Physics 141A
Problem Set 1
Due Monday, Oct. 8 (hand in in class).

October 2, 2018
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General: Organization, Web page, Labs, Syllabus: See the P141A Information Sheet, Syllabus, and Textbooks posted in 'Files' on Canvas

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Time Management and Study Groups You need to work with your study group. The problem sets will go faster if you discuss the problems, with friends/colleagues, and you will have a deeper understanding. However, the work you hand in **has to be your own**.[1]

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Problems with answers, and recycled problems There are a limited number of easily-solved mechanics problems, and so one can find answers to most by searching on the web. We trust you to instead work them yourself; ask the TA's, me, or fellow students for help if you need it. Using books is not only fair but recommended; however, you should write out the solution with the book closed to make sure you *know* how.

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Formulae: $\beta = v/c$; $\quad \gamma^2 = 1/(1-\beta^2)$; $\quad \beta^2 = (\gamma^2 - 1)/\gamma^2$

The invariant length of the 4-vector x_0, x_1, x_2, x_3: $|x^\mu| = \sqrt{x_0^2 - x_1^2 - x_2^2 - x_3^2}$

Lorentz Transformation for a 'Boost' of Frame F along the x direction relative to frame F':

$$t' = \gamma t + \beta\gamma x \tag{1}$$

$$x' = \beta\gamma t + \gamma x \tag{2}$$

$$y' = y \tag{3}$$

$$z' = z \tag{4}$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \tag{5}$$

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Reading: (quiz material- please read thoroughly- i.e. taking notes):
Taylor, Error Analysis Chapter 3, Principal Definitions and Equations (pp 77-79). (If the propagation of uncertainties is unfamiliar, you should read the entire chapter).
Strunk and White; Chapter I.

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Problems (10 points each). Please form a study group to work on these. Please do *not* plug in any numerical values until the end (!). Solutions will be provided, but not all will be graded.

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Problem 1. Practice with 3-vectors Consider the two vectors $\vec{A}= (3,1,-2)$ and $\vec{B}= (-2,-2,3)$ respectively:

1. Calculate the length of $\vec{A}$; (don't bother explicitly taking the square-root- it's quicker to leave the length-squared under the sqrt sign.).

[1] If you have any questions on where the line is I recommend getting and reading Charles Lipson's book, *Doing Honest Work in College*, UC Press. I also once required two straying students to read Egil Krogh's book *Integrity*.

2. Calculate the length of $\vec{A}+\vec{B}$;
3. Calculate the length of $\vec{A}-\vec{B}$;
4. Draw a diagram of the reference frame showing the x, y, and z axes and the position vectors $\vec{A}$ and $\vec{B}$.
5. On your diagram show $\vec{A}-\vec{B}$ and $\vec{A}+\vec{B}$;
6. Calculate $\vec{A}\cdot\vec{B}$
7. Calculate the angle between $\vec{A}$ and $\vec{B}$.
8. Calculate the projection of $\vec{A}$ on $\vec{B}$
9. Calculate $\vec{A}\times\vec{B}$
10. Find $(\vec{A}\times\vec{B})\cdot\vec{A}$
11. Find $(\vec{A}\times\vec{B})\times(\vec{A}\times\vec{B})$.

Problem 2. Indices Define "Index" in the context of vectors and matrices, and write down examples with 0, 1, 2, 3, and 4 indices, respectively (not trivial- discuss with your group).

Problem 3. Fundamentals of SR

1. Consider the 4-vectors $x^\mu = (t, x, y, z) = (-13, 0, 12, -5)$, $(0, 3, 6, -5)$, and $(6, 0, -3, -2)$ where time is measured in nsec and space coordinates in feet. What is the invariant length-squared of each?

2. Consider two events at space-time points $A^\mu = (15, 4, -16, 7)$ and $B^\mu = (2, 4, -4, 2)$ respectively, where time is measured in nsec and space coordinates in feet. What is the invariant distance-squared in space-time between them, $|B^\mu - A^\mu|$? What is the distance in space between them? In time?

3. Suppose event A happened at time $t_A = 16$ nsec rather than 15. What is the distance in space between A and B? In time?

4. Suppose event A happened at time $t_A = 14$ nsec rather than 15. What is the distance in space between A and B? In time?

5. In each of the above 3 examples can event A cause event B?

Problem 4. Cosmic Rays

Consider a muon (a heavy 'cousin' of the electron, identical in the form of its interactions with matter except effects due to its being 200 times heavier) created in the atmosphere by a cosmic ray coming from far away. Assume that in its own rest frame, this individual muon has a lifetime of $\tau = 2200$ nanoseconds (e.g. the distribution in how long muons live goes as $e^{-\frac{t}{\tau}}$, after which it decays to a muon neutrino and an electron/anti-neutrino pair). This muon is traveling with velocity $\beta = v/c = 0.9999995$ ($\gamma = 1000$) with respect to the Earth.

1. Draw a clear neat (i.e. not too small) diagram of the process in the muon rest frame and another diagram in your own frame (the frame of the Earth). Be sure to label the respective origins and axes.

2. Write down the 4-vector for the decay point in the coordinate frame of the muon.

3. Starting with the value of β, **calculate** the Lorentz factor γ for the transformation from the muon frame to the Earth frame.

4. Lorentz transform the 4-vector representing the decay point in the muon frame to get the 4-vector for the decay point in the Earth's frame.

5. How long is the lifetime as measured in the Earth's frame?

6. How far did the muon travel from where it was created to where it decayed in the Earth's frame?

7. Calculate the proper time (the invariant length of the 4-vector) from the coordinates of the decay event in both the muon and earth's frame.

Problem 5. Time Dilation Consider the first Einstein Gedanken experiment we discussed in lecture. A simple 'clock' is constructed on a very fast Metra Electric train by mounting an LED (light-emitting diode) and a photo-diode together inside the train on one wall, and a mirror on the wall across the train and directly opposite. The LED and photodiode are pointed at the mirror, and are electrically connected so that a reflected short LED pulse from the mirror triggers the photo-diode to make the LED flash. The result is that the LED flashes repeatedly at a fixed interval that corresponds to twice the light transit time across the width of the train. The train is an Express, moving at $\beta = 0.995, (\gamma = 10)$ relative to the 55-56-57th street station.

1. Set up the problem and define the relevant events in the frame of the train. (Be sure to draw a well-labeled clear diagram).
2. Transform the coordinates of each event into the Hyde Park (HP) frame.
3. Draw a carefully-labeled diagram of the geometry of the light path in the HP frame.
4. Find the time between flashes as seen in the HP frame.
5. Find the distance between flashes as seen in the HP frame.
6. Calculate the invariant length in space-time between the flashes in the HP frame.

Problem 6. Thoughts on Simultaneity Casals is on a train moving at speed corresponding to a Lorentz factor of $\gamma = 1000$ down a set of tracks past a platform on which Primrose is standing. Casals is in the middle of the train, i.e. equidistant-distant from both ends. Just as Casals is opposite Primrose (they can touch hands, e.g. take them to be so close as to be at the same point) both of them see two flashes of light that were produced by a light at each end of the train. Casals measures the length of the train to be L. Ignore the width of the train (i.e. the length is much longer than the width).

1. Draw a picture corresponding to the problem, and label the frames and axes.

2. Taking the origins of your two coordinate systems and clocks to be the point where Primrose and Casals are when they see the flash, write down the 4-vector corresponding to the position of each light when it flashed, in Casal's reference frame.

3. Use the Lorentz transformation to find the 4-vectors of each light when it flashed in Primrose's frame.

4. Primrose can calculate the length of the train from the following reasoning: the spatial separation of the two flashes is the distance the back of the train moved while the light was propagating plus the length of the train. In symbols,

$$\Delta x' = \beta \Delta t' + L' \qquad (6)$$

Find the length of the train as measured by Primrose, L', in terms of L and γ. (Note: you may need the identity $1/(\gamma^2) = (1 - \beta^2)$.)

5. Casals deduces that the two lights flashed simultaneously. In contrast, Primrose claims they had to have flashed at different times for him to have seen the flashes simultaneously. What is the time interval between the two lights flashing in Primrose's frame?" (perils of translating into English- another way to have asked is "Transform the two light-flashing events into Primrose's frame- what is the time difference?")