Midterm Review Problems

Physics 235 Fall Quarter 2004 H.J.Frisch HEP320 (702-7479)
Nov. 1, 2004 Office Hours: by appointment

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Reading: In addition to reviewing the assigned reading, I recommend looking for additional problems to solve in the recommended texts. Also, the questions and problems at the back of Chapters 8,9, and 10 in Eisberg and Resnick provide an excellent review of the atomic and molecular details. Past quizzes and problem sets are fair game.

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Problems: These are some problems that will help you review for the final. These do *not* have to be handed in.

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1. **Spin dynamics.** Consider an electron in a uniform magnetic field B in the $\hat{z}$ direction. If the initial state of the electron (quantized on the z axis) is:

$$|\psi(0)>= \begin{pmatrix} cos(\theta/2)e^{-i\phi/2)} \\ sin(\theta/2)e^{-i\phi/2)} \end{pmatrix}$$

   Find the probability of measuring the electron with spin in the $\hat{x}$ direction as a function of time.

2. **Addition of Angular Momenta.** Find the eigenvalues and eigenfunctions of J^2 and J_z in the case that J is the sum of $L = 1$ and $S = 1/2$. Please start with the highest J_z and use the lowering operator.

3. **Addition of Angular Momenta: the 3-3 Resonance.** Consider the three π-nucleon scattering processes $\pi^+ p \to \pi^+ p$, $\pi^- p \to \pi^- p$, $\pi^- p \to \pi^0 n$, where 'p' is a proton and 'n' a neutron. The scattering cross section is proportional to the square of the matrix element of some isospin operator H connecting initial and final states:

$$\sigma \equiv < \psi_f|H|\psi_i > \quad (1)$$

   Because isospin is conserved in the strong interaction, the matrix elements between I=1/2 and I=3/2 states are zero, leaving two non-zero matrix elements $M_{11} =< \psi_f(1/2)|H|\psi_i(1/2)$ and $M_{33} =< \psi_f(3/2)|H|\psi_i(3/2)$. Assume that $M_{33} > M_{11}$; calculate the ratio of the cross-sections for the three processes.

4. **Fine Structure in Hydrogen.** Draw and carefully label the energy levels of the hydrogen atom for n=1,2, and 3 including the (magnified) effect of fine-structure. Give the shifts in energy for each state due to the fine-structure effects.

5. **Hyperfine Structure in Hydrogen.** Starting from the Hamiltonian for the spin-spin interaction between the spins of the proton and the electron, $H^1 = A(S_1 \cdot S_2)$, find the resulting eigenstates of the $H^0 + H^1$, and the energy levels. Does the triplet or the singlet lie higher in energy? Making the (crude) approximation that $H^1 = (\mu_3 \cdot \mu_p)/a_0^3$, estimated the wave-length of the radiation for the triplet-singlet transition.

(OVER)

6. **Positronium.** Calculate the energy levels of positronium ignoring spin. Draw a term diagram for n = 1, 2, and 3.

7. **Fine Structure in Positronium.** Calculate the effect of the electron and positron spins on the above energy levels of positronium.

8. **Hyperfine Structure in Positronium.** Discuss.

9. **Zeeman Effect.** Very sparse clouds of hydrogen atoms exist in the galaxy, in which there is a weak magnetic field. Give the energies of each of the n=1 and n=2 states in this case, in which the degeneracies are broken by the spin and orbital angular momentum of the electron (ignore the hyperfine splitting.).

10. **Stark Effect.** Consider a Hydrogen atom in its ground state in a weak electric field $E^0\hat{z}$. What is the shift in the ground state energy due to the field? (This is a good 'quickie').

11. **Even Starker Effect.** Now consider the $n = 2$ states of the same H atom, which would be degenerate in the case of zero electric field. Find the new eigenstates and derive the energy shifts due to the electric field.

12. **Paschen-Back Effect.** Find the shifts in the energy levels up through n=3 for a hydrogen atom in a 10-Tesla magnetic field.

13. **Miscellaneous and Review.**

    (a) Describe the hyperfine structure of the ground state of Helium.
    (b) Write the electronic structure for the ground state of Carbon (Z =12).
    (c) Find the expectation values of $\sigma_1 \cdot \sigma_2$ in positronium.
    (d) For a spin-1/2 system show that $|\frac{1}{2}\,\frac{1}{2}><\frac{1}{2}\,\frac{1}{2}| + |\frac{1}{2}\ -\frac{1}{2}><\frac{1}{2}\ -\frac{1}{2}| = 1$
    (e) Write down the generator of translations in the x-direction.
    (f) Find and draw a picture of the spacing of the rotational energy levels of a diatomic molecule.