

Physics 141
Solutions for Quiz 2
Tuesday, October 28, 2008

Problem 1

A block of mass m rests on another block, of mass M , which rests on a frictionless table. The coefficient of friction between the two blocks is μ . Solve for the maximum horizontal force that can be applied to the upper block without the blocks slipping relative to each other.

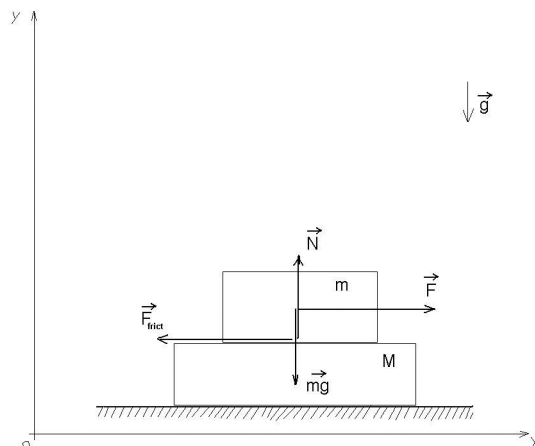


Figure 1: A system of the two blocks. The coefficient of friction between the two blocks is μ .

The horizontal force \vec{F} is applied to the upper block. The upper block does not slide when the both blocks move with the same acceleration, i.e. $a_M = a_m$. Vertical projections of forces acting on the second block are compensated by table's reaction force.

The second Newton's law for the upper block:

$$m\vec{g} + \vec{N} + \vec{F} + \vec{F}_{frict} = m\vec{a}_m \quad (1)$$

The second Newton's law for horizontal projections of forces acting on the lower block:

$$\vec{F}_{frict} = M\vec{a}_M \quad (2)$$

The friction force is $F_{frict} \leq N \cdot \mu$. The x- and y- projections of the forces acting on the upper block are $N = mg$ and $ma_m = F - F_{frict}$. We obtain a set of two equations:

$$F_{frict} \leq mg\mu \quad (3)$$

and

$$\frac{m}{M}F_{frict} = F - F_{frict}. \quad (4)$$

The force \vec{F} has the maximum value of

$$F^{max} = mg\mu \cdot \left(1 + \frac{m}{M}\right) \quad (5)$$

when the blocks are about to start slipping relative to each other.

Problem 2

Consider a block of mass m sitting on the (frictionless) top surface of a wedge of mass M , which in turn is free to move (again without friction) on a table.

0.) Draw a good (neat and well-labelled) picture.

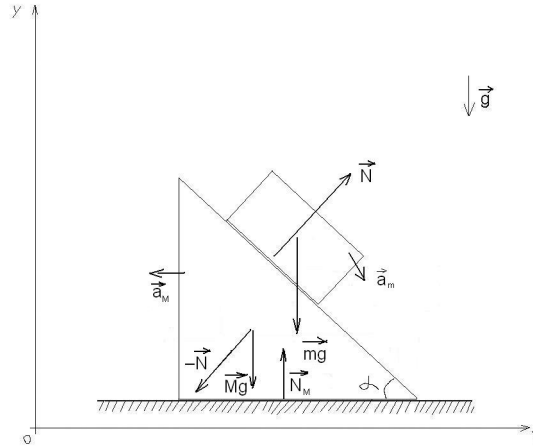


Figure 2: A system of a block sliding on a wedge. There is no friction.

1.) Set up the equations of motion for the 2 objects.

$$M\vec{a}_M = M\vec{g} + \vec{N}_M - \vec{N} \quad (6)$$

$$m\vec{a}_m = m\vec{g} + \vec{N} \quad (7)$$

2.) Set up the equation(s) of constraint.

$$(\vec{a}_m - \vec{a}_M) \cdot \vec{N} = 0 \quad (8)$$

$$\vec{a}_M \cdot \vec{N}_M = 0 \quad (9)$$

3.) Solve for the motion of the block and wedge assuming they are rest at $t = 0$ (do not worry about what happens when the block finally hits the table).

$$\vec{a}_M = \left(-g \frac{\sin(2\alpha)}{2} \frac{m}{M + m \sin^2(\alpha)}, 0\right) \quad (10)$$

$$\vec{a}_m = \left(g \frac{\sin(2\alpha)}{2} \frac{M}{M + m \sin^2(\alpha)}, -g \sin^2(\alpha) \frac{M + m}{M + m \sin^2(\alpha)} \right) \quad (11)$$

$$\vec{r}_M = \left(x_M^0 - g \frac{\sin(2\alpha)}{2} \frac{m}{M + m \sin^2(\alpha)} \frac{t^2}, y_M^0 \right) \quad (12)$$

$$\vec{r}_m = \left(x_m^0 g \frac{\sin(2\alpha)}{2} \frac{M}{M + m \sin^2(\alpha)} \frac{t^2}, y_m^0 - g \sin^2(\alpha) \frac{M + m}{M + m \sin^2(\alpha)} \frac{t^2} \right) \quad (13)$$

4.) Show that your solution makes sense in the limit $M \rightarrow \infty$.

$$\vec{a}_M \rightarrow (0, 0) \quad (14)$$

$$\vec{a}_m \rightarrow \left(g \frac{\sin(2\alpha)}{2}, -g \sin^2(\alpha) \right) \quad (15)$$

$$\vec{r}_M = (x_M^0, y_M^0) \quad (16)$$

$$\vec{r}_m = \left(x_m^0 g \frac{\sin(2\alpha)}{2} \frac{t^2}{2}, y_m^0 - g \sin^2(\alpha) \frac{t^2}{2} \right) \quad (17)$$