P234 HOMEWORK 6 SOLUTIONS

Q1. The energy width of the top quark is $\Delta E = 2GeV$. So using Hisenberg's uncertainty principle we have $\Delta t = \hbar/\Delta E = 3.3\times 10^{-25}s$. Assuming the momentum of the particle is the same as its energy we have $\Delta p \sim 90GeV/c$ so the length of travel using the position-momentum version of the Hisenberg uncertainty is $\Delta x \sim \hbar/\Delta P = 3.5 \times 10^{-16}m$.

Q2. Using the similar methods as in Q1 we have

$$\begin{array}{rll} \text{particle} & \text{lifetime} & \text{Energy width} \\ \pi^0 & 84 \times 10^{-18}\text{s} & 8\text{eV} \\ K_L^0 & 52 \times 10^{-9}\text{s} & 1.2 \times 10^{-8}\text{eV} \\ \mu & 2.2 \times 10^{-6}\text{s} & 3 \times 10^{-9}\text{eV} \\ \tau & 290 \times 10^{-15}\text{s} & 2.3 \times 10^{-3}\text{eV} \end{array} \quad (1)\text{–}(6)$$

Q3. The usual wavelengths given off in atomic processes are of the order of $\lambda_a \sim$ 500nm, while the radiation from nuclear processes are x-rays and so have wavelengths $\lambda_n \sim 1$ nm. So the momentum of the photon is $p = h/\lambda$ and the Hisenberg uncertainty principle gives the uncertainty in position $\Delta x \sim \hbar/p = \lambda/2\pi$. So the relative uncertainties of nuclear and atomic processes are $\Delta x_n/\Delta x_a = \lambda_n/\lambda_a \sim 10^-3$. While is the approximate ratio of nuclear and atomic radii.

Q4. We have two identical bosons in states $|\phi\rangle$ and $|\psi\rangle$ so the state is

$$|state'\rangle = |\psi\rangle|\phi\rangle + |\phi\rangle|\psi\rangle, \tag{7}$$

but as $\langle\phi|\psi\rangle \neq 0$. So the normalize state is

$$|state\rangle = \frac{1}{\sqrt{2+2|\langle\phi|\psi\rangle|^2}} \left(|\psi\rangle|\phi\rangle + |\phi\rangle|\psi\rangle\right). \tag{8}$$

Q5. Since we have three particles and two of the states are the same. So there are three possible states are $\{|334\rangle, |343\rangle, |433\rangle\}$. So the only possible completely symmetric state is

$$|state\rangle = \frac{1}{\sqrt{3}} \left(|334\rangle + |343\rangle + |433\rangle\right). \tag{9}$$

Q6. a) We have the general formula for two particles in an infinite square well is $E_{n1,n1} = (n_1^2 + n_2^2)\hbar^2\pi^2/(2mL^2)$. So if the energy of the two particle state is found to be $E = \hbar^2\pi^2/(mL^2)$ we have $n_1^2 + n_2^2 = 2$. Since for the infinite square well the energy quantum number ≥ 1, the only integer solutions to the above equation is $n_1 = 1$ and $n_2 = 1$. Therefore only a bosonic $|11\rangle$ is possible as be cannot form an antisymmetric state.

b) In this case we have equation that $n_1^2+n_2^2=5$. There are two integer solutions to this equation $n_1=1, n_2=2$ and $n_1=2, n_2=1$. So the bosonic and fermionic states are then

$$|boson\rangle = \frac{1}{\sqrt{2}}\left(|12\rangle + |21\rangle\right) \tag{10}$$

$$|fermion\rangle = \frac{1}{\sqrt{2}}\left(|12\rangle - |21\rangle\right) \tag{11}$$