CSR Microbunching: Gain Calculation

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Integral Equation for CSR Microbunching

- For bunching parameter $b$ at mod. wavelength $\lambda = \frac{2\pi}{k}$

$$b(k; s) = b_0(k; s) + \int_0^s ds' K(s', s)b(k'; s')$$

where $b_0(k; s)$ is the bunching without CSR

kernel $K(s', s) = ik(s)R_{56}(s' \rightarrow s)\frac{I(s')}{\gamma I_A}Z(k') \times \exp(...\varepsilon, \sigma_\delta...)$

- Given any initial condition (density and/or energy modulation), this determines the final microbunching

- Calculate gain $= b_{\text{final}} / b_{\text{initial}}$, comment on initial conditions
Staged Amplification

- Typical chicane

\[ \Delta L >> \text{dipole length } L_b \]

\[ R_{56}(s' \rightarrow s) \sim \begin{cases} \frac{L_b^3}{\rho^2} & \text{within the same dipole} \\ \frac{\Delta L L_b^2}{\rho^2} & \text{from one dipole to another} \end{cases} \]

- Ignore the induced bunching from energy modulation in the same dipole (Schneidmiller et al.)

- Consider staged amplification from dipole to dipole by setting \( K(s',s) = O(L_b/\Delta L) = 0 \) if \( s-s' < \Delta L \)
Iterative Solution

• Integral equation can be solved by two iterations

\[ b(k; s) = b_0(k; s) + \int_0^s ds' K(s', s)b_0(k'; s') \]

one-stage amplification

\[ + \int_0^s ds' K(s', s)\int_0^{s'} ds'' K(s'', s')b_0(k''; s'') \]

two-stage amplification (no compression and no emittance → Schneidmiller et al.)

\[ = b_0(k; s) + I_f(1 \rightarrow 3) + I_f(2 \rightarrow 3) + I_f^2(1 \rightarrow 2 \rightarrow 3) \]

dominant in low-gain high-gain
Berlin CSR Benchmark: No Compression

No emittance

\( \sigma_\delta = 3 \times 10^{-5} \)

\( \sigma_\delta = 0 \)
Berlin CSR Benchmark: Compression

\[ \varepsilon_n = 0 \, \mu m, \quad \sigma_\delta = 2 \times 10^{-6} \]
\[ \varepsilon_n = 1 \, \mu m, \quad \sigma_\delta = 2 \times 10^{-6} \]
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LCLS Bunch Compressors (BC)

<table>
<thead>
<tr>
<th></th>
<th>DL1</th>
<th>DL2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linac</strong></td>
<td></td>
<td></td>
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<tr>
<td><strong>SC wiggler</strong></td>
<td></td>
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</tr>
<tr>
<td><strong>DL1</strong></td>
<td>BC1</td>
<td>BC2</td>
</tr>
<tr>
<td><strong>Linac</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Linac</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Energy</strong></td>
<td>250 MeV</td>
<td>4.54 GeV</td>
</tr>
<tr>
<td><strong>Compressor current</strong></td>
<td>480 A</td>
<td>4000 A</td>
</tr>
<tr>
<td><strong>Norm. emittance</strong></td>
<td>1 mm</td>
<td>1 mm</td>
</tr>
<tr>
<td><strong>Energy spread</strong></td>
<td>$1.2 \times 10^{-5}$</td>
<td>$3 \times 10^{-6}$ ($3 \times 10^{-5}$)</td>
</tr>
</tbody>
</table>
1) 1–stage

2) 1+2 stage

• At low gain, one-stage amplification dominates

\( \sigma_\delta = 1.2 \times 10^{-5} \),  
\( \varepsilon_n = 1 \ \mu m \)

BC1 Gain of Density Modulation

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BC2 Gain of Density Modulation

1) $\sigma_\delta=3\times10^{-5}$, $\varepsilon_n=1\ \mu m$
2) $\sigma_\delta=3\times10^{-5}$, $\varepsilon_n=0\ \mu m$
3) $\sigma_\delta=3\times10^{-6}$, $\varepsilon_n=1\ \mu m$

Two-stage amplification dominates at high-gain

- Analytical solutions agree with the numerical solutions
  (from Heifets/Stupakov/Krinsky, dashed curves)

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Energy Modulation

• CSR Induced energy modulation in bends

\[ \Delta p(s) \approx -\int_0^s ds \frac{I(s')}{\gamma I_A} Z(s') b(s') \exp(...\varepsilon, \sigma_\delta,...) \]

• CSR microbunching can be generated by initial energy modulation of the bunch due to upstream wakefields (such as CSR)

\[ b^p = -ikR_{56} (\Delta p)_0 (\exp + \text{one - stage amplification}) \]

• Total gain of BC1+BC2 ⊗ gain in BC1 X gain in BC2 + energy modulation from BC1 converted to density gain in BC2
Reasonable agreement with P. Emma’s Simulation
**Dog Legs**

- Total gain of two chicanes can be large (>10) even though the gain in each chicane is small (<3)

- LCLS has more bends than BCs, Dog Legs (DLs) for beam transport (DL1+DL2)

- DLs typically have very small R56 (compared to BCs) ➔ ignore density gain but keep energy modulation (same approximation made within bends of a single chicane)

- DL1 energy modulation can be turned into BC1 density modulation through R56 of BC1, cascading through the whole system and leading to more gain in CSR microbunching
• Assume only initial density modulation before DL1

induced energy modulation at the exit of DL1 (in units of initial bunching)

Turns into BC1 density gain through R56 at end of BC1
Estimated Gain of All LCLS Bends

SC wiggler suppresses a short-wavelength, high-gain peak

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Comments on Initial Condition

• From shot noise

\[ b_{\text{eff}} \sim \frac{1}{\sqrt{N_{\text{coherence length}}}} \sim (10^{-4} \rightarrow 10^{-5}) \]

with a gain less than 100, this should be a small effect

• From sharp current spike

\[ b \approx \frac{N_{\text{spike}}}{N_{\text{total}}} \]

How big?

• Other sources of energy modulation (wakefields…)

• Watch out for numerical noise in simulations!
Conclusion

• CSR microbunching in a bunch compressor is studied using the iterative solution of the integral equation

• Initiated by density and/or energy modulation

• Cascading effects of multiple chicanes

• Gain curves agree with numerical solution and simulation

• Significant gain is found for the LCLS system (DL1+BC1+BC2+DL2), can be suppressed by increasing the uncorrelated energy spread through a SC wiggler