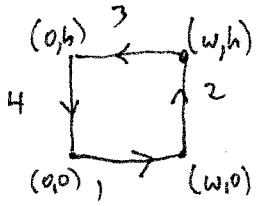
Physics 141
Quiz 3
Tuesday, Nov. 4, 2008 (Election Day!!)

Name: Solution

Problem 1 of 1 (Recommended time $\leq$ 15 minutes)

Consider a 2-dimensional vector field that describes a force $\vec{F}$ acting on a small object of mass m, where $\vec{F} = -y^2\hat{x} + x^2\hat{y}$.

a) Explicitly calculate (i.e. do the path integral) how much work is done pushing the mass counter-clockwise around a rectangular path for which the corners are (x, y) = (0,0), (w,0), (w,h), and (0,h)? (Assume it's done slowly, so ignore the kinetic energy given to the mass).



$$W = \oint \vec{F}\cdot d\vec{\ell} = \int_0^w 0\,dx + \int_0^h w^2\,dy + \int_w^0 -h^2\,dx + \int_h^0 0\,dy$$

$$W = w^2h + h^2w.$$

b) Now let us suppose that in addition to the force field F, the block is subjected to a frictional force as it slides on a surface around that path. If the coefficient of friction is μ, how much work due to friction is done pushing the block around the above path?

Now add $\vec{f} = -\mu mg\,\hat{\ell}$

$$\oint (\vec{F}+\vec{f})\cdot d\vec{\ell} = (w^2h + h^2w) + \oint -\mu mg\,\hat{\ell}\cdot d\ell$$

So the bit from friction is

$$W_f = \left[\int_0^w d\ell + \int_0^h d\ell + \int_0^w d\ell + \int_0^h d\ell\right](-\mu mg) = -(2w+2h)\mu mg$$