

Problem 1

$$\Omega^- = sss, M_{\Omega^-} = 1672 \text{ MeV}$$

1) Let's consider strong decays of Ω^- . There are three things to keep in mind:

- a) Strangeness (the # of s-quarks) must conserve
- b) The number of baryons should conserve
- c) The decays should be allowed kinematically
(the sum of decay product's masses should be less than the mass of Ω^-)

The Ω^- -baryon's strong decay should lead to another baryon and a few mesons. Let's list mesons and baryons with at least one s-quark (there is no need to consider \bar{s} -quarks).

$$\bar{K}^0 = \bar{d}s \quad K^- = \bar{u}s \quad M_K \approx 495 \text{ MeV}$$

$$\Lambda^0 = uds \quad M_\Lambda \approx 1116 \text{ MeV}$$

$$\Sigma^+ = uus \quad \Sigma^0 = uds \quad \Sigma^- = dds \quad M_\Sigma \approx 1190 \text{ MeV}$$

$$\Xi^0 = uss \quad \Xi^- = dss \quad M_\Xi \approx 1315 \text{ MeV}$$

These are the lightest mesons and baryons with s-quarks, therefore we need to consider only decays: $\Omega^- \rightarrow \Xi^0 + K^- + X$, $\Omega^- \rightarrow \Sigma^- + K^0 + K^- + X$ and $\Omega^- \rightarrow \Lambda^0 + K^+ + K^- + X$, where X can be any light mesons.

$$M_{\Xi} + M_K = 1890 \text{ MeV} > M_{\Omega} \Rightarrow \Omega \not\rightarrow \Xi + K + X$$

$$M_{\Sigma} + 2M_K = 2180 \text{ MeV} > M_{\Omega} \Rightarrow \Omega \not\rightarrow \Sigma + K + K + X$$

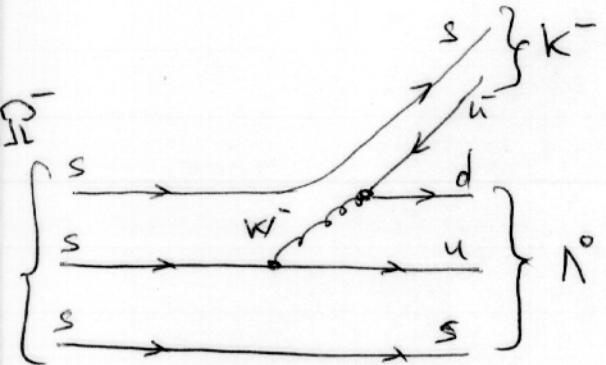
$$M_{\Lambda} + 2M_K = 2106 \text{ MeV} > M_{\Omega} \Rightarrow \Omega \not\rightarrow \Lambda + K + K + X.$$

All these strong decays are not kinematically allowed. $\Rightarrow \boxed{\Omega^- \text{ can not decay strongly.}}$

2) Let's consider a weak decay of Ω^- . We need to show that at least one such a decay is possible.

$$\Omega^- \rightarrow \Lambda^0 + K^-$$

$M_{\Lambda} + M_K = 1611 < 1672 = M_{\Omega} \Rightarrow$ The decay is kinematically allowed.



The decay also conserves charge and baryon number,

$$\Omega^-: J^P = \frac{3}{2}^+$$

$$\Lambda^0: J^P = \frac{1}{2}^+$$

$$K^-: J^P = 0^-$$

The final state $K^- \Lambda^0$ should have an orbital momentum of 1. This will conserve parity. The decay is also invariant under C and T. \Rightarrow The decay

$$\Omega^- \rightarrow \Lambda^0 + K^- \text{ (weak) is allowed.}$$

Problem 2.

$$|\psi(0)|^2 = ? \quad \Gamma_p, \Gamma_\omega, \Gamma_\phi = ?$$

$$|\psi(0)|^2 \sim v^{-1} \sim \left(\frac{4}{3} \pi R^3\right)^{-1}, \quad R \sim 0.6 \cdot 10^{-15} \text{ m}$$

$$|\psi(0)|^2 \approx \frac{3}{4\pi} \cdot \frac{1}{(0.6)^3 \cdot 10^{-45}} \text{ m}^{-3} \approx 1.1 \text{ fm}^{-3}$$

$$\Gamma_v = \frac{16\pi \alpha^2 Q^2}{m_v^2} |\psi(0)|^2 \quad v = p, \omega, \phi$$

$$Q^2 = 1 \sum a_i Q_i^2$$

a) $P = \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}) \quad Q^2 = \frac{1}{2} \quad M_p = 775 \text{ MeV}$

$$\alpha = \frac{1}{137} \quad 1 \text{ fm} = \frac{10^{-3}}{0.1975} \text{ MeV}^{-1} \Rightarrow |\psi(0)|^2 \approx \frac{1.1 \cdot (0.1975)}{10^9} \text{ MeV}^3$$

$$\Gamma_p \approx 1.8 \cdot 10^{-2} \text{ MeV} \approx 18.9 \text{ keV}$$

b) $\omega = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \quad Q^2 = \frac{1}{18} \quad M_\omega = 782 \text{ MeV}$

$$\Gamma_\omega \approx 2.1 \text{ keV}$$

c) $\phi = s\bar{s} \quad Q^2 = \frac{1}{9} \quad M_\phi = 1019 \text{ MeV}$

$$\Gamma_\phi \approx 2.4 \text{ keV}$$

Problem 3

$$\text{Br} (\Upsilon \rightarrow \gamma + \text{hadrons}) = 0.3\% \quad \Gamma_\gamma = 53 \text{ keV}$$

$$M_\chi \approx 9460 \text{ MeV}$$

$$\lambda_s = ?$$

$$\Gamma(\gamma G G) = \frac{2(\pi^2 - 9)}{9\pi} \cdot \lambda^2 \left(\frac{4\alpha_e}{3} \right)^4 \cdot M_\chi$$

$$\Gamma(\gamma G G) = \Gamma_\chi \cdot \text{Br}(\Upsilon \rightarrow \gamma G G)$$

$$\lambda_s = \frac{3}{4} \cdot \sqrt[4]{\frac{9\pi}{2(\pi^2 - 9)} \cdot \frac{\Gamma_\chi \cdot \text{Br}(\Upsilon \rightarrow \gamma G G)}{\lambda^2 \cdot M_\chi}}$$

$$\lambda \approx \frac{1}{137}$$

$$\lambda_s \approx 0.2$$

Problem 4

$$1^3S_1 \rightarrow 1^1S_0 \quad \Delta E \sim p_1 \cdot p_2$$

$p\bar{e}$, e^+e^- , $\mu^+\bar{\mu}$

$$\Delta E(p\bar{e}) = 1420 \text{ MHz}$$

$$\Delta E(\mu^+\bar{\mu}) = ? \quad \text{vs. } 4463.30 \text{ MHz}$$

$\frac{7}{16}$ + reduced mass for e^+e^- .

$$\Delta E(e^+e^-) = ? \quad \text{vs. } 203386.0 \text{ MHz.}$$

Why do we see a $\sim 1\%$ discrepancy?

$$\Delta E(\mu^+\bar{\mu}) = \Delta E(p\bar{e}) \cdot \frac{m_p}{m_\mu} = \Delta E(p\bar{e}) \cdot \frac{1.001 \cdot m_p}{2.793 \cdot m_\mu}$$

$$m_p = 938.27 \text{ MeV}$$

$$m_\mu = 105.66 \text{ MeV}$$

$$\Delta E(\mu^+\bar{\mu}) \approx 4519.27 \text{ MHz} \quad \text{The agreement is about } 1\%$$

$$\Delta E(e^+e^-) = \Delta E(p\bar{e}) \cdot \frac{7}{16} \cdot \frac{1}{2} \cdot \frac{m_e}{m_p}, \quad \frac{m_e}{m_p} \approx \frac{m_p}{2.793 \text{ me}}$$

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Due to reduced mass!

$$\Delta E(e^+e^-) = 204412 \text{ MHz, and it agrees well with } 203386 \text{ MHz.}$$

The 1% discrepancy is due to missing higher-order corrections to the magnetic moments.

Problem 5.

There is a typo in the Problem Set 4. The correct number of the table is 4.11 in Perkins.

$$\Delta E = ? \quad \frac{K}{m_n^2} = 50 \text{ MeV} \quad m_n = 363 \text{ MeV}, \quad m_s = 538 \text{ MeV}$$

$N(939)$:

$$\Delta E = -3 \frac{K}{m_n^2} = -150 \text{ MeV}$$

$\Lambda(1116)$: $\Delta E = -3 \frac{K}{m_n^2} = -150 \text{ MeV}$

$\Sigma(1193)$: $\Delta E = \frac{K}{m_n^2} - \frac{4K m_n}{m_n^2 m_s} = 50 - 200 \cdot \frac{363}{538} \approx -85 \text{ MeV}$

$\Xi(1193)$: $\Delta E \approx -85 \text{ MeV}$

$\Delta(1232)$: $\Delta E = \frac{3K}{m_n^2} \approx 150 \text{ MeV}$

$\Sigma(1384)$: $\Delta E = \frac{K}{m_n^2} + 2 \frac{K m_n}{m_n^2 m_s} = 50 + 100 \cdot \frac{363}{538} \approx 117 \text{ MeV}$

$\Xi(1533)$: $\Delta E \approx 117 \text{ MeV}$

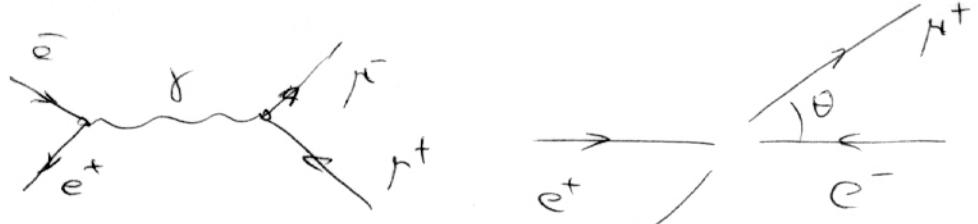
$\Omega(1672)$: $\Delta E = \frac{3K}{m_n^2} \cdot \frac{m_n^2}{m_s^2} = 150 \cdot \left(\frac{363}{538}\right)^2 \approx 68 \text{ MeV}$

Problem 6.

It appeared in the previous homeworked,

Problem 7.

$$e^+ e^- \rightarrow \mu^+ \mu^-$$



$$M_{if} = \frac{4\pi e^2}{q^2} \quad (\text{we neglect spins here})$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2 \hbar^2} \cdot |M_{if}|^2 \cdot \frac{p_f^2}{v_i v_f}$$

$$v_i = v_f = 2c \quad (p_f \gg m_e)$$

$$q^2 = -E_0^2 = -S, \quad p_f = E_0/2c$$

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^4}{\hbar^4} \cdot \frac{E_0^2}{4c^2 \cdot 4c^2} \cdot \frac{1}{E_p^4} \Rightarrow \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4S}$$

Now we need to take into account spins of the particles:

$$A_{RL \rightarrow RL} = \frac{1 + \cos\theta}{2} \quad A_{LR \rightarrow RL} = \frac{1 - \cos\theta}{2}$$

$$P(\theta) = A_{RL \rightarrow RL}^2 + A_{LR \rightarrow RL}^2 = \frac{1 + \cos^2\theta}{2} \quad (\text{for RL final state})$$

We have to double $P(\theta)$ to account for LR final state.

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4S} (1 + \cos^2\theta) \quad d\Omega = 2\pi d(\cos\theta)$$

$$S = \frac{\pi\alpha^2}{2} \cdot \int_{-1}^1 (1+x^2) dx = \frac{\pi\alpha^2}{2} \left(2 + \frac{2}{3}\right) = \frac{4\pi\alpha^2}{3}$$

To take into account for muon mass the cross section should be multiplied by $(1 + \frac{z^2}{2}) \sqrt{1-z^2}$, where

$$z = \frac{4m_\mu^2}{S}$$

Problem 8.

The static quark model is not taking into account the following effects:

- a) Relativistic
- b) sea-quarks (it considers only valence quarks)
- c) gluons.
- d) higher-order QCD corrections
- e) higher-order QED corrections.