Physics 141
Problem Set 1

Due Monday, Oct. 6 (hand in in class).

September 26, 2008
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General: Organization, Web page, Labs, Syllabus: See the P141 'Poop Sheet' handed out on Monday

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Reading: (quiz material- please read thoroughly- i.e. taking notes)
Taylor- Chapters 1 and 2
The problems below are solvable from your class notes.

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Study Groups You are strongly encouraged to work in groups- the problem sets will go faster if you discuss the problems, and you will have a much deeper understanding. However, the work you hand in has to be your own- you must NEVER copy anybody else's work!

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Formulae: $\beta = v/c$; $\gamma^2 = 1/(1-\beta^2)$

$$t' = \gamma t + \beta\gamma x \quad (1)$$
$$x' = \beta\gamma t + \gamma x \quad (2)$$
$$y' = y \quad (3)$$
$$z' = z \quad (4)$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \quad (5)$$

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Problem 1

1. Consider the 4-vector $x^\mu = (t, x, y, z) = (3, -1, 2, -1)$, where time is measured in nsec and space coordinates in feet. What is the invariant length $|x^\mu|$?

2. Consider the two 'events' $A^\mu = (0, 1, 2, 3)$ and $B^\mu = (-3, -1, 2, -3)$, where time is measured in nsec and space coordinates in feet. What is the invariant distance in space-time between them, $|B^\mu - A^\mu|$? What is the distance in space between them? In time?

3. State the Principle of Special Relativity in both the Weak or Strong forms.

4. Define an Inertial Frame (make it a complete description- i.e. state any 'law' you invoke.)

5. Give an example of a Inertial Frame, and the test you would make to prove that it is indeed Inertial.

Problem 2

Consider a muon (a heavy 'cousin' of the electron, identical in its interactions with matter except effects due to its being 200 times heavier) created in the atmosphere above 57th Street by a cosmic ray coming from the other side of our galaxy. In its own rest frame, this individual muon has a lifetime of $\tau = 2200$ nanoseconds (this is the average for muons), after which it decays to other particles. This muon is travelling with velocity $\beta = v/c = 0.99995$ ($\gamma = 100$) with respect to you.

1. Draw a picture of the process in the muon rest frame and another picture in your own frame (the frame of the Earth).

2. Write down the 4-vector for the decay point in the coordinate frame of the muon.

3. Lorentz transform this 4-vector to get the 4-vector for the decay point in your frame.

4. How long is the lifetime as observed by you (i.e. in your reference frame)?

5. How far is the decay point of the muon from where it was created, as observed by you?

6. Calculate the proper time (the invariant length of the 4-vector) from the coordinates of the decay event as seen by you.

Problem 3. Casals is on a of a train moving at speed $\beta = 0.99995$ ($\gamma = 100$) down a set of tracks past a platform on which Primrose is standing. Casals is in the middle of the train, i.e. equi-distant from both ends. Just as Casals is opposite Primrose (they can touch hands, e.g. take them to be so close as to be at the same point) both of them see two flashes of light which were produced by a light at each end of the train. Casals measures the length of the train to be L. Ignore the width of the train (i.e. the length is much longer than the width).

1. Draw a picture corresponding to the problem, and label the frames and axes.

2. Taking the origins of your two coordinate systems and clocks to be the point where Primrose and Casals are when they see the flash, write down the 4-vector corresponding to the position of each light when it flashed, in Casal's reference frame.

3. Use the Lorentz transformation to find the 4-vectors of each light when it flashed in Primrose's frame.

4. Primrose can calculate the length of the train from the following reasoning: the spatial separation of the two flashes is the distance the back of the train moved while the light was propagating plus the length of the train. In symbols,

$$\Delta x' = \beta \Delta t' + L' \tag{6}$$

   Find the length of the train as measured by Primrose, L', in terms of L and γ. (Note: you may need the identity $1/(\gamma^2) = (1 - \beta^2)$.)

5. Casals measures the two light flashes to be simultaneous. What is the time interval between the two measured by Primrose?