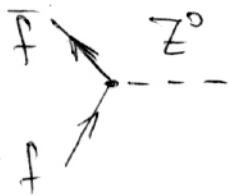


## Problem 1

Fermion coupling to the  $Z^0$  boson.



It is known from experiments that the weak interactions do not conserve C- and P-symmetries. As a result the coupling to left-handed and right-handed fermions are different. The coupling of the  $Z^0$ -boson to fermions has a "V-A" ("vector minus axial vector") form:

$$-\frac{ig_2}{2} \gamma^\mu (C_V^f - C_A^f \gamma^5), \quad g_2 = \frac{e}{\sin \theta_W \cdot \cos \theta_W}$$

$C_V^f$  and  $C_A^f$  are constants.

$f$	$C_V^f$	$C_A^f$
$\nu_e, \nu_\mu, \nu_\tau$	1/2	1/2
$e^-, \mu^-, \tau^-$	$-\frac{1}{2} + 2 \sin^2 \theta_W$	$-\frac{1}{2}$
$u, c, t$	$\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W$	1/2
$d, s, b$	$-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W$	$-\frac{1}{2}$

## Problem 2

$$M_W - ?$$


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The consistency between Fermi's and the electroweak models requires:

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2} \quad \text{and} \quad e = g \cdot \sin \theta_W$$

(it must be consistent with the electrodynamics too).

$$M_W = \sqrt{\frac{e^2 \sqrt{2}}{8 G \cdot \sin^2 \theta_W}}$$


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## Problem 3

$$M_Z = f(M_W) ?$$


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$\chi/\mu^{(3)}$  and  $B_\mu$  are mixed to construct  $M_W, M_Z, M_T$

$M_\chi = 0$ . As a consequence we have 5 equations:

$$M_Z^2 = M_W^2 \cdot \cos^2 \theta_W + M_B^2 \cdot \sin^2 \theta_W - 2 M_{BW}^2 \cdot \cos \theta_W \cdot \sin \theta_W$$

$$M_\chi^2 = 0$$

$$M_T^2 = M_W^2 \cdot \sin^2 \theta_W + M_B^2 \cos^2 \theta_W + 2 M_{BW}^2 \cdot \cos \theta_W \cdot \sin \theta_W$$

$$M_{ZT}^2 = 0 = (M_W^2 - M_B^2) \sin \theta_W \cos \theta_W + M_{BW}^2 (\cos^2 \theta_W - \sin^2 \theta_W)$$

↑

off-diagonal mass term

$$\Rightarrow \left\{ \begin{array}{l} M_2^2 = M_{K^+}^2 + M_B^2 \\ M_2^2 = M_{K^+}^2 \cdot \cos 2\theta_{K^+} - M_B^2 \cos 2\theta_{K^+} - 2 M_{B K^+}^2 \sin 2\theta_{K^+} \\ M_{B K^+}^2 \cdot \cos 2\theta_{K^+} = -\frac{1}{2} (M_{K^+}^2 - M_B^2) \cdot \sin 2\theta_{K^+} \end{array} \right.$$

$$\left\{ \begin{array}{l} M_2^2 = (M_{K^+}^2 - M_B^2) \cdot \cos 2\theta_{K^+} + (M_{K^+}^2 - M_B^2) \frac{\sin^2 2\theta_{K^+}}{\cos 2\theta_{K^+}} \Rightarrow \\ M_2^2 = \cos 2\theta_{K^+} = M_{K^+}^2 - M_B^2 \\ M_2^2 = M_{K^+}^2 + M_B^2 \end{array} \right. \Rightarrow 2 M_{K^+}^2 = M_2^2 (1 + \cos 2\theta_{K^+})$$

$$1 + \cos 2\theta_{K^+} = 2 \cos^2 \theta_{K^+} \Rightarrow M_{K^+}^2 = M_2^2 \cdot \cos^2 \theta_{K^+}$$

$$\boxed{M_{K^+} = M_2 \cdot \cos \theta_{K^+}}$$

### Problem 4.

Numerical values of  $e$ ,  $g$ , and  $g'$  - ?

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$$\lambda = \frac{1}{137} \quad \lambda = \frac{e^2}{4\pi} \Rightarrow e = \sqrt{\frac{4\pi}{137}} \approx 0.302$$

$$g = \frac{e}{\sin \theta_M} \quad \sin \theta_M \approx \sqrt{0.2397} \approx 0.619$$

$$g' = g \cdot \tan \theta_M \quad g' = \frac{e}{\cos \theta} = \frac{e}{\sqrt{1 - \sin^2 \theta}} \approx 0.347$$


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### Problem 5.

$$\frac{d\sigma}{dy} = f(\theta_M) ? \quad \text{for } \bar{\nu}_e e \text{ and } \bar{\nu}_e e$$


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The differential cross-sections are derived in Perkins  
(See p. 251).

$$\frac{d\sigma}{dy} (\bar{\nu}_e e \rightarrow \bar{\nu}_e e) = \frac{G_F^2 S}{\pi} \left[ \left( \frac{1}{2} + \sin^2 \theta_M \right)^2 + \left( \sin^2 \theta_M \right)^2 (1-y)^2 \right]$$

$$\frac{d\sigma}{dy} (\bar{\nu}_e e \rightarrow \bar{\nu}_e e) = \frac{G_F^2 S}{\pi} \left[ \left( \frac{1}{2} + \sin^2 \theta_M \right)^2 \cdot (1-y)^2 + \left( \sin^2 \theta_M \right)^2 \right]$$

# Problem 6.

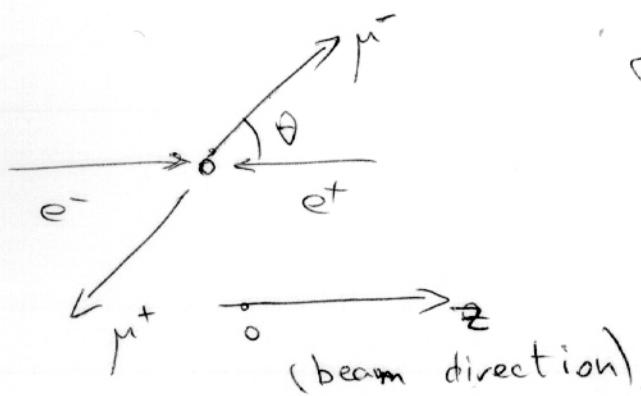
$$e^+ e^- \rightarrow \mu^+ \mu^- \quad s \gg M_Z^2$$

$A_{FB}$  - ?

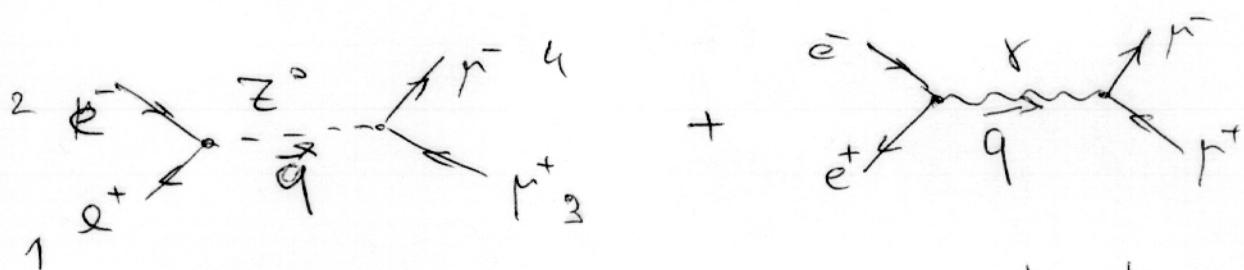
$$A_{FB} = \frac{\sigma_f - \sigma_B}{\sigma_f + \sigma_B}$$

$$\sigma_f = \int_0^{\pi/2} \left( \frac{d\sigma}{d\Theta} \right) d\Theta$$

$$\sigma_B = \int_{\pi/2}^{\pi} \left( \frac{d\sigma}{d\Theta} \right) d\Theta$$



We have to take into account 2 diagrams:



The only difference is the structure of the propagator, and couplings.

$$J_z^\mu = \frac{1}{\cos \theta_W} [\bar{e}_L \gamma^\mu (-\frac{1}{2} \sin^2 \theta_W) e_R + \bar{e}_R \gamma^\mu (\sin^2 \theta_W) e_L]$$

$$J_{EM}^\mu = \bar{e} \gamma_\mu (-i) \cdot e$$

$$M_2 = -\frac{g^2}{4(q^2 - M_2^2)} \cdot [\bar{u}(4)\gamma^\mu(c_v^\mu - c_A^\mu\gamma^5)\gamma(3)] \times [(\not{q}_\mu - \frac{q_\mu q_\nu}{M_2^2}) \cdot \star [\bar{\psi}(2)\gamma^\nu(c_v^\nu - c_A^\nu\gamma^5)u(1)]]$$

the  $\frac{q_\mu q_\nu}{M_2^2}$  component of the propagator cancels out due to Dirac equation for massless particles

(We can assume that  $m_e = m_\mu = 0$  since  $q \gg M_Z^2$ )

$$\Rightarrow M_2 = \frac{g^2}{4(q^2 - M_2^2)} [\bar{u}(4)\gamma^\mu(c_v^\mu - c_A^\mu\gamma^5)\gamma(3)] \times [\bar{\psi}(2)\gamma_\mu(c_v^\nu - c_A^\nu\gamma^5)u(1)]$$

$$c_v^\mu = c_v^\nu = -\frac{1}{2} + 2 \sin^2 \theta_W$$

$$c_A^\mu = c_A^\nu = -\frac{1}{2}$$

$$M_\chi = \frac{ge^2}{q^2} [\bar{u}(4)\gamma^\mu.\gamma(3)] \times [\bar{\psi}(2)\gamma_\mu u(1)]$$

$$g_e = e \quad g^2 = \frac{ge}{\cos \theta_W \cdot \sin \theta_W}$$

$$M = M_2 + M_\chi \quad q^2 \gg M_2^2$$

$$\sin^2 \theta_W \approx 0.2397$$

$$c_v \approx -0.0206 \quad c_A \approx -0.5$$

$4 \cdot \sin \theta_W \cdot \cos \theta_W \approx 1.708 \Rightarrow$  The contributions from  $\chi$  and  $\chi'$  are comparable. Both should be used.

$$M \sim \left[ \left( \frac{C_V^2}{4 \sin \theta_W \cdot \sin \theta_2} + 1 \right) \cdot (\bar{u}(4) \gamma^\mu \nu(3)) \cdot (\bar{d}(2) \gamma_\mu \gamma^5 u(1)) + \right.$$

$$\left. \frac{C_V C_A}{4 \sin \theta_W \cdot \sin \theta_2} \cdot (\bar{u}(4) \gamma^\mu \nu(3)) \cdot (\bar{d}(2) \gamma_\mu \gamma^5 u(1)) \right] +$$

$$+ \frac{C_V C_A}{4 \sin \theta_W \cdot \sin \theta_2} \cdot (\bar{u}(4) \gamma^\mu \gamma^5 \nu(3)) \cdot (\bar{d}(2) \gamma_\mu u(1)) +$$

$$+ \frac{C_A^2}{4 \sin \theta_W \cdot \sin \theta_2} \cdot (\bar{u}(4) \gamma^\mu \gamma^5 \nu(3)) \cdot (\bar{d}(2) \gamma_\mu \gamma^5 u(1)) \Big]$$

$$\frac{d\sigma}{d\Omega} \sim |M|^2$$

$$\frac{d\sigma}{d\Omega} \approx \left[ \left\{ (C_V^2 + A)^2 + 2(C_V C_A)^2 + C_A^2 \right\} \cdot (1 + \cos^2 \theta) - \right. \\ \left. - 8(C_A^2 + A) \cdot C_A^2 \cdot \cos \theta \right], \quad A = 4 \sin \theta_W \cdot \cos \theta_W$$

$(1 + \cos^2 \theta)$  is a symmetric term

$\cos \theta$  is an asymmetric term

$$d\Omega = 4\pi d\cos \theta \Rightarrow$$

$$A_{FB} \approx \frac{3}{8} \frac{8(C_V^2 + A) \cdot C_A^2}{(C_A^2 + C_V^2)^2 + 2C_V^2 A + A^2}$$

$$A = 1.708, \quad C_V \approx -0.0206 \quad C_A = -0.5$$

$$A_{FB} \approx 0.43$$