Physics 141

Problem Set 2

Due Monday, Oct. 13 (hand in in class).

Monday, Oct. 6, 2008
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General: Organization, Web page, Labs, Syllabus: See the (yellow) P141 'Poop Sheet'.

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First Midterm (!): The first of 2 Midterms will be Friday, Oct. 17. The material will be on what we have covered in class on Special Relativity, and your reading in Taylor on measurements and experimental uncertainties (including Significant Figures).

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Reading:Taylor, 'An Introduction to Error Analysis', Chapters 3 and 4
We will be working from class notes for SR, as I don't know of any text that treats the material simply but deeply. After that there will be assigned reading from Klepner, and later, Schey. However, if you wish to explore, Feynman has a chapter on SR, and there are lots of books in Crerar to browse. If you find one that uses the West Coast metric, $c = 1$, and Lorentz Transformations between frames and invariants to solve problems simply, please tell me.

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Study Groups You are strongly encouraged to work in groups. However, the work you hand in has to be your own- you must **NEVER** copy anybody else's work!

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Formulae: $\beta = v/c$; $\gamma^2 = 1/(1-\beta^2)$

$$t' = \gamma t + \beta\gamma x \qquad (1)$$

$$x' = \beta\gamma t + \gamma x \qquad (2)$$

$$y' = y \qquad (3)$$

$$z' = z \qquad (4)$$

$$|x^\mu|^2 = t^2 - x^2 - y^2 - z^2 \qquad (5)$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \qquad (6)$$

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Problem Solving: Please always work in symbols and only plug numbers in at the end where and if required. It usually pays to start by drawing a careful picture, if appropriate.

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Problem 1: Using the Lorentz Transformation- The Barn and the Pole 'Paradox':
Consider the following folk fable (Einstein): A runner carrying a 20-foot pole held horizontally runs rapidly toward a barn that is measured to be 30 feet long along the direction of the runner's motion in the rest frame of the barn. The barn has a door on each end that opens just as the front of the pole reaches it and shuts as the back of the pole arrives. Assume that she is *really* fast- $\gamma = 2$, $\beta = \sqrt{3}/2$. The 'paradox' is that an observer inside the barn sees the pole as Lorentz-contracted so that it was inside the barn with both doors closed. However the runner sees the barn as shorter than the pole, and so there's no way both doors could be closed at the same time with the pole inside. Who is right?

1. Draw a picture clearly showing the two coordinate frames, with origins and labeled axes, and the lengths of barn and pole defined as symbols (I suggest drawing the picture in the frame of the barn (primed frame in our convention of Primrose being the bystanding observer, and taking the origin when the front of the pole arrives at the first door.)

2. Write down the 4-vectors for the following four events in the runner's frame:

   (a) Front end of pole reaches barn front door;
   (b) Front end of pole reaches barn back door;
   (c) Back end of pole reaches barn front door;
   (d) Back end of pole reaches barn back door;

3. Write down the 4-vectors for the same four events in the frame of the barn (use the Lorentz Transformation, natch).
4. Using the numerical values for the lengths of the pole and barn, order the four events in time in the runner's frame.
5. Using the numerical values for the lengths of the pole and barn, order the four events in time in the barn's frame.
6. In each of the 2 frames make a short summary of the time sequence of the 4 events. Which narrative is 'correct', and is there a way to tell?

Problem 2: Derivation of Addition of Velocities
Consider a frame F moving in the x direction at velocity β_1 relative to a frame F', which is itself moving at velocity β_2 relative to a frame F''. Assume the velocities are in the same direction. Calculate the product of the two Lorentz transformations to derive the formula for the addition of velocities.

Problem 3: Addition of Velocities (Easy)

1. A rocket ship moving at β= 0.9 away from Earth launches a small ship in the same direction with β= 0.6 relative to the mother ship. What is the velocity measured from earth?
2. A rocket ship approaching Earth at β= 0.9 turns on a headlight pointed right at earth. Use the law of addition of velocities to calculate the velocity of the light from the headlight as seen on Earth.
3. A bottle is thrown forward from a moving car. If the car is moving at 90 feet/second and the bottle is thrown at 50 feet/second, how fast is is the bottle moving with respect to the ground?

Problem 4: Conservation of Energy and Momentum

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Please don't freak when reading this- it's much easier than poles and barns and flashing lights on trains.

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A Higgs boson is created at rest in a proton-antiproton collision at Fermilab. It decays into a b-quark and an anti-bquark. The mass of a Higgs boson is thought (by some) to be 115 GeV; the mass of a b-quark (which is equal to the mass of an anti-b-quark) is 4 GeV. The energies and momenta of the b-quark and the anti-bquark are equal (they have to be by momentum conservation). Find the momentum of the b-quark after the decay by making the following steps:

1. Write down the 4-vector for the Higgs boson in its rest frame before the decay (use M for the mass of the Higgs);
2. Write the 4-vector momenta for the b-quark and anti-quark in the Higgs rest frame after the decay (Use the symbols E_b and p_b, for example, for the energy and (magnitude of) momentum along the x axis for both the b and anti-b quarks.

3. Use conservation of energy to find the energy of the b-quark in the Higgs rest frame.
4. Use the invariance of the 'length' of the 4-vector of the b-quark to find the magnitude of the momentum $|\vec{p}|$ of the b-quark in the Higgs rest frame.
5. Plug in the numbers to get the momentum in GeV.

Extra Credit-Problem 5: the Twin 'Paradox' Two identical twins flip a coin as to who gets to go on a trip to a distant inter-galactic resort on Virgin Spaceways. The ship accelerates rapidly to $v = 0.9995c$ ($\gamma = 100$), goes for 1 year, and rapidly decelerates to land at the Galaxy Casino. After a brief visit, the process is reversed. Ignoring the time spent accelerating and visiting, the traveling twin is now 2 years older. How much time has elapsed on earth when she returns? Suppose a light flashes every New Year's (midnight Dec 31, local time) on Earth and another flashes also on New Year's (local time) on the spaceship. Draw a space-time diagram (t vs x) of the motion of the ship and the propagation of the light flashes. List the times when the twin on the ship sees flashes from Earth and when the twin on Earth sees flashes from the ship. (you are welcome to code this up if you program- but it's instructive to draw it by hand first.)