

Problems 1-4

The problems are discussed in Perkins (Chapter 5)
and in D. Griffiths (Chapters 6, 7, 8).

Problem 5

Derive

$$\int_0^1 [F_2^{ep}(x) - F_2^{en}(x)] \frac{dx}{x} = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx$$

We should keep in mind that $F_2^{ep}(x)$ and $F_2^{en}(x)$ are derived under the assumption of isospin invariance (which holds true in strong interactions).

However, $F_2^{ep}(x)$ and $F_2^{en}(x)$ are used for electroweak interactions and some discrepancies are to observe. In particular it is assumed that $u(x)$ in a proton is identical to $d(x)$ in a neutron (and it is just approximation).

Taking all this assumptions into consideration we can write:

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]$$

$$\frac{F_2^{en}(x)}{x} = \frac{4}{9}[d(x) + \bar{d}(x)] + \frac{1}{9}[u(x) + \bar{u}(x) + s(x) + \bar{s}(x)],$$

where $u(x)$, $\bar{u}(x)$, $d(x)$, and $\bar{d}(x)$ are taken for a proton.

$$\Rightarrow \int_0^1 (u(x) - \bar{u}(x)) dx = 2 \quad \int_0^1 (d(x) - \bar{d}(x)) = 1$$

(the number of valence (u and d) quarks in proton)

$$\begin{aligned} \frac{F_2^{ep}(x) - F_2^{en}(x)}{x} &= \frac{1}{3} [u(x) + \bar{u}(x)] - \frac{1}{3} [d(x) + \bar{d}(x)] \\ &= \frac{1}{3} [u(x) - \bar{u}(x) + 2\bar{u}(x)] - \\ &\quad - \frac{1}{3} [d(x) - \bar{d}(x) + 2\bar{d}(x)] \end{aligned}$$

Therefore:

$$\begin{aligned} \int_0^1 \frac{F_2^{ep}(x) - F_2^{en}(x)}{x} dx &= \frac{1}{3}(2-1) + \frac{2}{3} \int_0^1 (\bar{u}(x) - \bar{d}(x)) dx = \\ &= \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx \end{aligned}$$

Problem 6

Verify:

$$\int_0^1 x F_3^{DN}(x) \frac{dx}{x} = \int_0^1 [u_v(x) + d_v(x)] dx = 3$$

$$u_v(x) = u(x) - \bar{u}(x) \quad d_v(x) = d(x) - \bar{d}(x)$$

$$F_3^{DN} = u(x) + d(x) - \bar{u}(x) - \bar{d}(x)$$

$$F_3^{DN} = u_v(x) + d_v(x), \text{ where } \int_0^1 u_v(x) dx \text{ and}$$

$\int_0^1 d_v(x) dx$ are the numbers of valence quarks
in a nucleon.

We should keep in mind that $x F_3^{DN}(x)$ is derived under assumption that u- and d-quarks are identical (isospin invariance) and we know that it's not absolute. Also, $u(x)$, $d(x)$, $\bar{u}(x)$, and $\bar{d}(x)$ are taken for proton $\Rightarrow \int_0^1 u_v(x) dx = 1 \quad \int_0^1 d_v(x) dx = 1$

Therefore: $\int_0^1 x F_3^{DN}(x) \frac{dx}{x} = 3 = \int_0^1 [u_v(x) + d_v(x)] dx$.

Problem 7

e^+e^- -collider 2 bunches, 2 collisions per revolution

$$R = 10 \text{ m} \quad I = 10 \text{ mA} \quad A = 0.1 \text{ cm}^2$$

a) $L = ?$ (in $\text{cm}^{-2} \text{s}^{-1}$)

b) $\sigma_w = ?$ $B_r (\omega \rightarrow e^+e^-) = \alpha^2$ Rate = ? $m_\omega = 783 \text{ MeV}$
 $J_\omega = 1.$

a) $L = \frac{N_e^2 \cdot f}{A}$ $f = 2 \cdot \left(\frac{2\pi R}{c} \right)^{-1}$

$$\begin{cases} I = n A C Q \\ N_e = n \cdot A \cdot 2\pi R \end{cases}$$

since we have 2 bunch crossings
per revolution

$$N_e = \frac{I \cdot 2\pi R}{C \cdot Q} \Rightarrow L = \frac{2}{\pi} \cdot \frac{C}{2\pi R} \cdot \left(\frac{I \cdot 2\pi R}{C Q} \right)^2$$

$Q = e$ is the charge of electron.

$$L = \frac{2}{\pi} \left(\frac{I}{e} \right)^2 \cdot \frac{2\pi R}{C} = \frac{4\pi R}{A \cdot C} \cdot \left(\frac{I}{e} \right)^2$$

$$L \approx 1.64 \cdot 10^{28} \text{ cm}^{-2} \text{ s}^{-1}$$

b) $\sigma_w = \frac{4\pi \alpha^2 (2J+1)}{(2s_e+1)^2} \cdot B_r \quad s_e = \frac{1}{2} \quad J = 1$

$$B_r = \alpha^2 \quad \alpha = \frac{\hbar}{p} \quad p = \frac{m_\omega}{2} \quad \cancel{B_r \approx \frac{1}{137}}$$

$$\sigma_w = \frac{4\pi}{4} \cdot 3 \left(\frac{2\hbar}{m_\omega} \right)^2 \cdot B_r , \quad \sigma_w \approx 1.278 \cdot 10^{-30} \text{ cm}^2$$

Therefore Rate $\approx L \cdot \sigma_w \approx 110$ per hour

Problem 8.

$$q_u(x) = A(1-x)^3 \quad q_{\bar{u}}(x) = B(1-\bar{x})$$

$A, B = \text{const}$

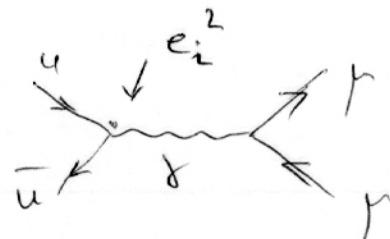
$$u\bar{u} \rightarrow \mu^+\mu^- \quad \pi p - \text{collisions}$$

$$\tilde{s} = \frac{m^2}{s} = x \cdot \bar{x} \quad s \text{ is the square of the pion-proton cms energy.}$$

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi \alpha^2}{3s}$$

$$s = m^2$$

in case of



$$m^2 = s \cdot x \cdot \bar{x}$$

$$\frac{d\sigma(p\bar{n} \rightarrow \mu^+\mu^-)}{dm} = - \frac{8\pi \alpha^2}{3m^3} \cdot \sum AB \cdot e_i^2 \cdot \int_0^1 dx \int_0^1 d\bar{x} \cdot \delta(m^2 - s \cdot x \cdot \bar{x})$$

$$\cdot (1-x)^3 \cdot (1-\bar{x})$$

after integrating over \bar{x} :

$$\frac{d\sigma(p\bar{n} \rightarrow \mu^+\mu^-)}{dm} = - \frac{8\pi \alpha^2}{3m^2} \int_{\frac{m^2}{s}}^1 dx (1-x)^3 \cdot \left(1 - \frac{m^2}{sx}\right)$$

There's no way it can give $(1 - \frac{m^2}{s})^5$