

Problem 1

Write down 4-momenta of the electron and the proton.

Before collision:

$$\mathbf{p}_e = (E_e, E_e, 0, 0) \quad E_e = 20 \text{ GeV}$$

$$\mathbf{p}_p = (mc^2, 0, 0, 0) \quad mc^2 = 0.938 \text{ GeV}$$

After collision:

$$\mathbf{p}'_e = (E'_e, E'_e \cdot \cos\alpha, E'_e \cdot \sin\alpha, 0) \quad \alpha = 5^\circ$$

$$\mathbf{p}'_p = (E'_p, p'_p \cdot \cos\phi, p'_p \cdot \sin\phi, 0)$$

The collision is elastic: $\mathbf{p}_e + \mathbf{p}_p = \mathbf{p}'_e + \mathbf{p}'_p$

i.e.

$$\begin{cases} E_e + mc^2 = E'_e + E'_p \\ E_e = E'_e \cdot \cos\alpha + p'_p \cdot \cos\alpha \\ E'_e \cdot \sin\alpha + p'_p \cdot \sin\phi = 0 \\ E'_p = p'_p + (mc^2)^2 \end{cases} \Rightarrow$$

$$\begin{cases} E'_p = E_e + mc^2 - E'_e \\ p'^2_p = (E_e - E'_e \cdot \cos\alpha)^2 + (E'_e \cdot \sin\alpha)^2 \\ E'^2_p = p'^2_p + (mc^2)^2 \end{cases}$$

$$E'_e = \frac{E_e \cdot mc^2}{mc^2 + E_e(1 - \cos\alpha)} \quad E'_e \approx 18.5 \text{ GeV}$$

$$\mathbf{q} = \mathbf{p}_e - \mathbf{p}'_e = (E_e - E'_e, E_e - E'_e \cdot \cos\alpha, -E'_e \cdot \sin\alpha, 0)$$

$$q^2 = -2E_e \cdot E'_e (1 - \cos\alpha) \approx -2.81 \text{ GeV}$$

$$\Delta r = \frac{\hbar}{q} \approx 0.74 \text{ fm}$$

Problem 2

$$\text{Power} = \text{Area} * \text{Flux} * \text{Energy}$$

$$\text{Energy} = 3 \text{ GeV} = 3 \cdot 1.602 \cdot 10^{-10} \text{ J}$$

$$\text{Area} = 4\pi \cdot R^2, \quad R = 6400 \text{ km} = 6.4 \cdot 10^8 \text{ cm}$$

$$\text{Flux} = 1 \text{ cm}^{-2} \text{s}^{-1}$$

$$\text{Power} = 4 \cdot 3.1416 \cdot (6.4)^2 \cdot 3 \cdot 1.602 \cdot 10^6 \text{ W}$$

$$\text{Power} = 2.47 \cdot 10^9 \text{ W} = 2.47 \text{ MW}$$

Problem 3

(a) The ground state of $H^+\mu^-$ is lower than that of H^+e^- :

$$\left\{ \begin{array}{l} E_b(H^+\mu^-) \sim -\frac{m_\mu m_p}{m_\mu + m_p} \approx -m_\mu \\ E_b(H^+e^-) \sim -\frac{m_e m_p}{m_e + m_p} \approx -m_e \end{array} \right. \Rightarrow E_b(H^+\mu^-) < E_b(H^+e^-)$$

(b) Similarly, the binding energy of $D^+\bar{\nu}$ is higher than of $H^+\bar{\nu}$

$$E_b(D^+\bar{\nu}) \sim -\frac{m_p}{1 + \frac{m_p}{m_D}} \quad E_b(H^+\bar{\nu}) \sim -\frac{m_p}{1 + \frac{m_p}{m_H}}$$

(c) Separation in H_2 molecule is about $7.4 \cdot 10^{-11} \text{ m}$.

~~The~~ muon "shields" the nucleus much better than an electron.

$$a_0 r \approx \frac{\hbar}{cd} \frac{m_D + m_p}{m_D \cdot m_p} = a_0 \frac{m_e}{m_p} \cdot \frac{m_D + m_p}{m_p}$$

Bohr radius

$$a_0 \approx 5.3 \cdot 10^{-10} \text{ m} \Rightarrow a_0 r \approx 2.7 \cdot 10^{-11} \text{ cm}$$

(d) "Muon - Catalyzed fusion"

$D^+\bar{\nu}\mu^-e^-$ molecule has much smaller separation than $D^+\bar{\nu}e^-e^-$

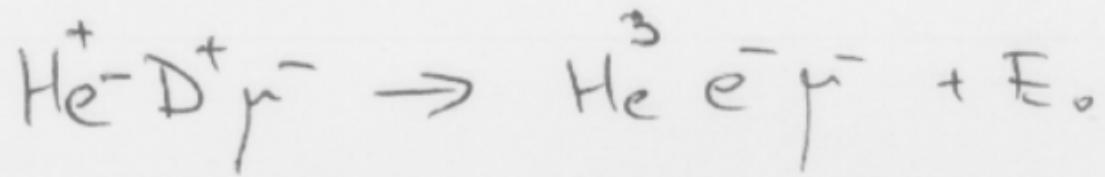
This greatly increases ~~the~~ probability of forming He^3 .

$$M(D) \approx 2.01355 \text{ u}$$

$$M(H^3) \approx 3.01603 \text{ u}$$

$$1 \text{ u} \approx 931 \text{ MeV/c}^2$$

$$M(H^3) \approx 3.00783 \text{ u}$$



$$E_\gamma \approx M(D) + M(H) - M(He^3) \approx 5 \text{ MeV}$$

Problem 4

$$\rho: J=1, M_\rho = 770 \text{ GeV}/c^2$$

$$\pi: J=0.$$

$$\text{Show: } \rho^\pm \rightarrow \pi^\circ \pi^\pm, \rho^0 \rightarrow \pi^+ \pi^- , \rho^0 \rightarrow \pi^0 \pi^0$$

The decays are kinematically allowed since $M_\pi \sim 135 \text{ MeV}$ and $M_\rho > 2 M_\pi$

Also we should guarantee conservation of the angular momentum. This means that the final states with two pions must have $\ell=1$ (the orbital angular momentum). However, two π° can not form a state with $\ell=1$. π° 's are bosons and their wavefunction is identical under interchange, but a function for $\ell=1$ is antisymmetric.

~~Please note that the following decays are allowed~~

$$\rho^\pm \rightarrow \frac{1}{\sqrt{2}} (|\pi^> |\pi^\pm> - |\pi^\pm> |\pi^>) \quad \text{and}$$

$$\rho^0 \rightarrow \frac{1}{\sqrt{2}} (|\pi^+> |\pi^-> - |\pi^-> |\pi^+>).$$

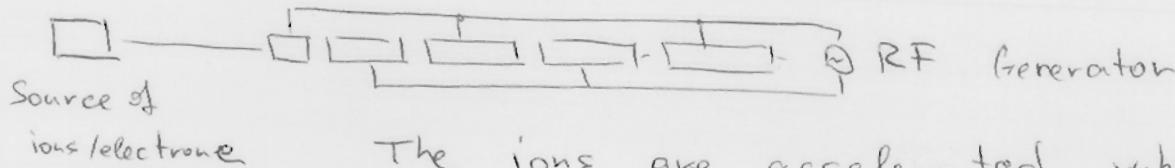
Problem 5.

We should consider a set of conservation laws for each process:

	Energy	Momentum	Electric Charge	Lepton Flavors	Quark flavors
$\pi^0 \rightarrow e^+ e^-$	$135 \rightarrow 0.5 + 0.5$ OK	OK	OK $0 - 0 + 1 - 1$	$0 = 0$	$0 \rightarrow 0$ OK
$e^- + p \rightarrow n + \bar{\nu}_e$	OK (collision)	OK	OK $-1 + 1 = 0$	$l \rightarrow l$	$0 \rightarrow 0$ OK
$p^+ \rightarrow e^+ e^- \bar{e}^+$	OK $106 \rightarrow 1.5$	OK	OK $+1 = +1 - 1 + 1$	muon: $1 \rightarrow 0$ Electron: $0 \rightarrow +1 - 1 + 1$ NO	$0 \rightarrow 0$ OK
$K^0 + n \rightarrow \Lambda + \bar{\pi}^0$	$49.6 + 1$ $\rightarrow 111.6 + 140$ Collision	OK	$0 = 0$ OK	$0 \rightarrow 0$ OK	$d\bar{s} + u\bar{d} \rightarrow u\bar{s} + u\bar{u}$ $S = +1 \quad S = -1$ $\Delta S = 2 \quad \text{NO}$
$\Xi^0 \rightarrow \Lambda + \bar{\pi}^0$	$1315 \rightarrow 111.6 + 140$ OK	OK	$0 = 0$ OK	$0 \rightarrow 0$ OK	$u\bar{s} \rightarrow u\bar{s} + u\bar{u}/d\bar{d}$ $\Delta S = 1 \Rightarrow \underline{\text{Electroweak}}$

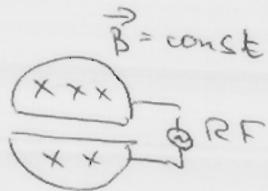
Problem 6.

- (a) Electrons, emitted from a hot cathode are accelerated towards anode. Then electrons pass out of the electron gun through a hole in the anode
- (b) A Van de Graaff accelerator uses electrostatic potential to accelerate ions. The potential is generated with a moving dielectric belt, spherical capacitors, and ~~two~~^A
- (c) Alvarez linear accelerator



The ions are accelerated when they are passing through oscillating electric field. The frequency is constant but separation of the cavities is adjusted to be in phase with the beam.

(d) Cyclotron.



$B = \text{const}$ The beam is moving in a spiral orbit in constant magnetic field. The electric field accelerates the beam each time it traverses the gap between the cavities.

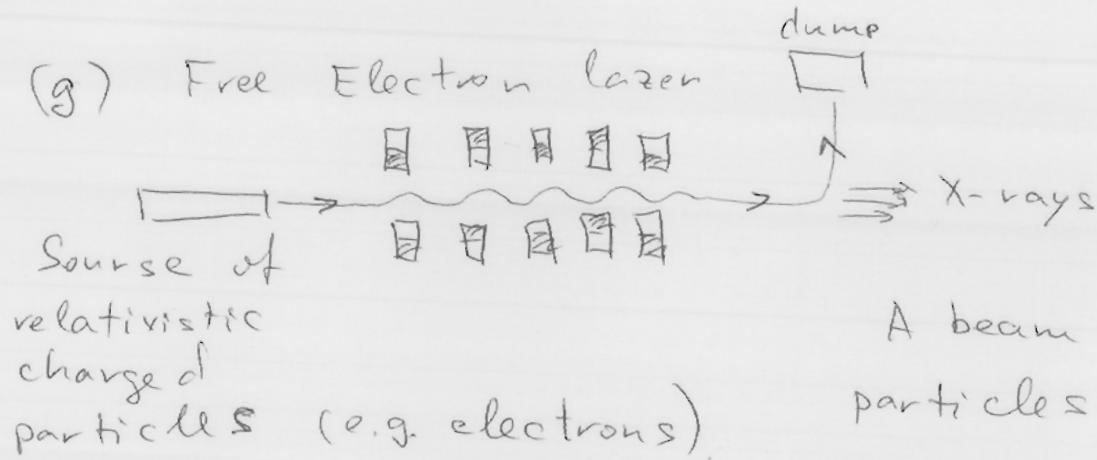
(e) Betatron.

The beam is moving in a fixed circular orbit in increasing magnetic field. The acceleration of the beam is performed with induced electric field.

(f) Synchrotron.

The beam is moving in a fixed circular orbit in magnetic field. Acceleration is performed with a set of RF cavities which operate in phase with the beam.

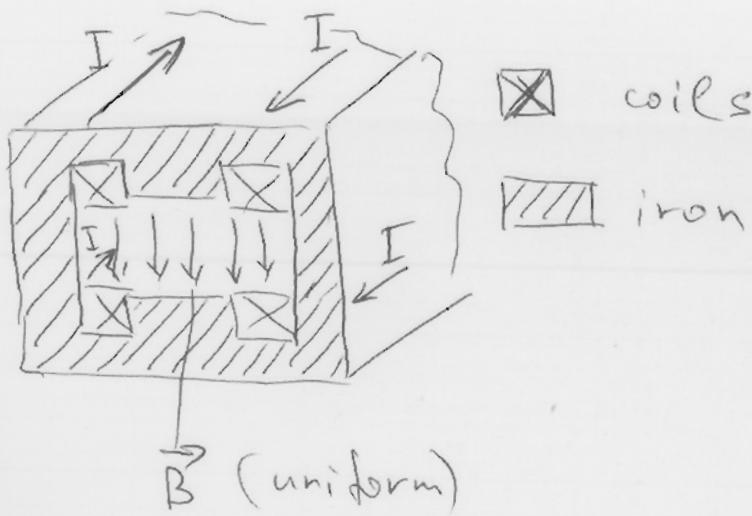
(g) Free Electron laser



A beam of charged particles emits coherent electromagnetic radiation when moving in a sinusoidal field.

Problem 7.

(a) A dipole magnet



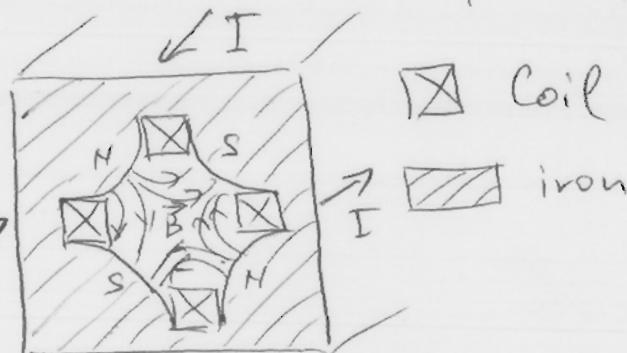
(b) A cos-theta winding dipole magnet.



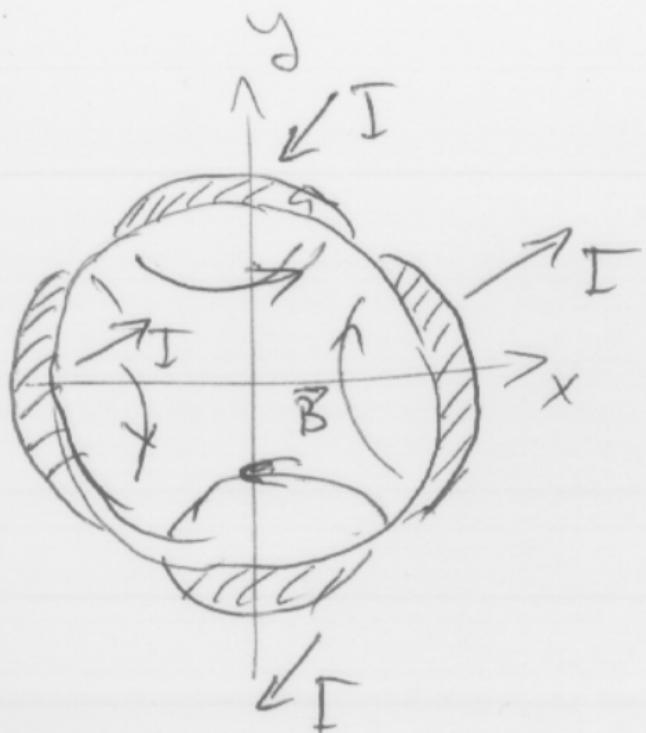
The current density is proportional to $\cos\theta$.

The magnetic field is uniform and constant inside the cylinder.

(c) Old-fashioned quadrupole magnet.



(d) A superconducting quadrupole magnet.

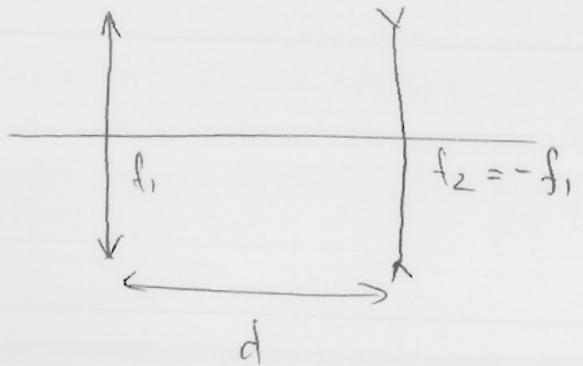


The current density is proportional to $(3\cos^2\theta - 1)$

Problem 8

A dipole magnet focuses in one plane and defocuses in the other plane.

Let's consider a system of 2 lenses:



$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$
$$\Rightarrow \frac{1}{f} = \frac{d}{f_1^2}$$

You can see that a focusing and defocusing lenses work as a focusing lens overall.

Therefore a set of dipole magnets can focus a particle beam in the both planes.

Problem 9.

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ \frac{\delta p}{p} \end{pmatrix} \quad x' = \frac{\partial x}{\partial z} \quad y' = \frac{\partial y}{\partial z}$$

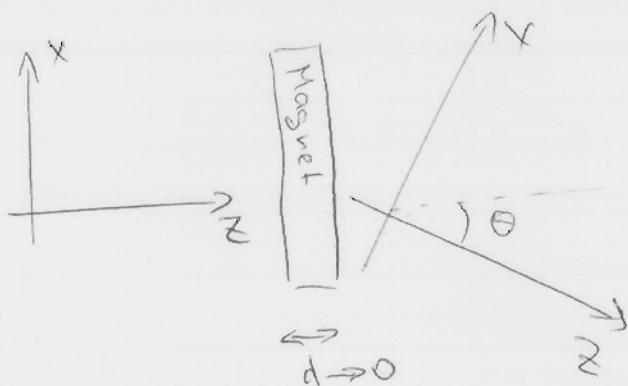
(a) drift space. (Lengt = L)

$$x' \rightarrow x' , \quad y' \rightarrow y' , \quad x \rightarrow x + L \cdot x' , \quad y \rightarrow y + L \cdot y'$$

$$\frac{\delta p}{p} \Rightarrow \frac{\delta p}{p}$$

$$\begin{pmatrix} 1 & 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

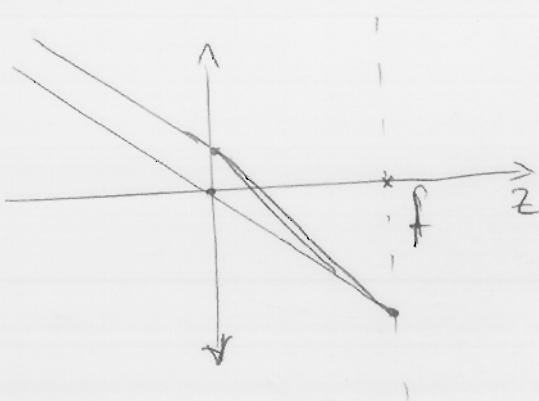
(b) Bend magnet



$$\begin{aligned} \frac{\delta p}{p} &\rightarrow \frac{\delta p}{p} \\ x &\rightarrow x \\ y &\rightarrow y \\ y' &\rightarrow y' \\ x' &\rightarrow x' - \theta \frac{\delta p}{p} \end{aligned}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\theta \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) a focusing lens of focal length f



$$x \rightarrow x$$

$$y \rightarrow y$$

$$x' \rightarrow x' - \frac{x}{f}$$

$$y' \rightarrow y' - \frac{y}{f}$$

(The sign is opposite
for a dipole magnet)

$$\frac{\delta P}{P} \rightarrow \frac{\delta P}{P}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -f^{-1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -f^{-1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$