Physics 141
Quiz 5
Tuesday, Nov. 25, 2008

Name:

2 Problems (turn page over) (Recommended time $\leq$ 20 minutes)

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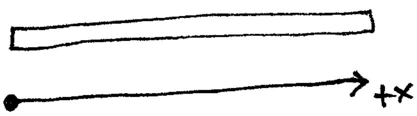
Problem 1: 10 minutes/10 points Calculate the moment of inertia of a rod of length L and mass M about one end.

Assume constant mass density $\lambda = \frac{M}{L} = \frac{dM}{dL}$.



Moment of inertia is $I = \int r^2 dm = \int_0^L x^2 (\lambda dx)$

$$I = \lambda \left[\frac{1}{3} x^3 \right|_0^L = \left(\frac{M}{L}\right)\left(\frac{1}{3}L^3\right)$$

$$\boxed{I = \frac{1}{3} M L^2}$$

Problem 2: 10 minutes/10 points

a) A can of bean soup (solid inside- no internal degrees-of-freedom) rolls down a ramp from a height h. The can is cylindrical, with mass M and radius R. How fast is it going when it gets to the bottom of the ramp? (ignore the moment of inertia of the metal of the can- treat the can as a solid uniform body, as the soup weighs much more than the can. Also assume $h >> R$)

Initial Energy: $Mgh = E_i$ (Assume initially at rest, so only potential energy)

Final Energy (translation + rotation): $\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = E_f$

We have $I = \frac{1}{2}MR^2$. As it rolls, $\omega = v/R$. Then via energy conservation,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(v^2/R^2\right)$$

$$= \frac{3}{4}Mv^2 \Rightarrow v^2 = \frac{4}{3}gh \Rightarrow \boxed{v = 2\sqrt{gh/3}}$$

b) Now suppose the can is boullion (e.g. chicken stock- basically salty water- low viscosity), so that the contents do not gain any angular motion as the can rolls. How fast is the can of bouillion going when it gets to the bottom? (again, ignore the moment of inertia of the can).

The final energy is now only $E_f = \frac{1}{2}Mv^2$

so $Mgh = \frac{1}{2}Mv^2 \Rightarrow \boxed{v = \sqrt{2gh}}$