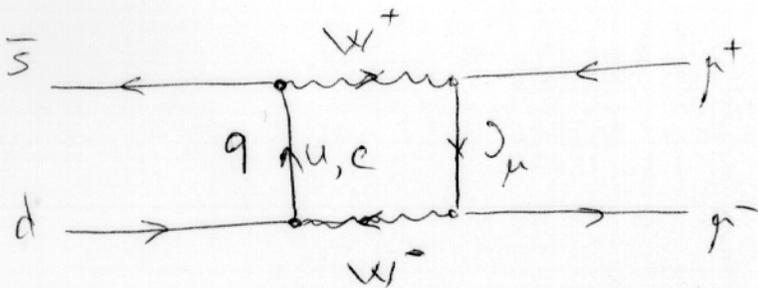
$K_4^0 \rightarrow \mu^+ \mu^-$ including up and charm quarks

$$\Gamma \sim \left(\frac{m_c}{m_u}\right)^N, \quad N = ?$$

$$K^0 = d\bar{s} \quad \overline{K^0} = \bar{d}s$$

$$K_4^0 = \frac{1}{\sqrt{2}} (K^0 - \overline{K^0})$$

Let's consider $K^0 = d\bar{s}$ for now.



$$M \sim \int d^4 q \left(\frac{1}{q^2 - m_u^2} - \frac{1}{q^2 - m_c^2} \right) \sim \left\{ \frac{1}{m_u^2} - \frac{1}{m_c^2} \right\}$$

The integrals can be estimated with dimensional regularization.

$$\Gamma \sim |M|^2 \sim \left(\frac{m_c}{m_u}\right)^4 \Rightarrow N = 4$$

Problem 3.

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

The unitary requirement is $V^\dagger V = 1$.

As a result we get:

$$V_{ud} \cdot V_{ub}^* + V_{cd} \cdot V_{cb}^* + V_{td} \cdot V_{tb}^* = 0 \quad \text{for an off-diagonal element.}$$

The measured values of the elements of the CKM matrix do not have proper complex phases.

We are going to use Wolfenstein's parametrization to verify the equation numerically.

$$V_{ud} = 1 - \frac{\lambda^2}{2} \quad V_{ub} = A \lambda^3 (\rho - i\eta)$$

$$V_{cd} = -\lambda \quad V_{cb} = A \lambda^2$$

$$V_{td} = A \lambda^3 (1 - \rho - i\eta) \quad V_{tb} = 1$$

$$V_{ud} \cdot V_{ub}^* + V_{cd} \cdot V_{cb}^* + V_{td} \cdot V_{tb}^* = \left(1 - \frac{\lambda^2}{2}\right) \cdot A \lambda^3 (\rho + i\eta) - A \lambda^3 + A \lambda^3 (1 - \rho - i\eta)$$

$$= A \lambda^3 \cdot \left\{ \left(1 - \frac{\lambda^2}{2}\right) (\rho + i\eta) - 1 + 1 - \rho - i\eta \right\} = -A \frac{\lambda^5}{2} \cdot (\rho + i\eta)$$

$$A \approx 0.8 \pm 0.1 \quad \lambda \approx 0.221 \pm 0.002 \quad \rho, \eta \sim 1$$

$$\frac{A \lambda^5}{2} \approx 2 \cdot 10^{-4} \quad \text{and it's close to } 0.$$

Problem 4

Compare CP violation in K decays and B decays.

a) K decays.

$K^0 = d\bar{s}$ and $\bar{K}^0 = \bar{d}s$ are mass and strangeness eigenstates

The CP eigenstates are:

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \quad CP = +1$$

$$|K_L\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP = -1$$

$$K_S \rightarrow \pi^0\pi^0 \text{ or } \pi^+\pi^- \quad CP = +1 \quad (\text{final-state})$$

$$K_L \rightarrow \pi^+\pi^-\pi^0 \quad CP = -1$$

The CP violation happens when $K_L \rightarrow \pi^+\pi^-$
 $CP = -1 \quad CP = +1$

Let's say:

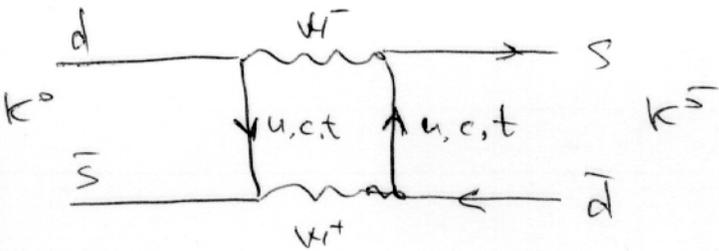
$$\begin{cases} CP |K_2\rangle = -|K_2\rangle \\ CP |K_1\rangle = +|K_1\rangle \end{cases} \Rightarrow \begin{aligned} K_S &= \frac{1}{\sqrt{1+\epsilon^2}} (K_1 + \epsilon K_2) \\ K_L &= \frac{1}{\sqrt{1-\epsilon^2}} (K_2 + \epsilon K_1) \end{aligned}$$

$$\epsilon \ll 1$$

ϵ is a parameter which quantifies CP-violation

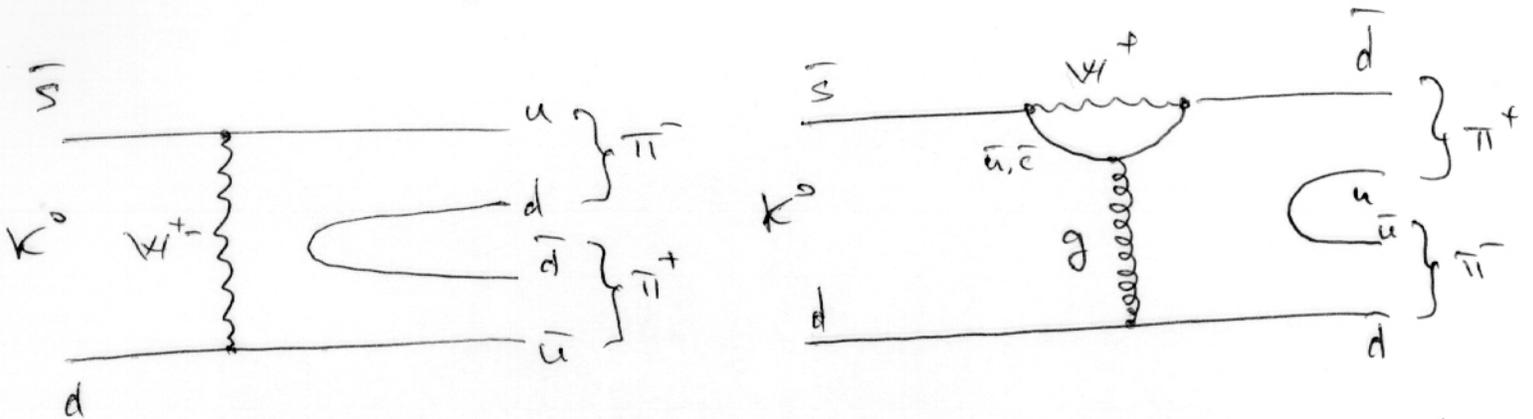
It was found that $\epsilon \approx 2.3 \cdot 10^{-3}$

The biggest source of the CP-violation in K-decays is caused by the box diagram:



It is called indirect CP-violation.

Also there is a contribution from direct CP violation decays:



The amplitude of the direct CP violation, ϵ' , is significantly smaller than that of indirect:

$$\frac{\epsilon'}{\epsilon} \approx 2.2 \cdot 10^{-3}$$

b) B decays

There is no indirect CP-violation due to: $\frac{\Gamma(B_s^0) - \Gamma(B_d^0)}{\Gamma(B_s^0) + \Gamma(B_d^0)} \approx 0$

(it is $\frac{\Gamma(K_s^0) - \Gamma(K_L^0)}{\Gamma(K_s^0) + \Gamma(K_L^0)} \approx 1$ due to limited phase-space $m_K \approx 3m_\pi$)

As a result we observe only direct CP-violation on the rate of 1.

Problem 5.

Quantum numbers of the Higgs boson.

$$Q = T^3 + Y \quad (Q = -1 \text{ for an electron})$$

Y = hyper charge

T^3 is the third component of isospin.

The Standard Model Higgs boson interacts with left- and right-handed fermions.

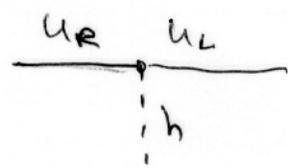
The left-handed fermions are doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad Y = -\frac{1}{2} \quad \text{or} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad Y = +\frac{1}{6}$$

and the right-handed are singlets.

$$u_R \quad Y = +\frac{2}{3} \quad e_R^- \quad Y = -1$$

Let's consider an interaction like:



$$Y = -1 \quad Y = -\frac{1}{2}$$

\Rightarrow The Higgs boson must have:

$$Y = +\frac{1}{2} \quad T^3 = -\frac{1}{2}$$

$$Q = 0$$

Problem 6.

This is an essay question.