

Problem 1

$$H^0 \rightarrow b\bar{b}$$

$$M_b = 5 \text{ GeV}$$

$$M_H = 120 \text{ GeV}$$

$$P_b = ?$$

$$P_H = (M_H, 0, 0, 0)$$

$$P_b = (E_b, p_b, 0, 0)$$

$$P_{\bar{b}} = (E_b, -p_b, 0, 0)$$

$$E_b = \frac{M_H}{2}$$

$$\text{and } E_b^2 - p_b^2 = M_b^2$$

$$\Rightarrow p_b = \sqrt{\left(\frac{M_H}{2}\right)^2 - M_b^2} = \sqrt{60^2 - 5^2}$$

$$p_b \approx 59.8 \text{ GeV}$$

Problem 2.

$$M_H = 165 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV}$$

$$H^0 \rightarrow W^+ W^-, \quad P_W = ?$$

Similarly to the 1st problem:

$$P_W = \sqrt{\left(\frac{M_H}{2}\right)^2 - M_W^2} = \sqrt{(82.5)^2 - (80.4)^2}$$

$$P_W = 18.5 \text{ GeV}$$

Problem 3.

$$W \rightarrow e \nu \quad P_T^{\max}, P_T^{\min} - ?$$

$$P_W = 18.5 \text{ GeV} \quad M_W = 80.4 \text{ GeV}$$

Since $M_W \gg m_e$ the electron mass can be neglected.

$P_T \geq 0$ and $\boxed{P_T^{\min} = 0}$ when the both decays ($W \rightarrow \nu W$ and $W \rightarrow e \nu$) are along the beam-line.

The transverse momentum of the electron is maximum when the decays of $W \rightarrow \nu W$ and $W \rightarrow e \nu$ are perpendicular to the beam-line and electron is moving in the same direction as the W -boson.

$$P_W = (\sqrt{M_W^2 + P_{W,T}^2}, P_{W,T}, 0, 0)$$

$$P_e = (P_e, P_e, 0, 0)$$

$$P_\nu = (P_\nu, -P_\nu, 0, 0)$$

$$P_e + P_\nu = P_W \Rightarrow$$

$$\begin{cases} P_e - P_\nu = P_W \\ P_e + P_\nu = \sqrt{M_W^2 + P_{W,T}^2} \end{cases} \Rightarrow P_e = \frac{1}{2} (P_{W,T} + \sqrt{M_W^2 + P_{W,T}^2})$$

$$\boxed{P_T^{\max} = P_e = 50.5 \text{ GeV}}$$

Problem 4.



$$P_\pi \sim 1 \text{ GeV}, \quad \sigma \approx 1 \text{ mb} = 10^{-31} \text{ m}^2$$

$$\Gamma_K \sim (10^{-10} \text{ s})^{-1}$$

$$\frac{\alpha_{\text{prod}}}{\alpha_{\text{dec}}} \sim ?$$

a) $1+2 \rightarrow 3+4$

$$d\sigma = |M|^2 \frac{\hbar^2 S'}{4 \sqrt{(p_1 p_2)^2 - (m_1 m_2 c^2)^2}} \left[\left(\frac{cd^3 \vec{p}_3}{(2\pi)^3 2E_3} \right) \cdot \left(\frac{cd^3 \vec{p}_4}{(2\pi)^3 2E_4} \right) \right] \cdot (2\pi)^4 S^4 (p_1 + p_2 - p_3 - p_4)$$

$$\Rightarrow \sigma \sim \frac{|M|^2}{P_\pi^2}$$

S' corresponds to the statistics of the final state

b) $1 \rightarrow 2+3$

$$d\Gamma \approx \frac{2\pi}{2\hbar m_1} |M|^2 \cdot S' \cdot \left[\left(\frac{cd^3 \vec{p}_2}{(2\pi)^3 2E_2} \right) \cdot \left(\frac{cd^3 \vec{p}_3}{(2\pi)^3 2E_3} \right) \right] \cdot (2\pi)^3 S^4 (p_1 - p_2 - p_3)$$

$$\Rightarrow \Gamma \sim \frac{|M|^2}{m_1}$$

statistical factor

Therefore, $\left(\frac{\alpha_{\text{prod}}}{\alpha_{\text{dec}}} \right)^2 \sim \frac{\sigma \cdot P_\pi^2}{\Gamma m_1}$

$$\Gamma = 6.56 \cdot 10^{-15} \text{ GeV}$$

$$\sigma \approx 2.56 \text{ GeV}^{-2}$$

$$m_1 = M_\pi \sim 1 \text{ GeV}$$

$$\Rightarrow \frac{\alpha_{\text{prod}}}{\alpha_{\text{dec}}} \sim 10^7$$

Problem 5

$$\pi^+ p \rightarrow \Delta^{++} \rightarrow \pi^+ p$$

$$\Gamma = 150 \text{ MeV} \quad M_{\Delta^{++}} = 1.62 \text{ GeV} \quad J = \frac{1}{2}$$

$$s_{\pi} = 0 \quad s_p = \frac{1}{2}$$

$$\sigma_{\max} = ?$$

$$\sigma_{\max} = \frac{4\pi \lambda^2 (2J+1)}{(2s_{\pi}+1) \cdot (2s_p+1)} = 4\pi \lambda^2$$

$$\lambda = \frac{\hbar c}{pc}$$

p is obtained in the center-of-momentum frame.



$$\begin{cases} M_{\Delta^{++}} = E_{\pi} + E_p \\ E_p^2 = p^2 + M_p^2 \\ E_{\pi}^2 = p^2 + M_{\pi}^2 \end{cases} \Rightarrow M_{\Delta^{++}} = \sqrt{p^2 + M_p^2} + \sqrt{p^2 + M_{\pi}^2}$$

$$p^2 = \left(\frac{M_{\Delta}^2 + M_{\pi}^2 - M_p^2}{2M_{\Delta}} \right)^2 - M_{\pi}^2$$

$$M_{\Delta} = 1620 \text{ MeV}, \quad M_p = 938 \text{ MeV}, \quad M_{\pi} = 140 \text{ MeV}$$

$$p = 526 \text{ MeV}/c = 0.526 \text{ GeV}/c$$

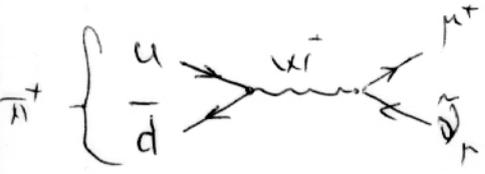
$$\hbar c = 0.1975 \text{ GeV} \cdot \text{fm}$$

$$\sigma_{\max} = 4\pi \left(\frac{\hbar c}{pc} \right)^2 \approx 1.77 \text{ fm}^2 = 17.7 \text{ mb}$$

Problem 6.

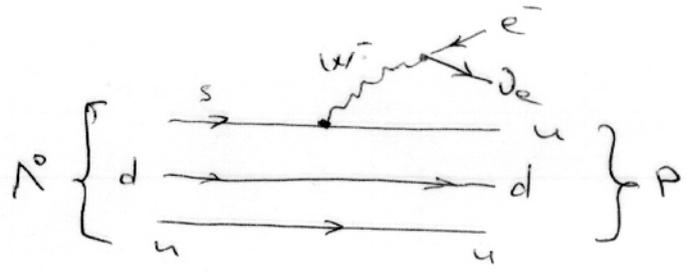
Perkins 2.5

a) $\pi^+ \rightarrow \mu^+ + \nu_\mu$



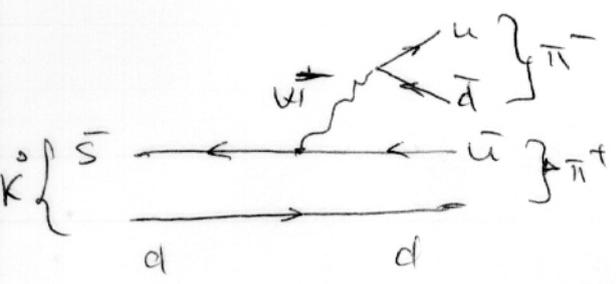
b) $\Lambda \rightarrow p + e^- + \nu_e$

$\Lambda = uds$
 $p = uud$

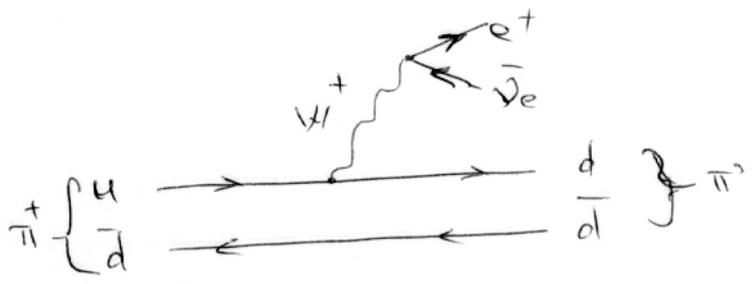


c) $K^0 \rightarrow \pi^+ \pi^-$

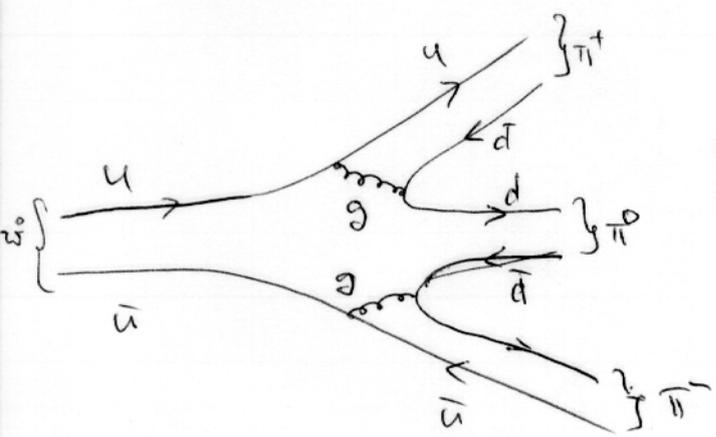
$K^0 = d\bar{s}$



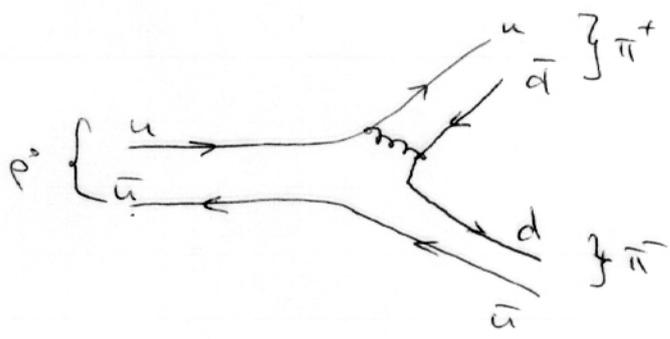
d) $\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$



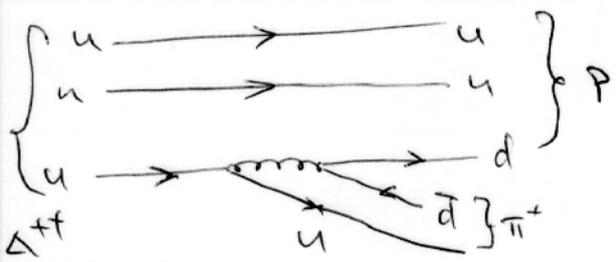
e) $\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$



f) $\rho^0 \rightarrow \pi^+ \pi^-$



g) $\Delta^{++} \rightarrow \pi^+ + p$



Problem 7.

The additive quark model works under assumption that the hadron-hadron amplitude is the sum of the scattering amplitudes of the constituent quarks.

$$\sigma(u+u) = \sigma(u+d) = \sigma_1$$

$$\sigma(s+u) = \sigma(s+d) = \sigma_2$$

$$a) \quad \sigma(\Lambda+p) = \sigma(p+p) + \sigma(K^-+n) - \sigma(\pi^-p)$$

$$\Lambda = uds$$

$$p = uud$$

$$\sigma(\Lambda+p) = \sigma(uds + uud) = 3\sigma_2 + 6\sigma_1$$

$$\sigma(p+p) = 3\sigma_1$$

$$\sigma(K^-+n) = 3\sigma_2 + 3\sigma_1$$

$$K^- = \bar{u}s$$

$$\sigma(\pi^-p) = 6\sigma_1$$

$$\pi^- = \bar{u}d$$

$$\sigma(p+p) + \sigma(K^-+n) - \sigma(\pi^-p) = 6\sigma_1 + 3\sigma_2 = \sigma(\Lambda+p)$$

$$b) \sigma(\Sigma^- + p) = \sigma(p+p) + \sigma(K+p) - \sigma(\pi^- + p) + 2[\sigma(K^+ + n) - \sigma(K^+ + p)]$$

$$p = uud$$

$$\Sigma^- = dds$$

$$K^+ = u\bar{s}$$

$$\sigma(\Sigma^- + p) = 3\sigma_2 + 6\sigma_1$$

$$\sigma(p+p) = 9\sigma_1$$

$$\sigma(K+p) = 3\sigma_2 + 3\sigma_1$$

$$\sigma(\pi^- + p) = 6\sigma_1$$

$$\sigma(K^+ + n) = \sigma(K^+ + p) = 3\sigma_1 + 3\sigma_2$$

Therefore:

$$\begin{aligned} \sigma(p+p) + \sigma(K+p) - \sigma(\pi^- + p) + 2[\sigma(K^+ + n) - \sigma(K^+ + p)] &= \\ = 9\sigma_1 + 3\sigma_2 + 3\sigma_1 - 6\sigma_1 + 2[3\sigma_1 + 3\sigma_2 - 3\sigma_1 - 3\sigma_2] &= \\ = 3\sigma_2 + 6\sigma_1 = \sigma(\Sigma^- + p) \end{aligned}$$

$$c) \sigma(\Sigma^- + n) = \sigma(p+p) + \sigma(K^- + p) - \sigma(\pi^- + p)$$

$$\sigma(\Sigma^- + n) = 3\sigma_2 + 6\sigma_1$$

$$\begin{aligned} \sigma(p+p) + \sigma(K^- + p) - \sigma(\pi^- + p) &= 9\sigma_1 + 3\sigma_1 + 3\sigma_2 - 6\sigma_1 = \\ = 6\sigma_1 + 3\sigma_2 = \sigma(\Sigma^- + n) \end{aligned}$$

Problem 8.

1) Expressions for the magnetic moments.

$$p = uud$$

It is derived earlier in the chapter that

$$\mu_p = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d. \quad \text{The same as for proton under interchange of } u \text{ and } d \text{ labels,}$$

We will use this result for the rest of the particles.

$$n = udd$$

$$\mu_n = \frac{4}{3} \mu_d - \frac{1}{3} \mu_u$$

$$\Lambda = uds$$

$$I = 0, \quad J = \frac{1}{2}, \quad P = +$$

↓

u and d must be in state with $J = 0 \Rightarrow \mu_\Lambda = \mu_s$

$$\Sigma^+ = uus \Rightarrow \mu_{\Sigma^+} = \frac{4}{3} \mu_u - \frac{1}{3} \mu_s$$

$$\Sigma^- = dds \Rightarrow \mu_{\Sigma^-} = \frac{4}{3} \mu_d - \frac{1}{3} \mu_s$$

$$\Sigma^0 = uss \Rightarrow \mu_{\Sigma^0} = \frac{4}{3} \mu_s - \frac{1}{3} \mu_u$$

$$\Sigma^- = dds \Rightarrow \mu_{\Sigma^-} = \frac{4}{3} \mu_d - \frac{1}{3} \mu_s$$

$$\Omega^- = sss, \quad J = \frac{3}{2} \Rightarrow \mu_{\Omega^-} = 3 \mu_s$$

2) Baryon Moments, (p and n).

$$p: uud \quad \mu_p = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d$$

$$m_u = m_d = \frac{m_p}{3}$$

$$\mu = \frac{e\hbar}{2Mc}$$

$$q_u = \frac{2}{3}e$$

$$q_d = -\frac{1}{3}e$$

$$\text{Therefore } \mu_u = \frac{2e\hbar}{3 \cdot 2m_u c} = 2\mu$$

$$\mu_d = -\frac{1}{3} \frac{e\hbar}{2m_d c} = -\mu$$

$$\mu_p = \frac{4}{3} \cdot 2 - \frac{1}{3}(-1) = 3$$

$$n: ddu$$

$$\mu_n = \frac{4}{3} \mu_d - \frac{1}{3} \mu_u = -\frac{4}{3} - \frac{2}{3} = -2$$

3) Anomalous moments of μ_p and μ_n in the old-school nucleon model.

a) The nucleon is a Dirac particle (part-time)

$$\mu_p = 1, \quad \mu_n = 0 \text{ (no charge)}$$

b) The nucleon is a pointlike core with a charged circulating pion in a P-state.

proton: π^+ is circulating around neutral core: $\mu_p \approx 2 \frac{m_p}{m_\pi}$

neutron: π^- is orbiting around negatively-charged core:

$$\mu_n \approx -2 \cdot \frac{m_p}{m_\pi} \Rightarrow \text{Averaging over time can give the}$$

proper values of the magnetic moments.

Problem 9

J/ψ (3100)

$$\Gamma = 0.087 \text{ MeV}$$

$$88\% = \text{Br}(\text{J}/\psi \rightarrow \text{hadrons})$$

$$\Gamma(3G) = \frac{2(\pi^2 - 9)}{3\pi} \left(\frac{4}{3} \alpha_s\right)^6 \cdot m$$

$$m = 3100 \text{ MeV}$$

$$\Gamma(3G) = \Gamma \cdot \text{Br}(\text{J}/\psi \rightarrow \text{hadrons}) = \Gamma \cdot 0.88$$

$$\alpha_s = \frac{3}{4} \left(\frac{9\pi}{2(\pi^2 - 9)} \cdot \frac{\Gamma \cdot 0.88}{m} \right)^{1/6}$$

$$\alpha_s \approx 0.204$$